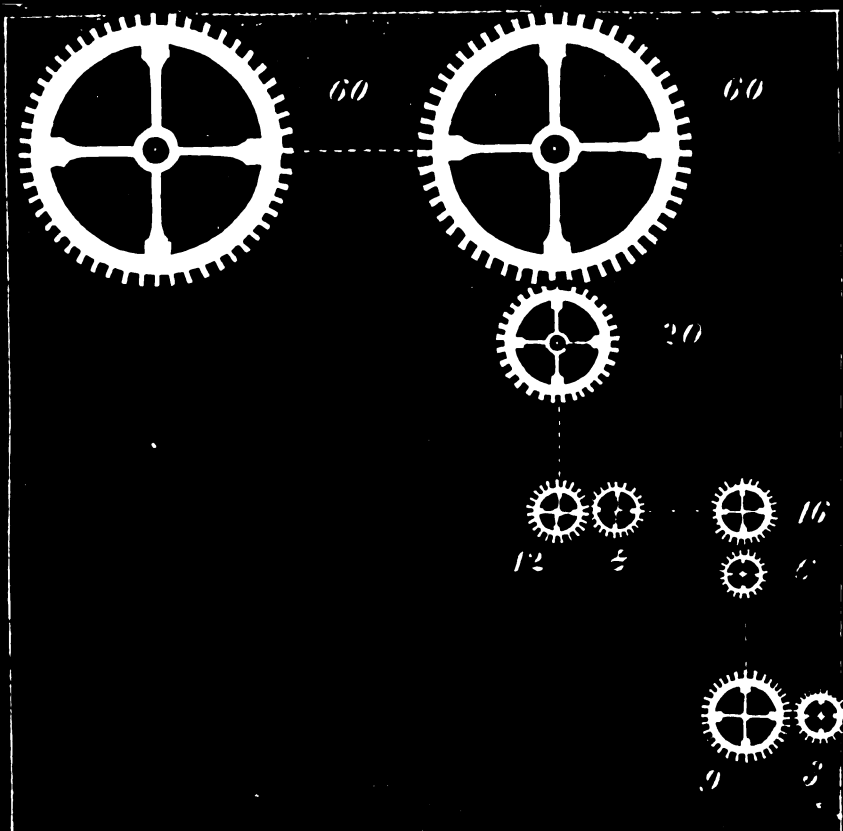

The speedy calculator

Benjamin Naylor

Purchase Mrs. Chas. H. Burleigh
June 4, 1938

M. E. Burleigh

THE

SPEEDY CALCULATOR:

BEING A

SYSTEM OF ARITHMETIC

DESIGNED TO ABRIDGE THE LABOUR OF THE LEARNER, AND TO EXPAND HIS INTELLECTUAL FACULTIES.

BY BENJAMIN NAYLOR,

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PREFACE.

The design of a Systematic Education is, generally, to develope and invigorate all the intellectual faculties, and especially to instruct the pupil in those particular departments of science which the duties and business transactions of his life are supposed to require. Methods of teaching particular branches of school learning, which employ the more general and higher powers of the understanding, answer at once all the ends of discipline and instruction. Besides, nothing so much promotes facility and thoroughness of attainment as those exercises which call into action the reasoning powers, in connection with the efforts of memory. Problems solved, and tasks performed by rules, which hide their reasons from the student, however they may happen to answer mere business ends, have the effect of converting him into an automaton; and, as he loses all the charm of those general laws and relations which a thinking system unfolds at every step of his progress, the study must, in the same proportion, lose both its highest advantages and attractiveness to him. A knowledge of arithmetic is constantly in demand for practical purposes; it enters largely into the speculative sciences, and is indispensable as a preliminary to the study of all the other branches of mathematics. Considerable improvement has been made of late years in the manner of teaching it; but it is still embarrassed with methods and formulas, merely mechanical, and extremely laborious, besides wanting in the important qualities which attract the learner and discipline his understanding.

The system here presented to the public, it is believed, answers all these more general ends of scientific education, and has, besides, the very great merit of abridging the mechanical part of the solutions so much as to dispense with full five sixths of the figures that are ordinarily employed. This is not only a saving of time and labour, but what is of vast importance in arithmetical solutions, it greatly diminishes the liabilities to mistakes, and at the same time facilitates their detection.

This work is submitted to the public with the confidence that it is a very great practical improvement, and that it not only greatly increases the pleasures of the study, but renders the science easily attainable to those who find it extremely difficult to master by the mechanical rules and unreasoning methods so generally in use.

The first part of the work being intended for beginners, the mechanical part of the solutions are performed in the usual manner, but as the pupil advances, the exercises are so arranged as to gradually prepare him to understand and appreciate the improved method.

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ARITHMETICAL CHARACTERS OR SIGNS.

The sign + (*plus*) placed between numbers signifies that they are to be added together, thus, $4+8$ make 12.

The sign — (*minus*) signifies subtraction, thus, $12-4$ leaves 8, that is, 12 less 4 leaves 8.

The sign = signifies equal, as 12 inches = 1 foot; that is, 12 inches are equal to one foot.

The sign × signifies multiplication, as $6\times8=48$; that is, 6 multiplied by 8 is equal to 48.

The sign ÷ signifies division, as $48\div8=6$; that is, 48 divided by 8 is equal to 6.

Division is also represented thus, $\frac{54}{9}=6$; that is, 54 divided by 9 is equal to 6.

Numbers placed in this manner, $\overline{14-6}+9$ show that different operations are to be performed, thus $\overline{14-6}+9$ signifies that 6 is to be taken from 14, and 9 added to the remainder. The line at the top is called a *vinculum*, and connects all the numbers over which it is drawn.

These ( ) are called parentheses, and these [ ] brackets; they are frequently used instead of a vinculum, thus, $(14-6)+9=17$, or $(14-6)\times9=72$.

A figure or number with a small figure placed over it, thus, 8^2 signifies that the figure or number is to be raised to such power as the small figure indicates, for instance, 8^2 signifies the second power of 8, or that 8 is to be multiplied by itself, thus, $8^2=8\times8=64$; and $8^3=8\times8\times8=512=$ the third power of 8; $8^4=$ the fourth power of 8, &c.

The sign $\sqrt{}$ prefixed to a number shows that the square root is to be extracted.

This sign $\sqrt[3]{}$ prefixed to a number shows that the third or cube root is to be extracted.

The square root is also denoted thus $9^{\frac{1}{2}}$ and the cube root thus $(27)^{\frac{1}{3}}$.

EXAMPLES.

How do you read the expression, $12+8+6+4=30$?

Ans. 12 plus 8, plus 6, plus 4 is equal to 30.

How do you read the expression, $14+9+5-12=16$?

Ans. 14 plus 9, plus 5, minus 12, is equal to 16.

How do you read the expression $(16+8-11)\times4=52$?

Ans. 16 plus 8, minus 11, multiplied by 4, is equal to 52.

How do you read the expression, $(9\times4+14)\div5=10$?

Ans. 9 multiplied by 4, plus 14, divided by 5 is equal to 10.

How do read the expression, $(48\div4)\times7)-16=68$?

Ans. 48 divided by 4, the quotient multiplied by 7, and 16 subtracted from the product leaves 68.

How do you read the expression, $\frac{36}{9}\times12+14=62$?

Ans. 36 divided by 9, the quotient multiplied by 12, and 14 added to the product, make 62.

How do you read the expression, $4\times6\times5+8\times3-2\times8=128$?

Ans. 4 times 6 times 5, plus 8 times 3, minus 8 times 2, is equal to 128.

How do you read the expression, $\frac{72}{9}+12\times5-18+7=57$?

Ans. 72 divided by 9, plus 12 times 5, minus 18, plus 7, is equal to 57.

How do you read the expression, $\overline{13+23-12}\times3+8=80$?

Ans. 13 plus 23, minus 12, multiplied by 3, plus 8, is equal to 80.

How do you read the expression, $\frac{7^2\times3+9}{6}=25$?

Ans. The second power of 7 multiplied by 3 plus 9, divided by 6 is equal to 25.

How do you read the expression $\frac{(26-6)\times4}{(7\times5-3)}=5$

Ans. 26 minus 6, multiplied by 4, divided by 7 times 3 minus 5, is equal to 5.

THE SPEEDY CALCULATOR.

THE office of ARITHMETIC is to illustrate the principles and powers of numbers, and the art of computing thereby.

In order to make arithmetical calculations with facility, a thorough knowledge of NOTATION, NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION, is indispensable.

Notation teaches the writing of numbers in numerical characters; and numeration is the reading of them.

NUMERATION TABLE.

Hundreds of billions,	Tens of billions,	Billions,	Hundreds of millions,	Tens of millions,	Millions,	Hundreds of thousands,	Tens of thousands,	Thousands,	Hundreds,	Tens,	Units,
											4
										2	4
									3	2	4
								5	3	2	4
							6	5	3	2	4
						7	6	5	3	2	4
					8	7	6	5	3	2	4
				9	8	7	6	5	3	2	4
			1	9	8	7	6	5	3	2	4
		2	1	9	8	7	6	5	3	2	4
	3	2	1	9	8	7	6	5	3	2	4
4	3	2	1	9	8	7	6	5	3	2	4

Numeration can be explained by the teacher, with the aid of the black-board, so much better than by means of the most copious examples and explanations given in the book, that it is deemed inexpedient to occupy more space than merely to introduce the subject to the learner.

12th place, Hundreds of billions,	11th place, Tens of billions,	10th place, Billions,	9th place, Hundreds of millions,	8th place, Tens of millions,	7th place, Millions,	6th place, Hundreds of thousands,	5th place, Tens of thousands,	4th place, Thousands,	3d place, Hundreds,	2d place, Tens,	1st place, Units,	
											4	Four.
										4		Four tens or forty.
									4			Enumerate.
								4				Enumerate.
							4					
						4						
					4							
				4								
			4									
		4										
	4											
4												
4	4	4	4	4	4	4	4	4	4	4	4	Enumerate.
										6	6	Enumerate.
									6	6	6	Enumerate.
								6	4	6	6	Enumerate.
							7	8	5	4	6	
				3	4	9	8	7	2	7	6	
5	6	8	9	4	5	7	2	3	6	4	9	
						7	2	4				Enumerate.
			4	7	6	9						Enumerate.
	4		3			7			2			

Write in figures, eight thousand seven hundred and thirty-six. Ninety-four thousand five hundred and fifty-three. Three millions nine thousand four hundred and forty-eight.

Nine hundred and ninety-nine millions, nine hundred and ninety-nine thousand, nine hundred and ninety-nine.

Note.—In writing numbers, every place not occupied by a figure must be occupied by a cipher. Thus, to write seven hundred and nine, as there are no tens, a cipher must occupy the place of tens, as 709. Should we omit the cipher and write 79, the 7 would occupy the ten's place, and would not express seven hundreds, but 7 tens.

MENTAL OPERATIONS IN ADDITION.

1. A BOY bought a knife for 16 cents, and a pint of nuts for 6 cents; how much did he give for both?

2. A man bought a firkin of butter for 12 dollars, and three barrels of cider for 8 dollars; how much did he give for the whole?

3. A boy had 14 cents, and his father gave him 6 more; how many had he then?

4. A man bought three cows, for the first he gave 13 dollars, for the second 7 dollars, and for the third 12 dollars; how much did he give for all?

5. A man bought a barrel of flour for 7 dollars, and three cords of wood for 15 dollars; what did he give for the whole?

6. John gave 11 nuts to one boy, 9 to another, and had 12 left; how many had he at first?

7. Joseph bought some oranges for 17 cents, a book for 13 cents, and a knife for 16 cents; how much did he give for the whole?

8. A farmer sold 18 bushels of wheat to one man, 12 bushels to another, and 13 bushels to a third; how many bushels did he sell to all?

9. A man bought a cow for 28 dollars, some corn for 12 dollars, and a pig for 6 dollars; what did he give for the whole?

10. A drover bought 23 sheep of one farmer, 17 of another, and 16 of a third; how many did he buy of all?

ADDITION OF SIMPLE NUMBERS.

ADDITION is the operation by which several numbers are united in one. The number thus obtained is called the *sum* or *amount*.

A man wishing to pay for a farm, found he had in one drawer of his bureau 7 one dollar bills, 8 ten dollar bills, and 3 hundred dollar bills. In another drawer, 4 one dollar bills, 6 ten dollar bills, and 7 hundred dollar bills. In a third drawer,

5 one dollar bills, 9 ten dollar bills, and 5 hundred dollar bills. In a fourth drawer, 6 one dollar bills, 4 ten dollar bills, 6 hundred dollar bills, and 1 thousand dollar bill, all of which he gave for the farm; what was the whole amount?

Thousand dollar bills	Hundred dollar bills	Ten dollar bills	One dollar bills
	3	8	7
	7	6	4
	5	9	5
1	6	4	6
3	3	9	2

The sum of the one dollar bills amounts to 22, this is equal to 2 ten dollar bills and 2 ones,—put the 2 one dollar bills under the ones, and count the other 2 with the tens,—this column amounts, with the two from the first, to 29 tens, this is equal to 2 hundred dollar bills and 9 tens,—put the nine tens under the tens, and count the 2 hundred with the hundreds, or third column, and the sum is 23 hundreds, which is equal to 2 thousands and 3 hundreds, put the 3 hundreds under the hundreds and count the 2 with the thousands, making 3 thousands, and we find the whole amounts to three thousand three hundred and ninety-two dollars.

RULE.

Write the numbers to be added, so that units stand under units, tens under tens, hundreds under hundreds, &c. Then begin at the units or right hand column and add up the figures in that column, if the sum can be expressed by one figure, write it down under the column of units, but if it amounts to 10 or more, put down the right hand figure only, and count the left hand figure, with the second column, and do the same with every other column, except the last or left hand column, the whole amount of which must be set down.

EXAMPLES.

Add together 246, 372, and 468.
Place the numbers under each other thus,

	246
	372
	468
Sum	1086

To prove addition, begin at the top of the units column

and count downwards, carrying the left-hand figure of each column to the next column, as before; if the sum thus obtained be the same as the first, the work may be supposed to be right.

(1.)	(2.)	(3.)
2634	7950	12345
9826	1267	9876
1247	846	1234
4368	473	3456
Sum 18075	Sum 10536	764
		Sum 27675

A cross formed by two lines, one horizontal and the other perpendicular, thus, + signifies that the numbers connected by it are to be added. It is called *plus*, (a Latin word signifying *more*,) or *and*, thus, $8+6$ are 14; that is, 8 plus 6 are 14; or 8 and 6 are 14.

Two horizontal lines thus, = mean equal, or as much as, thus, $8+6=14$; that is, 8 and 6 are equal to 14; or 8 and 6 are as much as 14.

The following table should be thoroughly learned.

3 and 2 are how many?
3 and 4 are how many?
3 and 5 are how many?
3 and 6 are how many?
3 and 9 are how many?
3 and 15 are how many?
3 and 14 are how many?
3 and 19 are how many?
19 and 3 are how many?
14 and 3 are how many?
15 and 3 are how many?
9 and 3 are how many?
12 and 3 are how many?
6 and 3 are how many?
5 and 3 are how many?
4 and 3 are how many?
2 and 3 are how many?
5 and 3 are how many?

5 and 4 are how many?
5 and 5 are how many?
5 and 6 are how many?
5 and 8 are how many?
5 and 9 are how many?
5 and 12 are how many?
5 and 19 are how many?
5 and 14 are how many?
5 and 23 are how many?
5 and 28 are how many?
5 and 33 are how many?
5 and 38 are how many?
38 and 5 are how many?
33 and 5 are how many?
28 and 5 are how many?
23 and 5 are how many?
14 and 5 are how many?

How many are 6 and 4?
How many are 8 and 4?
How many are 9 and 4?
How many are 12 and 6?
How many are 19 and 7?
How many are 26 and 7?
How many are 33 and 7?
How many are 47 and 7?

How many are 6+7+4? Ans. 17.
How many are 3+4+5+6? 18
How many are 8+2+7+9? 26
How many are 12+8+6? 26
How many are 14+6+5+7? 32
How many are 8+9+6+4? 27
How many are 3+12+5+8? 28
How many are 19+7+2+3+5? 36
How many are 24+6+8+5+3? 46
How many are 9+6+7+2+4?
How many are 4+6+5+9+7+3?
How many are 14+8+6+9+2+5?
How many are 1+2+3+4+5+6+7?
How many are 9+8+7+6+5+4+3+2+1?
How many are 12+8+4+6?
How many are 13+7+9+5? = 109
How many are 11+9+2+7+8?
How many are 16+4+9+3+7?
How many are 15+5+7+9+4?
How many are 17+3+9+8+6?
How many are 16+8+9+7+5?
How many are 4+8+2+6+3?
How many are 19+8+4+7+9+2?
How many are 23+7+8+6+9?
How many are 36+7+9+2+8?
How many are 47+6+8+9+5+4?

(4.)	(5.)	(6.)	(7.)	(8.)
9123	47689	42345	324	12679
8746	54321	63100	986	87654
8257	67890	54321	123	90123
2564	23654	987	432	17654
7893	1235	62	567	23456
121	976	36		987
432	442	41		
32136	123	160892		
	196330			

1. A boy gave some nuts to his companions, to one he gave 16, to another 17, to another, 18, to another 19; how many did he give to the whole?

2. Bought a cow for 37 dollars, some sheep for 46 dollars, a horse for 85 dollars, a saddle for 26 dollars; how much did the whole cost? Ans. $194.

3. A lady bought a comb for 37 cents, some muslin for 75 cents, a pair of shoes for 96 cents, some lace for 87 cents, and some cotton yarn for 69 cents; what did the whole cost? Ans. $3.64.

4. A drover bought of one man 48 sheep, of another 64, of another 37, of another 78, and another 156; how many did he buy altogether? Ans. 383 sheep.

5. If you buy a yoke of oxen for 76 dollars, a cart for 5 dollars, four cows for 96 dollars, and a plough for 15 dollars; how much must you pay for the whole? Ans. $192.

6. A merchant owes one man in Philadelphia $975, another $1325, to one man in New York he owes $480, to another $647, and to a man in New Orleans he owes $843; what is the amount of all his debts? Ans. 4270.

7. A dealer in lumber has 37,276 feet in one pile of boards, 13,834 feet in another, 47,693 in a third pile, and 98,417 feet in a fourth, and 27,689 feet lying on his wharf; how many feet does he own altogether? Ans. 224,909 feet.

8. A man bought 6 loads of hay, the first weighed 2348 pounds, the second 1849 pounds, the third 2418 pounds, the fourth 1789 pounds, the fifth 1945 pounds, and the sixth 1648 pounds; how many pounds did the whole weigh?
11,997 pounds.

9. If a farmer raise in one year 297 bushels of wheat, 125 of rye, 754 of Indian corn, 127 of barley, and 235 of oats; what is his crop? Ans. 1538.

10. A certain farmer raised 178 bushels of barley, twice as many bushels of wheat, 243 bushels of corn, and 96 of rye; how many bushels of grain did he raise? Ans. 873.

11. The State of Maine contains 35,000 square miles, New Hampshire 9,491, Vermont 8,000, Massachusetts 7,800, Rhode Island 1,225, Connecticut 4,764; how many square miles in the six New England States? Ans. 66,280 sq. m.

12. New York comprises 47,000 square miles, New Jer-

sey 8,320, Pennsylvania 46,000, Delaware 2,100, Maryland 9,356; how many square miles in those five States?

Ans. 112,776.

13. A merchant sold in one day 74 yards of muslin, 28 of linen, 36 of calico, and 69 of chintz; how many yards did he sell in all? Ans. 207 yards.

14. If a man travel 28 miles on Monday, 32 on Tuesday, 36 on Wednesday, 34 on Thursday, 38 on Friday, and 40 on Saturday; how many miles did he travel during the week? Ans. 208 miles.

15. A lady bought a comb for 37 cents, some tape for 13 cents some pins for 3 cents, and some thread for 12 cents; how much did she give for the whole?

NOTE.—When no answers are given the questions are to be answered *orally*.

16. Bought a ton of coal for 7 dollars, a barrel of flour for 6 dollars, two cords of wood for 9 dollars, and a stove for 18 dollars; how much did the whole cost?

17. If you pay 19 cents for a slate, 11 cents for some paper, and 14 cents for a book; what will the whole cost?

18. William has 26 peaches, his brother gave him 14 more, and his sister gave him 7; how many had he then?

19. Suppose I owe 26 dollars to A., 14 dollars to B., 13 dollars to C., 7 dollars to D., and 8 dollars to E.; how much do I owe in all?

20. Bought four books, for the first I gave 30 cents, for the second 40 cents, for the third 20 cents, and for the fourth 60 cents; what did the whole cost?

In this question you need only add the 3, 4, 2, and 6 together, and then put the 0 to the right of the sum.

21. Four boys went a fishing, the first caught 30 fishes, the second 40, the third 60, and the fourth 80; how many did they all catch?

22. How many are 8 and 4, and 6 and 12?

23. What is the sum of the following numbers, 67, 76, 82, 147, 768, 3462, 45, 96, 742, and 97? Ans. 5582.

24. William had 16 cents and 36 marbles, John had 23 cents and 27 marbles, Charles had 28 cents and 34 marbles; how many cents, and how many marbles have they all together? Ans. 67 cents and 97 marbles.

25. What is the sum of three hundred and forty-seven,—eight thousand seven hundred and eighty-four,—two hundred and sixty-nine,—twenty-eight thousand six hundred and ninety-two,—nine hundred and thirty-nine thousand,—three hundred and twenty-eight? Ans. 977,420.

26. The distance from Washington to Baltimore is 38 miles, from Baltimore to Philadelphia 93, from Philadelphia to New York 87 miles, from New York to Albany 145 miles; what is the distance from Washington to Albany? Ans. 363 miles.

27. According to sacred history, the time from the creation of the world to the deluge, was 1656 years, thence to the building of Solomon's temple 1344 years, thence to the birth of Christ 1004; how old is the world at the present time, A. D. 1849? Ans. 5853 years.

28. A man bought a horse for 75 dollars, a chaise for 184 dollars, and a set of harness for 48 dollars, he then sold his horse for 140 dollars, his chaise for 160 dollars, and his harness for 36 dollars; what did he pay for the whole, and what did he get for the whole? Paid $307, Received $336 } Ans.

QUESTIONS IN SUBTRACTION.

TO BE PERFORMED MENTALLY.

1. Joseph had 16 pears, and he gave 5 of them to William; how many had he left, and how many more than William had he then?

2. Dick had 24 apples, he gave to one of his companions 6, to another 8, and to another 3; how many had he left?

3. A boy had 23 cents, and lost 7 of them; how many had he left?

4. A boy having 30 cents bought a pound of cherries for 8 cents, and an orange for 6 cents; how many cents had he left?

5. In three bags there are 28 dollars, in the first there are

9 dollars, and in the second 7 dollars; how many dollars in the third?

6. A man bought a sleigh for 17 dollars, and paid 9 dollars to have it repaired and painted, and then sold it for 30 dollars; how much did he gain by the transaction?

7. A man bought a watch for 27 dollars, and sold it for 39 dollars; how much did he gain?

8. A boy bought a box for 16 cents, and paid 6 cents to have it painted, and then sold it for 30 cents; how much did he gain?

9. James had 12 cents, and wished to buy a knife worth 20 cents; how many more cents does he want?

10. Robert had 42 apples, he gave 12 to one companion, and 8 to another, and when he had given some to another he had 9 left; how many did he give to the third boy?

11. A man owed 60 dollars, at one time he paid 18 dollars, at another 12, at another 16; how much does he still owe?

12. A man owes 46 dollars and has but 37; how much more does he want to pay the debt?

13. A farmer agreed to pay 17 dollars for a cow, he paid 9 dollars down; how much does he still owe?

14. A boy sold a knife for 16 cents, which was 7 cents more than it cost him; how much did he pay for it?

15. A boy had 17 nuts, another boy gave him 9, another 6, and another enough to make the number 40; how many did the last boy give him?

16. A man has 18 dollars in one drawer, 12 in another, and 13 in another, he owes a debt of 36 dollars; how much will he have left after paying this debt?

17. A man owed 70 dollars, at one time he paid 24 dollars, at another 12, at another 14, and lastly, he paid the remainder of the debt all but 7 dollars; what was the last payment?

1. 12 less 8 are how many, that is, if you take 8 from 12 how many will remain?

2. 14 less 6 are how many?

3. 18 less 7 are how many?

4. 21 less 9 are how many?

5. 20 less 8 are how many?
6. 6 and 4 and 7 less 5 are how many?
7. 12 and 8 and 11 less 9 are how many?
8. 27 and 8 and 5 less 12 are how many?
9. 6 and 9 and 8 and 4 and 12 less 14 are how many?
10. 24+6+13+7 less 20 are how many?
11. 14+6+12+9+7 less 13 are how many?

SUBTRACTION OF SIMPLE NUMBERS.

By subtraction we ascertain how much greater one number is than another, or what remains when a less number is taken from a greater, or the difference between any two numbers. The greater number is called the *minuend*, and the less, or the number to be subtracted, the *subtrahend*, and the result of the operation is called the difference or remainder.

RULE.

Write the less number under the greater, with units under units, tens under tens, &c., then commence at the right hand or units place, and take each figure in the lower number from the one immediately above it; when the lower figure is greater than the one above it, call the upper 10 more than it is, and the next lower figure 1 more than it is.

EXAMPLES.

To prove subtraction add the less number to the difference, and their sum, if the work has been correctly performed, will be equal to the greater number.

From	946237
Take	234823
Difference	711414
	946237

In this example we take 3 from 7 and 4 remains, then 2 from 3 and 1 remains, but how shall we take 8 from 2? simply by calling the 2 10 more than it is, that is 12, and then take 8 from 12 and 4 remains,—now call the next

lower figure to the left of the 8, 1 more than it is, and say, 5 from 6 leaves 1, then 3 from 4 leaves 1, and 2 from 9 leaves 7.

	(1.)		(2.)
From	684792	From	1263247
Take	163451	Take	846184
Difference or rem.	521341	Difference	417063
Proof	684792	Proof	1263247

	(3.)	(4.)	(5.)
From	4602738	146180049236	276234612
Take	1461384	98024034618	143762346
Remainder	3141354	48156014618	
Proof	4602738		

(6.)	(7.)	(8.)
29613204446602	123456789012	8462173764
481436721566	32109876543	6543210987

(9.)	(10.)	(11.)
47789001213	476234560	743062156
18239001214	276342164	471949128

12. How much greater is 426 than 234? Ans. 192.
13. What will remain if 47 be taken from 84? Ans. 37.
14. What is the difference between 96 and 68? Ans. 28.
15. William has 62 cents, and Charles 27; how many has William more than Charles?
16. Mount St. Elias is 17,900 feet high, Popocatapetl is 17,735 feet high; how much higher is Mount St. Elias than Popocatapetl? And. 165 feet.
17. Water Volcano is 12,620 feet high, and Mount Hecla is 5,530 feet high; what is the difference in the height of the two volcanoes? Ans. 7,090 feet.
18. The Black Mountains in North Carolina are 6,476

feet high, and the White Mountains in New Hampshire are 6234 feet; how much higher are the Black Mountains than the White? Ans. 242 feet.

19. Mount Sorata, the highest mountain in South America, is 25,400 feet high, Mount Blanc, the highest mountain in Europe, is 15,533 feet high; what is the difference in the height of these mountains? Ans. 9867 feet.

20. In 1840 there were produced in the State of Ohio 16,571,601 bushels of wheat, and in Pennsylvania 13,213,077 bushels; how many bushels did Ohio produce more than Pennsylvania? Ans. 3,358,524 bushels.

21. Tennessee produced in 1840, 44,986,088 bushels of Indian corn, and Kentucky 39,847,120 bushels; how many did Tennessee produce more than Kentucky? Ans. 5,138,968 bushels.

22. The lead mines of Wisconsin produced in 1840, 15,129,350 lbs. of lead, those of Illinois 8,755,000 lbs.; how many pounds did Wisconsin produce more than Illinois? Ans. 6,374,350 lbs.

23. A drover bought of one man 42 sheep, of another 36, and of a third 54, he then sold 76; how many had he left? Ans. 56 sheep.

24. A man bought a horse for 85 dollars, and paid 17 dollars for keeping him, he let him out enough to receive 29 dollars and then sold him for 64 dollars; did he gain or lose, and how much?

25. Borrowed 640 dollars, of which I have paid 375 dollars; how much do I still owe? Ans. $265.

26. Columbus discovered America in 1492; how many years is it since?

27. If I buy 489 oranges for 912 cents, and then sell 136 for 196 cents; how many oranges will I have left, and what will they cost me?

28. How much must I add to 467 to make the number 2324? Ans. 1857.

Suppose there are a certain number of books lying on a desk, and I put 12 more on, and then take 12 off, the number remaining on the desk will evidently be the same as it was before I put the 12 on. The first operation is addition, and the other subtraction. The first is increase, the other

decrease; the first makes greater and the other makes less.

Again, suppose there are a certain number of books lying on a table, and you put 14 more on, and then take off 18, and on counting the remainder you find there are 28 on the table; how many were there at first? Now to ascertain the number, take the 28 which are now on the table and restore the 18 which you took off, that is, add 18 to the 28, this makes 46, and the 14 which you put on must be taken off, that is, subtracted, this leaves 32, the required number; or we may reason differently, thus—as you took 18 off and put but 14 on, you have taken off 4 more than you put on, so you must add 4 to the 28 to get the number at first on the table, 32 books.

In working such questions you will find that wherever a number has been added, you must subtract the same number, and where a number has been subtracted you must add an equal number to the given number.

Sometimes several numbers are added and several subtracted, the quickest way of working such questions is to set all the numbers which have been subtracted under the last mentioned number, and add the whole together, then all the numbers added in the question should be collected in one sum, which subtract from the other sum.

29. A man put some money into the bank, and after it had been there several weeks he drew out 275 dollars, a few days after this, he put in 480 dollars, at another time he drew out 240 dollars and still had 1235 dollars remaining in bank; what sum did he put in at first? Ans. $1270.

30. A miller put the wheat which he raised on his own farm into his mill without measuring it, he then purchased 240 bushels of a neighbouring farmer and put it in the same bin with his own, out of which he ground and sold 190 bushels, and then bought 137 bushels, which he put with the rest, after this he ground and sold 280 bushels and then measured what remained in the bin, and found he had 128 bushels left; how many bushels did he raise on his own farm? Ans. 221 bushels.

31. A man being asked how many books were in his library, said, if I were to take 87 out, and then put 45 in,

and again take 128 out, and afterwards put in 43, I should then have 879; how many did his library contain?

Ans. 1006.

32. Suppose a snail should crawl 18 feet up a pole one night, and slide down 9 feet during the day, the next night it crawled 24 feet upwards but slides down 8 feet during the day, the third night it goes up 23 feet and it slides down 11 feet the next day, on the fourth night it reached the top having gone up 29 feet; what was the height of the pole?

Ans. 66 feet.

Add all the upward distances together and take from the sum, the sum of the downward distances.

33. A farmer sold 37 sheep out of his flock and the next day bought 75, shortly after this he sold 84, and then had 29 left; how many had he at first? Ans. 75.

MULTIPLICATION.

1. If a peach cost 3 cents; what will 4 peaches cost?
2. What cost 4 oranges at 5 cents a-piece? Why, say if one orange cost 5 cents, 4 oranges will cost 4 times 5 cents.
3. What cost 3 pounds of raisins at 6 cents a pound? Why?
4. What cost 6 yards of tape at 2 cents a yard? Why?
5. What cost 5 pounds of sugar at 9 cents a pound? Why?
6. What are 3 barrels of flour worth at 6 dollars a barrel? Why?
7. What will 8 pounds of beef cost at 7 cents a pound? Why?
8. What will 8 reams of paper cost at 4 dollars a ream? Why?
9. There are 4 quarts in one gallon; how many quarts in 7 gallons?
10. There are 8 pints in one gallon; how many pints in 6 gallons?
11. What will 7 pair of shoes cost at 3 dollars a pair? Why?
12. What will 8 pounds of rice cost at 4 cents a pound?

MULTIPLICATION TABLE.*

2 times		3 times		4 times		5 times		6 times		7 times	
1	are 2	1	are 3	1	are 4	1	are 5	1	are 6	1	are 7
2	4	2	6	2	8	2	10	2	12	2	14
3	6	3	9	3	12	3	15	3	18	3	21
4	8	4	12	4	16	4	20	4	24	4	28
5	10	5	15	5	20	5	25	5	30	5	35
6	12	6	18	6	24	6	30	6	36	6	42
7	14	7	21	7	28	7	35	7	42	7	49
8	16	8	24	8	32	8	40	8	48	8	56
9	18	9	27	9	36	9	45	9	54	9	63
10	20	10	30	10	40	10	50	10	60	10	70
11	22	11	33	11	44	11	55	11	66	11	77
12	24	12	36	12	48	12	60	12	72	12	84
13	26	13	39	13	52	13	65	13	78	13	91
14	28	14	42	14	56	14	70	14	84	14	98
15	30	15	45	15	60	15	75	15	90	15	105
16	32	16	48	16	64	16	80	16	96	16	112
17	34	17	51	17	68	17	85	17	102	17	119
18	36	18	54	18	72	18	90	18	108	18	126
19	38	19	57	19	76	19	95	19	114	19	133
20	40	20	60	20	80	20	100	20	120	20	140
21	42	21	63	21	84	21	105	21	126	21	147
22	44	22	66	22	88	22	110	22	132	22	154
23	46	23	69	23	92	23	115	23	138	23	161
24	48	24	72	24	96	24	120	24	144	24	168
25	50	25	75	25	100	25	125	25	150	25	175
26	52	26	78	26	104	26	130	26	156	26	182
27	54	27	81	27	108	27	135	27	162	27	189
28	56	28	84	28	112	28	140	28	168	28	196
29	58	29	87	29	116	29	145	29	174	29	203
30	60	30	90	30	120	30	150	30	180	30	210

* The above table should be thoroughly committed to memory before the learner proceeds further.

NOTE.—This table should be written with chalk upon the black board, and recited in concert by the whole class, first from top to bottom, then from bottom to the top of the column. The exercise should

MULTIPLICATION TABLE CONTINUED.

8 times		9 times		10 times		11 times		12 times	
1 are	8	1 are	9	1 are	10	1 are	11	1 are	12
2	16	2	18	2	20	2	22	2	24
3	24	3	27	3	30	3	33	3	36
4	32	4	36	4	40	4	44	4	48
5	40	5	45	5	50	5	55	5	60
6	48	6	54	6	60	6	66	6	72
7	56	7	63	7	70	7	77	7	84
8	64	8	72	8	80	8	88	8	96
9	72	9	81	9	90	9	99	9	108
10	80	10	90	10	100	10	110	10	120
11	88	11	99	11	110	11	121	11	132
12	96	12	108	12	120	12	132	12	144
13	104	13	117	13	130	13	143	13	156
14	112	14	126	14	140	14	154	14	168
15	120	15	135	15	150	15	165	15	180
16	128	16	144	16	160	16	176	16	192
17	136	17	153	17	170	17	187	17	204
18	144	18	162	18	180	18	198	18	216
19	152	19	171	19	190	19	209	19	228
20	160	20	180	20	200	20	220	20	240
21	168	21	189	21	210	21	231	21	252
22	176	22	198	22	220	22	242	22	264
23	184	23	207	23	230	23	253	23	276
24	192	24	216	24	240	24	264	24	288
25	200	25	225	25	250	25	275	25	300
26	208	26	234	26	260	26	286	26	312
27	216	27	243	27	270	27	297	27	324
28	224	28	252	28	280	28	308	28	336
29	232	29	261	29	290	29	319	29	348
30	240	30	270	30	300	30	330	30	360

be continued from time to time, taking one column for a lesson, until the whole is thoroughly learned.

The best way to determine whether a column is thoroughly learned by the pupils, is for the teacher to rub out the products, and then with a rod to point promiscuously in the left-hand column, and if the class can keep up a good concert whilst going over the numbers thus, it may be taken for granted that the lesson is pretty well learned.

13. What will 9 pounds of sugar cost at 18 cents a pound?
14. What cost 12 yards of cloth at 7 dollars a yard? Why?
15. If a man can earn 13 dollars a month; how much can he earn in 8 months?
16. What will 16 oranges cost at 3 cents a piece? Why?
17. In one gallon there are 8 pints; how many pints in 13 gallons? Why?
18. Which is the greater 8 times 13, or 13 times 8?
19. If a you read 14 pages in a day; how many pages could you read, at that rate, in 9 days? Why?
20. How much will 16 melons cost at 9 cents a-piece? Why?
21. If a man can walk 24 miles in one day, how many miles can he walk in 6 days? Why?
22. What will 12 pounds of butter cost at 18 cents a pound?
23. What are 8 dozen eggs worth at 15 cents a dozen? Why?
24. There are 8 quarts in a peck; how many quarts in 14 pecks?
25. If one ton of hay cost 17 dollars what will 6 tons cost? Why?
26. If a pound of honey cost 14 cents, what will 7 pounds cost? Why?
27. If a steamboat will go 13 miles in one hour, how far will it go in 12 hours? Why?
28. If a labourer can earn 16 dollars in a month, how much can he earn in 11 months? Why?
29. There are 4 gills in a pint, and 8 pints in a gallon; how many gills in 2 gallons?
30. There are 12 inches in one foot; how many inches in 9 feet? Why?
31. There are 1760 yards in one mile; how many yards in 7 miles,—one of these numbers being greater than any in the multiplication table, we must use the slate and multiply one figure at a time.

	Yds.
	1760
Thus,	7
	12320

Here we say, 7 times 0 make 0, then 7 times 6 make 42, that is, 42 tens, which is equal to 4 hundred and 2 tens, put down the 2 under the 6 tens, and then say, 7 times 7 make 49 hundreds, and 4 hundreds make 53

hundreds, 5 thousands and 3 hundreds, put down the 2 hundreds under the 7 hundreds, and then say, 7 times 1 make 7 thousands, and 5 thousands make 12 thousands.

By multiplication we ascertain what a number amounts to when repeated a given number of times; thus, 3 times 4 make 12, that is, 4 repeated 3 times make 12. The number to be multiplied is called the *multiplicand*, the number we multiply by is called the *multiplier*, the result of multiplication is called the *product*. The multiplier and multiplicand are also called *factors*, or either of them a *factor*.

RULE FOR MULTIPLICATION.

Write the multiplier under the multiplicand, placing units under units, tens under tens, &c.

Commence at the right hand and multiply each figure in the multiplicand separately, and place the right hand figure of each product under the figure in the multiplicand from which it arose, and carry the left hand figure as in addition.

If there is more than one figure in the multiplier, multiply by each figure separately, and write its product in a separate line, placing the right hand figure of each product directly under the figure by which you multiply, after which add to them the several products, the sum will be the required product.

EXAMPLES.

1. Multiply 4623 by 8,

Multiplicand	4623
Multiplier	8
Product	36984

2. Multiply 3764 by 34,

	3764
	34
	15056
	11292
Product	127976

3. Suppose 674 be the multiplicand and 24 the multiplier, what is the product?

	674
	24
	2696
	1348
Product	16176

This character = signifies equal, as 1 dollar =100 cents.

+ addition, as 4+12=16.

× multiplication, as 8×6=48.

4. Multiply 4684 by 12	Product	=	56208
5. Multiply 276 by 47	Product	=	12972
6. Multiply 4764 by 326	Product	=	1553064
7. Multiply 2453 by 19	Product	=	46607
8. Multiply 4628 by 247	Product	=	1143116

9. What will 96 barrels of flour cost at 7 dollars a barrel? Ans. 662 dollars. Why? Say, if it takes 7 dollars to buy one barrel, the whole cost will be seven times as many dollars as there are barrels.

10. There are 24 hours in one day; how many hours in 7 days? Ans. 168 hours. Why? Say, if there are 24 hours in one day, in 7 days there will be 7 times 24 hours.

11. Suppose 4673 be one factor, and 3764 another; what is the product?

PROOF OF MULTIPLICATION.

Add together all the figures in the product, and observe the number of 9s contained in the sum, and put down the excess; do the same with each factor, then multiply together the excess of the factors and cast out the nines from the product. If the excess in this case be the same as that in the first product the work may be considered right. It is proper to observe, however, that there is a possibility of an error being in the work, though it proves correct by this process, but as this error must be just 9, in the units, tens, or hundreds, &c., it is not likely to occur very often.

12. Multiply 7438 ×4 by 3652

```
    7438 ×4
    3652 ×7
   -------
   14876 | 28
  37190  |
 44628   |
22314    |
-------- ----
27163576   1
```

The sum of the figures in the multiplicand is 22, this makes two nines, and an excess of 4, in the multiplier there is one nine, and an excess of 7, their product is 28= three nines, and an excess of 1, the same as that in the product. Counting the nines in the product we say, the 7 and 2 make 9, then 6 and 3 make 9, we have yet 1, 5, 7, and 6, these make two nines, and an excess of 1.

13. What cost 48 bushels of corn at 56 cents a bushel? Ans. 2688 cents. Why? Say, if one bushel cost 56 cents, 48 bushels will cost 48 times 56 cents.

NOTE.—If you point off two figures to the right for cents, all to the left of the point will be dollars; thus, 2688 cents = $26.88, read 26 dollars and 88 cents.

14. What are 247 bushels of wheat worth at 96 cents a bushel? Ans. $237.12. Why? Say, if one bushel cost 96 cents, the whole price will be 96 times as many cents as there are bushels.

15. In a certain corn field there are 248 rows, and 87 hills in each row; how many hills are there in the field? Ans. 21576 hills. Why? Say, if there are 87 hills in one row, there must be 87 times as many hills as there are rows.

16. There are 69 pieces of cloth, each piece containing 112 yards; how many yards are there in all? Ans. 7728 yards. Why? If there are 112 yards in one piece, in 69 pieces there will be 69 times 112 yards.

17. What will 421 bushels of wheat cost at 135 cents a bushel? Ans. $568.35. Why?

18. Suppose 24 men can build a certain wall in 36 days; in how many days can one man do it? Ans. 864 days. Why? If it takes 36 days for 24 men to build a wall, it will take 24 times 36 days for one man to do it.

19. How many days' work can 16 men do in 29 days? Ans. 464 days' work. Why? Since one man can do 29 days' work in 29 days, 16 men can do 16 times 29 days' work in the same time.

20. There are 24 hours in a day; how many hours in 75 days? Why?

21. There are 52 weeks in a year; how many weeks in 37 years? Why?

22. Bought 23 bales of cloth, each containing 46 pieces, and each piece 25 yards; how many yards in all? Ans. 26450 yards. Why?

23. A merchant bought 235 yards of cloth for which he paid 276 cents a yard; what did the whole amount to. Ans. $648.60. Why?

24. What will 784 chests of tea cost, at 69 dollars a chest? Ans. $54096. Why?

25. What cost 28 bushels of potatoes at 45 cents a bushel? Why?

26. What will 56 yards of cloth cost at 235 cents a yard? Ans. $131.60. Why?

27. A merchant bought one piece of cloth containing 69 yards, at $2.75 per yard, and another containing 56 yards, at $3.84 per yard; what did the whole cost? Ans. $404.79.

28. Suppose I buy 27 bags of coffee, each containing 126 pounds, and then sell 1678 pounds; how much will I have left? Ans. 1724 pounds.

29. Multiply 68 by 37, and then add 426 to the product. Ans. 2942.

30. Multiply 46 by 38, and 76 by 23, and add the products together. Ans. 3496.

31. Multiply 75 by 39, and 64 by 43, and subtract the less product from the greater. Ans. 173.

32. A man who had $156, bought 14 barrels of flour at $6 a barrel, 4 cords of wood at $4, a cord, and 9 yards of cloth at $3 a yard; how much money had he left? Ans. $29.

DIVISION.

QUESTIONS TO BE ANSWERED ORALLY.

1. A farmer paid 24 dollars for some sheep, at 3 dollars a-piece; how many sheep did he buy? Why? Since 3 dollars will buy one sheep, 24 dollars will buy as many sheep as there are threes in 24.

2. A man paid 28 dollars for some wood, giving 4 dollars a cord; how many cords did he buy? Why? Since 4 dollars will buy one cord, 28 dollars will buy as many cords as there are 4s in 28.

3. How many yards of cloth, at 7 dollars a yard can you buy for 56 dollars? Ans. 8 yards. Why?

4. If a man travel 3 miles in a hour, how many hours will he be in travelling 39 miles? Ans. 13 hours. Why?

5. How many pounds of sugar, at 3 cents a pound, can you buy for 72 cents? Why?

6. If a man can earn 16 dollars in one month, in how many months can he earn 80 dollars? Why?

7. A man had 91 dollars, which he gave for cloth, paying 7 dollars a yard; how many yards did he buy? Why?

8. A boy had 72 nuts, which he gave to his companions, each boy receiving 6 nuts; how many boys were there? Why?

9. How many lead pencils could you buy for 42 cents if they are sold at 6 cents a-piece? Why? Say, if 6 cents will pay for one pencil, as many times 6 as there are in 42, so many pencils 42 cents will pay for.

10. In a certain orchard there are 117 trees, and 13 trees in each row; how many rows are there? Why? Say, since it takes 13 trees to make one row, as many times 13 as there are in 117, so many rows will 117 trees make.

11. If you divide 48 apples equally among 8 boys, how many will each boy have? Why? Say, since it would take 8 apples to give each boy one, as many times 8 as there are in 48, so many apples will each boy receive.

12. If you have 65 cents to pay for beef, at 5 cents a pound, how many pounds can you buy? Why?

13. Suppose 12 men are to receive 156 dollars for doing a certain piece of work, how much will each man receive? Why?

14. A man sold some sheep at 5 dollars a-piece, and received 80 dollars for the whole; how many sheep were there? Why?

15. A man bought 12 gallons of brandy for 36 dollars; how much was that a gallon? Why? Since it would take 12 dollars to pay one dollar a gallon, the price will be as many times one dollar as there one 12s in 36.

16. A man received 132 dollars for working 12 months, how much was that a month? Why?

17. If you had 75 dollars, how many barrels of flour could you buy at 5 dollars a barrel? Why?

18. How many pounds of coffee at 16 cents a pound can you buy for $1.28? Why?

19. When hay is 13 dollars a ton, how many tons can you purchase for 78 dollars? Why?

20. If an orange is worth as much as 5 apples, how many

oranges are 60 apples worth? Why? Since 5 apples are worth one orange, as many times 5 as there are in 60, so many oranges are 60 apples worth.

21. If one man can do a piece of work in 40 days, in how many days can 5 men do the same? Why? Since 5 men can do 5 days' work in one day, as many times 5 as there are in 40, so many days would 5 men require to do 40 days' work.

22. A man paid 98 cents for a horse to ride 14 miles; how much was that a mile? Why?

23. If a horse eats 8 quarts of oats in one day, in how many days will he eat 96 quarts? Why?

24. A man has 84 gallons of beer, which he wished to put into kegs, each of which would hold 7 gallons; how many kegs would he require? Why? Since it requires one keg to hold 7 gallons, he will require as many kegs as there are 7s in 84.

25. 14 men agreed to pay in equal shares for a horse worth 126 dollars; how much did each pay? Why? Since it would take 14 dollars for each man to pay 1 dollar, as many times 14 dollars as there are in 126, so many dollars each man must pay.

26. If 4 yards of cloth will make a cloak, how many cloaks will 56 yards make? Why?

27. If you had a vessel containing 96 gallons of wine, having a pipe which discharges 8 gallons in an hour; in how many hours would it discharge the 96 gallons? Why?

28. If a man perform a journey in 72 hours, how many days will he require to do it if he travel 9 hours a day? Why?

29. In 36, how many times 4? how many times 9, 12, 6?

30. 48 are how many times 8, 12, 16, 4, 3, 6?

31. 24 are how many times 8, 6, 3, 4, 12?

32. 72 are how many times 8, 9, 12, 18, 24, 36, 2, 3, 4, 6?

33. 96 are how many times 8, 12, 16, 24, 4?

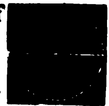
34. 84 are how many times 7, 12, 14, 28, 3, 4?

35. 144 are how many times 12, 16, 18, 8, 9, 24, 6?

36. 80 are how many times 8, 10, 16, 5, 4, 20?

37. 60 are how many times 5, 6, 12, 15, 20?

MENTAL ARITHMETIC.



1. If an orange cost 6 cents, what will one-half of it be worth?

2. What is one-half of 6?

What do you understand by the term one-half? Ans. If any thing, or any number be divided into two equal parts, one of these parts is called one-half.

How many halves make the whole of any thing?

3. 3 is what part of 6? Ans. 3 is the half of 6, because two 3s make 6, and two halves make the whole.

4. If a cord of wood cost 8 dollars, what will half a cord cost? What is one-half of 8? How do you find one-half of any number? Ans. divide it by 2.

5. What is one half 12, of 14, of 16, of 24, of 26, of 28?

6. How many times 2 in 7? Ans. 3 twos and 1 over.

7. What part of 2 is 1? Ans. 1 is one-half of 2.

8. What is one-half of 9? Ans. 4 and one-half, $4\frac{1}{2}$.

9. What is $\frac{1}{2}$ of 5? Ans. $2\frac{1}{2}$.

10. What is $\frac{1}{2}$ of 13, of 17, of 19, of 25, of 27, of 29?

11. If a peach cost 3 cents, what is one-third of it worth? What is one-third of three?

12. What are we to understand by the term one-third? Ans. If any thing or any number be divided into three equal parts, one of those parts is called one-third, and two of them two-thirds.

13. If a yard of cloth cost 6 dollars, what is one-third of it worth? What is one-third ($\frac{1}{3}$) of 6, two-thirds ($\frac{2}{3}$) of 6?

14. How do you find one-third ($\frac{1}{3}$) of any number? Ans. divide it by 3.

15. What is one-third ($\frac{1}{3}$) of 12? two-thirds of 12?

16. What part of 12 is 4, is 8? Why is 4, $\frac{1}{3}$ of 12? Ans. because three times 4 make 12, and three-thirds make the whole of any thing, or any number.

17. What is $\frac{1}{3}$ of 6, of 9, of 12, of 15, of 18, of 24, of 27, of 30?

18. What is $\frac{1}{3}$ of 36, $\frac{2}{3}$ of 36, $\frac{2}{3}$ of 6, $\frac{2}{3}$ of 18, $\frac{2}{3}$ of 15, $\frac{2}{3}$ of 9, $\frac{2}{3}$ of 39?

19. 2 is $\frac{1}{3}$ of what number? What number is 8 the $\frac{2}{3}$ of? 13, 14, 15?

20. Suppose 8 is $\frac{2}{3}$ of some number, what is $\frac{1}{3}$ of that number? Ans. 4. Why? Say if 8 be two parts, the half of 8 will be one part, and 4 is the half of 8.

21. Suppose 12 is $\frac{2}{3}$ of some number, what is $\frac{1}{3}$ of that number? Why?

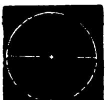
22. If 16 be $\frac{2}{3}$ of some number, what is $\frac{1}{3}$ of that number? Why? What is the whole, or $\frac{3}{3}$ of the number?

23. 18 is $\frac{2}{3}$ of what number? 20 is, 24 is, 22 is?

24. What is $\frac{2}{3}$ of 27? First find $\frac{1}{3}$; thus, 3 in 27, 9 times; then 9 is $\frac{1}{3}$ of 27, and two 9s, or 18 is $\frac{2}{3}$ of 27.

25. What is one-half ($\frac{1}{2}$) of 24, of 28, $\frac{1}{3}$ of 36, $\frac{2}{3}$ of 36?

26. What is $\frac{1}{3}$ of 39, $\frac{2}{3}$ of 39, $\frac{2}{3}$ of 42, $\frac{1}{2}$ of 56?



27. If an orange cost 4 cents, what is one-fourth of it worth?

28. What are we to understand by the term one-fourth ($\frac{1}{4}$) and ($\frac{3}{4}$)? Ans. If any thing, or any number, be divided into 4 equal parts, one of those parts is called one-fourth ($\frac{1}{4}$), two of them, two-fourths ($\frac{2}{4}$), three of them ($\frac{3}{4}$). What is $\frac{1}{4}$ of 8, of 12, of 20, of 24, of 28? How many fourths make the whole of any number?

29. How do you find one-fourth of any number? Ans. by dividing it by 4. What is $\frac{1}{4}$ of 36, of 48, of 32?

30. 7 is $\frac{1}{4}$ of what number? 9 is $\frac{1}{4}$ of what number? 13 is $\frac{1}{4}$ of what number?

31. 16 is $\frac{1}{4}$ of what number? 14, 18, 17, 15, 19, 23, 24, 25?

32. If 18 be $\frac{3}{4}$ of some number, what is $\frac{1}{4}$ of that number? Ans. 6. Why? Say, if 18 be 3 parts, one-third of 18 must be one part, and $\frac{1}{3}$ of 18 is 6.

33. 24 is $\frac{3}{4}$ of what number? 36 is, 42 is, 15 is?

34. If 27 be $\frac{3}{4}$ of some number, what is that number? First find $\frac{1}{4}$ by dividing 27 by 3.

35. If you have $\frac{3}{4}$ of a number, how do you get $\frac{1}{4}$? Ans. divide the number by 3,—after you get $\frac{1}{4}$, how do you get the whole, or $\frac{4}{4}$?

36. If 36 be $\frac{3}{4}$ of some number, what is that number? Ans. 48. Why? Say, if 36 be 3 parts, $\frac{1}{3}$ of 36, or 12 must be one part; and as the number contains 4 of those parts, 4 times 12 must be the number; now 4 times 12 make 48.

37. What is $\frac{1}{4}$ of 20, of 36, of 40, of 60, of 72, of 76, of 96?

38. What is $\frac{3}{4}$ of 12, $\frac{3}{4}$ of 36, $\frac{3}{4}$ of 56, $\frac{3}{4}$ of 72, $\frac{3}{4}$ of 48, of 76?

39. What is $\frac{1}{2}$ of 36, $\frac{1}{3}$ of 21, $\frac{1}{4}$ of 24, $\frac{1}{2}$ of 34, $\frac{1}{3}$ of 42, $\frac{1}{4}$ of 60?

40. What is $\frac{1}{2}$ of 58, $\frac{2}{3}$ of 21, $\frac{2}{3}$ of 39, $\frac{3}{4}$ of 44, $\frac{3}{4}$ of 16, $\frac{3}{4}$ of 20?

41. If 2 pounds of sugar cost 28 cents, what will one pound cost? Ans. 14 cents. Why? Since 2 pounds cost 28 cents, one pound will cost $\frac{1}{2}$ of 28 cents, and 14 is $\frac{1}{2}$ of 28.

42. If 3 bushels of apples cost 72 cents, what will one bushel cost? Ans. 24 cents. Why? Since 3 bushels of apples cost 72 cents, one bushel will cost $\frac{1}{3}$ of 72 cents, and $\frac{1}{3}$ of 72 is 24.

43. If 4 yards of muslin cost 72 cents, what will one yard cost? Ans. 18 cents. Why? Since 4 yards cost 72 cents, one yard will cost $\frac{1}{4}$ of 72 cents, and $\frac{1}{4}$ of 72 is 18.

44. If 3 bushels of apples cost 69 cents, what will 2 bushels cost? Ans. 46 cents. Why? Since 3 bushels cost 69 cents, one bushel will cost $\frac{1}{3}$ of 69 cents, which is 23 cents, and 2 bushels will cost $\frac{2}{3}$ of 69 cents, or twice 23 cents = 46 cents.

45. If 3 yards of linen cost 84 cents, what will 2 yards cost? Why?

46. If 4 yards of cotton cloth cost 72 cents, what will 3 yards cost? Why?

47. If 2 bushels of corn cost 84 cents, what will 7 bushels cost? Ans. $2.94. Why? Since 2 bushels cost 84 cents, 1 bushel will cost $\frac{1}{2}$ of 84, or 42 cents, and 7 bushels will cost 7 times $\frac{1}{2}$ of 84, or 7 times 42 cents.

48. If $\frac{3}{4}$ of a yard of silk, cost 87 cents, what will $\frac{1}{4}$ of a yard cost? Ans. 29 cents. Why? Since 3 parts cost 87 cents, 1 part will cost $\frac{1}{3}$ of 87 cents, and $\frac{1}{3}$ of 87 is 29.

49. If $\frac{3}{4}$ of a yard of cotton cloth cost 18 cents, what will a yard cost? Ans. 24 cents. Why? Since 3 parts cost 18 cents, 1 part will cost $\frac{1}{3}$ of 18 cents, which is 6 cents, and 4 parts will cost 4 times $\frac{1}{3}$ of 18 cents, or 4 times 6 cents = 24 cents.

50. What is to be understood by the term $\frac{3}{4}$? Ans. If any thing or any number, be divided into 4 equal parts, 3 of these parts are called $\frac{3}{4}$ of that thing or number; or we may express it more briefly by saying 3 of 4 equal parts.

51. If a pound of honey cost 32 cents, what will $\frac{3}{4}$ of a pound cost? Why?

52. If $\frac{3}{4}$ of a bushel of corn cost 39 cents, what is a bushel worth? Why? What are 8 bushels worth?

53. If $\frac{3}{4}$ of a barrel of flour cost 6 dollars, what will 12 barrels cost? Why?

54. If a pine-apple be worth 20 cents, and it be cut into 5 equal pieces, what will one piece be worth?

55. If any thing or any number be divided into 5 equal parts, what do we call one of those parts? Ans. one-fifth. How many fifths make the whole of any thing or of any number? Ans. five.

56. What is one-fifth ($\frac{1}{5}$) of 20? Ans. 4. Why? Because 5 times 4 make 20, and 5 times $\frac{1}{5}$ make the whole of any thing or of any number.

57. How do you find $\frac{1}{5}$ of any number? Ans. divide it by 5. What is $\frac{1}{5}$ of 35, of 40, of 60, of 75, of 15, of 30, of 45?

58. What is $\frac{1}{5}$ of 80, $\frac{2}{5}$ of 80, $\frac{4}{5}$ of 80, $\frac{3}{5}$ of 45, $\frac{2}{5}$ of 10, $\frac{4}{5}$ of 60?

59. If 32 be $\frac{4}{5}$ of some number, what is $\frac{1}{5}$ of that number? Ans. 8. Why? Since 32 make 4 parts of a certain number, $\frac{1}{4}$ of 32 must be 1 part of that number, and $\frac{1}{4}$ of 32 is 8.

60. If you divide a number by 5, what part of the number do you get, if you divide by 4, by 3, by 2?

61. What is $\frac{1}{2}$ of 28, of 36, $\frac{1}{3}$ of 12, of 42, $\frac{1}{4}$ of 60, of 36, $\frac{1}{5}$ of 85?

62. If 42 be $\frac{3}{5}$ of some number, what is that number? Ans. 70. Why? Say, if 42 be 3 parts of some number, then $\frac{1}{3}$ of 42 must be 1 part of that number, and as the number consists of 5 parts, 5 times $\frac{1}{3}$ of 42 will make the whole of the number, $\frac{1}{3}$ of 42 is 14, and 5 times $\frac{1}{3}$ of 42 is 5 times 14=70.

63. 36 is $\frac{4}{5}$ of what number? Why?

64. 18 is $\frac{2}{5}$ of what number, $\frac{3}{5}$ of what number? Why?

65. 24 is $\frac{3}{5}$ of what number, is $\frac{2}{3}$ of what number? Why?

66. 36 is $\frac{2}{5}$ of what number, is $\frac{4}{5}$, is $\frac{3}{5}$, is $\frac{3}{4}$, is $\frac{1}{2}$, of what number?

67. What is $\frac{1}{5}$ of 80, $\frac{3}{5}$ of 80, $\frac{3}{4}$ of 80, $\frac{1}{2}$ of 80, $\frac{3}{5}$ of 45, $\frac{2}{3}$ of 48? Why?

68. 48 is $\frac{1}{2}$ of what number, $\frac{1}{3}$, is $\frac{1}{4}$, is $\frac{1}{5}$, is $\frac{4}{5}$, is $\frac{3}{4}$ of what number?

69. If 5 yards of muslin cost 70 cents, what part of 70 cents will 1 yard cost, will 2 yards, 3 yards, 4 yards? What is $\frac{1}{5}$ of 70, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ of 70?

70. 5 pounds of cheese cost 40 cents, what will 1 pound cost, 2 pounds, 3, 4 pounds? Why?

71. If $\frac{3}{5}$ of a yard of calico cost 27 cents, what will a yard cost, 4 yards, 10 yards, 7 yards, 3 yards? Why?

72. If a bushel of oats cost 35 cents, what will $\frac{3}{5}$ of a bushel cost, $\frac{4}{5}$ of a bushel? Why?

73. If a man travel 28 miles in $\frac{4}{5}$ of a day, how far will he travel in a day, in 8 days? Why?

74. If 45 bushels of corn grow on an acre of land, how much will grow on $\frac{3}{5}$ of an acre, $\frac{4}{5}$ of an acre, $\frac{2}{3}$ of an acre?

75. If a bushel of wheat cost 90 cents, what is $\frac{1}{2}$ a bushel worth, $\frac{2}{3}$ of a bushel, $\frac{4}{5}$ of a bushel?

76. A man being asked how many sheep he had, said that $\frac{3}{5}$ of the number made 57; how many had he?

77. A man being asked the age of his eldest son, said his youngest son's age was 18 years, which was three-fifths ($\frac{3}{5}$) of the age of his eldest son, what was his eldest son's age?

78. A man bought two cows, for one he gave 24 dollars, which was $\frac{3}{4}$ of what he gave for the other, what did he give for the other?

79. A man bought a cow, an ox, and a horse, for the cow he gave 27 dollars, which was $\frac{3}{4}$ of what he gave for the ox, and the price of the ox was $\frac{2}{5}$ as much as the price of the horse; what was the price of the ox, and also of the horse?

80. John bought a slate, an Arithmetic, a Reader, and a Dictionary; for the slate he gave 16 cents, which was $\frac{1}{2}$ of what he gave for his Arithmetic, and the price of the Arithmetic was $\frac{2}{3}$ of what he gave for his Reader, and the price of the Reader was $\frac{3}{5}$ of what he gave for his Dictionary; what did he give for his Arithmetic, for his Reader, and for his Dictionary?

81. If a melon cost 6 cents, and it be cut into 6 equal pieces, what will one piece be worth? What is one of 6 equal parts called? Ans. $\frac{1}{6}$. 2 of 6 equal parts, 3, 5 of 6 equal parts? How many sixths are there in the whole of any thing, or of any number?

82. How do you find one-sixth of any number? divide it by 6, that is, find how many times 6 is contained in it.

83. What is $\frac{1}{6}$ of 24, of 48, of 36, of 30, of 18, of 72, of 96, of 78, of 54?

84. What is $\frac{5}{6}$ of 18, of 36, of 60, of 72, of 42, of 84, of 48, of 96, of 54, of 78?

85. Suppose 72 dollars are to be divided among six men, what part of 72 dollars must one man receive, 2 men, 3 men, 5 men? How many dollars will one man receive, 5 men?

86. If $\frac{5}{6}$ of a certain number be 45, what is $\frac{1}{6}$ of that number? Ans. 9. Why? Since 5 parts of a number make 45, 1 part will be $\frac{1}{5}$ of 45, and $\frac{1}{5}$ of 45 is 9.

87. 14 is $\frac{1}{6}$ of what number, 16 is, 12 is, 15 is, 25 is, 28 is?

88. 30 is $\frac{5}{6}$ of what number? Ans. 36. Why? Since 30 is 5 parts, $\frac{1}{5}$ of 30 must be one part, and as the number consists of 6 parts, 6 times $\frac{1}{5}$ of 30 is the number; now $\frac{1}{5}$ of 30 is 6, and 6 times $\frac{1}{5}$ of 30 is 6 times 6=36.

89. 40 is $\frac{5}{6}$ of what number, 65 is, 35 is, 75 is, 60 is, 70 is, 80 is?

90. What is $\frac{1}{6}$ of 60, $\frac{1}{5}$ of 60, $\frac{1}{4}$ of 60, $\frac{1}{3}$ of 60, $\frac{1}{2}$ of 60, $\frac{2}{3}$ of 60, $\frac{3}{4}$ of 60?

91. What is $\frac{1}{2}$ of 84, $\frac{3}{4}$ of 84, $\frac{2}{5}$ of 40, $\frac{4}{5}$ of 60, $\frac{5}{6}$ of 72, $\frac{5}{6}$ of 96?

92. If a yard of linen cost 54 cents, what will $\frac{1}{6}$ of a yard cost, $\frac{5}{6}$ of a yard? Why?

93. If $\frac{5}{6}$ of a day's labour be worth 70 cents, what will $\frac{1}{6}$ of a day be worth, what will a day's labour be worth? Why?

94. If $\frac{5}{6}$ of a bushel of apples cost 35 cents, what will a bushel cost, what will 12 bushels cost? Why?

95. If a yard of muslin cost 18 cents, what will 2 yards and $\frac{5}{6}$ of a yard cost?

96. A man sold a cow for 20 dollars, which was only

$\frac{5}{6}$ of what she cost him; how much did she cost him? Why?

97. A man bought a watch for 42 dollars, and sold it for $\frac{5}{6}$ of what it cost him; how much did he sell it for? Why?

98. A boy bought a knife, a handkerchief, and a book, the knife cost 35 cents, which was $\frac{5}{6}$ of what the handkerchief cost, and the price of the handkerchief was $\frac{3}{4}$ as much as the price of the book; what did the handkerchief, and what did the book cost?

99. A man bought a goose, a turkey, a sheep, and a pig. for the goose he gave 60 cents, which was $\frac{5}{6}$ of what he gave for the turkey:—the turkey cost $\frac{2}{5}$ as much as the sheep, and the sheep $\frac{3}{4}$ as much as the pig; what did the turkey, the sheep, and also the pig cost?

100. A man gave 35 cents for his breakfast, which was $\frac{5}{6}$ of what he gave for his dinner; what did he give for his dinner?

101. A man being asked how many sheep he had, answered, that he had them in 4 pastures, in the first, there were 48, $\frac{3}{4}$ as many in the second, which was $\frac{4}{5}$ of the number in the third, and that the number in the third was $\frac{5}{6}$ as many as were in the fourth; how many sheep were in the 2d, 3d, and 4th pastures, and how many had he altogether?

102. If a cocoa nut cost 21 cents, and it be cut into 7 equal pieces; what will one piece be worth? What is one of 7 equal parts called, 2 of 7 equal parts, 3, 4, 5, 6 of 7 equal parts?

103. What is $\frac{1}{7}$ of 21, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{6}{7}$ of 21?

104. How do you find one-seventh of any number? Ans. Divide it by 7, that is, find how many times 7 is contained in the number.

105. What is $\frac{1}{7}$ of 63, of 84, of 98, of 112, of 77, of 28, of 56, of 91?

106. What is $\frac{1}{7}$ of 35, $\frac{2}{7}$ of 35, $\frac{4}{7}$ of 42, $\frac{6}{7}$ of 56, $\frac{5}{7}$ of 84, $\frac{3}{7}$ of 91?

107. 12 is $\frac{1}{7}$ of what number, 9 is, 19 is, 16 is, 8 is, 25 is, 27 is $\frac{1}{7}$ of what number?

108. 28 is $\frac{4}{7}$ of what number, 36 is $\frac{6}{7}$ of what number? Why?

109. 48 is $\frac{3}{7}$ of what number, 45 is $\frac{5}{7}$ of what number?

Why? Since 45 is 5 parts of a certain number, $\frac{1}{5}$ of 45 must be 1 part, and as the number consists of 7 parts, 7 times $\frac{1}{5}$ of 45 is the number; now $\frac{1}{5}$ of 45 is 9, and 7 times $\frac{1}{5}$ of 45 is 7 times 9=63.

110. 24 is $\frac{3}{7}$ of what number, 28 is $\frac{4}{7}$ of, 40 is $\frac{5}{7}$ of, 60 is $\frac{3}{7}$ of what number?

111. If a bushel of apples cost 56 cents, what will $\frac{6}{7}$ of a bushel cost? Ans. 48 cents. Why? Since a bushel cost 56 cents, $\frac{1}{7}$ of a bushel will cost $\frac{1}{7}$ of 56 cents, and $\frac{6}{7}$ of a bushel will cost 6 times $\frac{1}{7}$ of 56 cents=6 times 8=48 cents.

112. If a yard of silk cost 98 cents, what will $\frac{4}{7}$ of a yard cost, $\frac{3}{7}$, $\frac{6}{7}$, $\frac{5}{7}$, $\frac{2}{7}$ of a yard?

113. Bought $\frac{3}{7}$ of a ton of hay, for 9 dollars, what is a ton worth at that rate?

114. Sold a watch for 45 dollars, which was only $\frac{5}{7}$ of what it cost me; what did it cost me? Why?

115. If a man can travel 20 miles in $\frac{4}{7}$ of a day, how many miles can he travel in a day, in 3 days, in 8 days?

116. Bought a calf for 8 dollars, which was $\frac{2}{7}$ of what I gave for a cow; what was the price of the cow?

117. A man being asked his age, said that the age of his youngest son was 12 years, which was $\frac{3}{7}$ of the age of his eldest son, and that his eldest son's age was $\frac{2}{7}$ of his own age; what was the age of the eldest son, and what was the father's age?

118. In a certain town there are 4 schools, in the first there are 27 scholars, which is $\frac{3}{4}$ of the number in the second, and the second contains only $\frac{4}{7}$ as many as the third, and the third but $\frac{3}{5}$ as many as the fourth; how many in the second, third, and fourth respectively?

119. John, William, Peter, and Oliver, gave some money to a beggar, John gave him 42 cents, William $\frac{2}{3}$ as much as John, Peter $\frac{3}{4}$ as much as William, and Oliver $\frac{4}{7}$ as much as Peter; how much did each of the three give him, and how much did all give him?

120. A man sold a yoke of oxen for 80 dollars, which was $\frac{5}{6}$ of what he gave for them, when he bought them he paid for them with sheep, at 4 dollars a-piece; how many sheep did he give?

121. If a citron melon cost 8 cents, and it be

cut into 8 equal parts, what will one piece be worth, what is one of 8 equal parts called, 3 of 8 equal parts, 5, 7 of 8 equal parts ?

122. How do you find one-eighth of any number ? Ans. Divide the number by 8, or find how many times 8 is contained in it.

123. What is one-eighth ($\frac{1}{8}$) of 16, of 24, of 72, of 96, of 56, of 48, of 32, of 136, of 120, of 64, of 40, of 80, of 144, of 176 ?

124. What is $\frac{3}{8}$ of 24, $\frac{5}{8}$ of 72, $\frac{7}{8}$ of 24, $\frac{3}{8}$ of 96, $\frac{3}{8}$ of 56, $\frac{3}{8}$ of 40 ?

125. 12 is $\frac{1}{8}$ of what number, 16 is, 24 is, 18 is, 15 is, 13, 9, 25 is $\frac{1}{8}$ of what number ?

126. 35 is $\frac{7}{8}$ of what number, 63 is, 91 is, 42, 84, 21 is ?

127. 45 is $\frac{5}{8}$ of what number, 54 is $\frac{3}{8}$ of what number, 84 is $\frac{3}{8}$ of what number, 25 is $\frac{5}{8}$, 40 is $\frac{5}{8}$, 65 is $\frac{5}{8}$, 15 is $\frac{3}{8}$ of what number ?

128. A man bought $\frac{7}{8}$ of a cord of wood for $3.50; how much was that a cord ?

129. Sold $\frac{3}{8}$ of a yard of silk for 48 cents; what is that a yard ?

130. If a bushel of wheat cost $1.20, what is $\frac{5}{8}$ of a bushel worth, $\frac{3}{8}$ of a bushel, $\frac{7}{8}$ of a bushel ?

131. Bought $\frac{7}{8}$ of a ton of logwood, for 28 dollars; what is that for a ton, for 5 tons, 7 tons ?

132. If $\frac{5}{8}$ of a yard of flannel cost 40 cents, what will $\frac{3}{4}$ of a yard cost, $\frac{1}{2}$ of a yard ?

133. If $\frac{2}{3}$ of a pound of coffee cost 16 cents, what will $\frac{5}{6}$ of a pound cost, $\frac{3}{8}$ of a pound, $\frac{3}{4}$ of a pound, $\frac{1}{2}$ of a pound ?

134. If 8 sheep cost 24 dollars, what part of 24 dollars will 1 sheep cost, 3 sheep, 7 sheep ?

135. If 8 oranges cost 40 cents, what will 3 oranges cost, 5 oranges, 7 oranges, 20 oranges ?

136. Sold a watch for 30 dollars, which was only $\frac{5}{8}$ of what it cost me when I bought it, I paid for it with silk at 3 dollars a yard; how many yards did I give ?

137. 24 is $\frac{3}{4}$ of how many times 4, how many times 8 ?

138. 21 is $\frac{7}{8}$ of how many times 6, how many times 8, 4, 12 ?

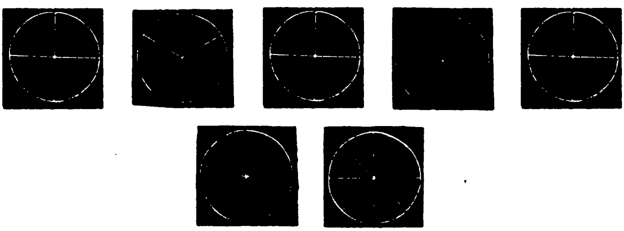
139. 30 is $\frac{5}{8}$ of how many times 12, 8, 6, 4 ?

140. 50 is $\frac{5}{7}$ of how many times 10, how many times 14?

141. Samuel, John, Thomas, William, and James were counting their marbles, found that Samuel had 24, which was but $\frac{2}{3}$ of the number John had, William had $\frac{5}{6}$ as many as John, but John had only $\frac{4}{5}$ as many as James, and James $\frac{3}{4}$ as many as Thomas; how many had each, and how many had they all together?

142. A man bought 8 yards of cloth, and sold it for 24 dollars which was only $\frac{3}{4}$ of what it cost him; how much did it cost him a yard?

143. What are we to understand by the terms one-half ($\frac{1}{2}$) one-third ($\frac{1}{3}$) two-thirds $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{6}{7}$, $\frac{1}{7}$, $\frac{1}{8}$, &c.? Ans. When any thing or any number is divided into two equal parts, one of those parts is called one-half of that thing or number. If it is divided into 3 equal parts, one of the parts is called one-third, and 2 of them two-thirds. If it is divided into 4 equal parts, one of the parts is called one-fourth ($\frac{1}{4}$), 2 of them $\frac{2}{4}$, 3 of them $\frac{3}{4}$. Go on and define, in like manner, all the above expressions.



144. If a watermelon be worth 18 cents, and it be cut into 9 equal parts, what will one part be worth; that is, what will one-ninth ($\frac{1}{9}$) be worth? What do you understand by one-ninth, $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, &c.? How many ninths make the whole of any thing or number?

145. How do you get $\frac{1}{9}$ of any number? Ans. Divide it by 9, or find how many nines are contained in it?

146. What is $\frac{1}{9}$ of 27, of 63, of 72, of 36; of 99, of 144, of 162, of 108, of 117, of 135, of 171?

147. What is $\frac{4}{9}$ of 72, $\frac{2}{9}$ of 63, $\frac{8}{9}$ of 27, $\frac{5}{9}$ of 45, $\frac{7}{9}$ of 108?

148. If 45 be $\frac{5}{9}$ of some number, what is that number?

149. 32 is $\frac{4}{9}$ of what number, is $\frac{8}{9}$ of what number?

150. If 20 be $\frac{5}{9}$ of some number, what is $\frac{3}{4}$ of that number, $\frac{2}{3}$ of that number, $\frac{1}{6}$ of that number?

151. If a bushel of potatoes cost 72 cents, what will $\frac{5}{8}$ of a bushel cost, $\frac{7}{8}$ of a bushel, $\frac{5}{8}$ of a bushel, $\frac{2}{3}$ of a bushel, $\frac{1}{2}$ of a bushel?

152. A man being asked the age of his wife, replied, that the age of his youngest son was 12 years, which was $\frac{2}{3}$ of the age of his daughter, and that his daughter's age was $\frac{3}{4}$ of the age of his eldest son, and that his eldest son's age was $\frac{4}{9}$ of his own age, and that his wife's age was $\frac{5}{6}$ of his own age, what was his wife's age?

153. Two boys talking of their ages, one said he was 15 years old, Well, said the other, $\frac{4}{5}$ of your age is just $\frac{2}{3}$ of my age, how old am I?

154. Two boys counting their money, one said he had 28 cents, Well, said the other, $\frac{3}{7}$ of your money is just $\frac{2}{9}$ of mine, how much have I?

155. 1 is what part of 2, of 3, of 4, of 5, of 6, of 7, of 8, of 9, of 10, of 11, of 12, of 13, of 14, of 15, of 16, of 17, of 18, of 19, of 20, &c., &c.

156. 3 is how many times 2? Ans. one and a half times 2; 5 are how many times 2, 7 are how many times 2, 9 are, 11, 13, 15, 17, 19?

157. 7 are how many times 3? Ans. 2 times 3 and $\frac{1}{3}$ of 3, 8 are 2 times 3, and $\frac{2}{3}$ of 3, 9 are how many times 3, 10 are, 11, 12, 13, 14 15, 16, 17, 18, 19, 20, 21, 22?

158. 8 are how many times 4, 9 are, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27?

159. 10 are how many times 5, 11 are, 12, 13, 14, 15, 16, 18, 27, 30, 32, 35, 38, 42, 75, 80, 87?

160. 12 are how many times 6, 13 are, 14, 15, 16, 17, 18, 19, 24, 29, 49, 75, 87, 96, 63, 83, 91, 97, 85?

161. How many times is 8 contained in 967464? We here find that 8 is contained in 9 once, leaving a remainder of 1, suppose this 1 placed to the left of the 6, making 16, we then find that 8 is contained in 16, 2 times, leaving no remainder, then finding that 8 is not contained in 7, we place

$$\begin{array}{r} 8)\underline{967464} \\ 120933 \end{array}$$

a 0 under the 7, and next find the number of 8s in 7 with the 4 joined to it, making 74, this contains 8, 9 times, leaving a remainder of 2, we place the 9 under the 4, and suppose the 2 placed to the left of the 6, making 26, this contains 8 3 times, leaving a remainder of 2, we place the 3 under the 6, and suppose the 2 placed to the left of the 4, making 24, this contains 8, 3 times, so we place the 3 under the 4.

162. How many times is 4 contained in 953612?

163. How many times is 12 contained in 678432?

164. How many times is 7 contained in 5947?

DIVISION.

By division we find how many times one number is contained in another. It is the *converse* of *multiplication.* The product and one factor being given, and the other resulting from the operation. By division we can also divide a number into as many equal parts as the number we divide by contains units;—the result of the operation expressing the value of one of those parts:—Thus, if we divide 24 by 6, the result is 4, one of the 6 equal parts which 24 can be divided into; then (4+4+4+4+4+4+) that is, 24 is divided into as many equal parts as (6) the number we divide by contains units.

The number to be divided is called the *dividend.*

The number we divide by is called the *divisor.*

The result of the operation is called the *quotient.*

When a remainder occurs in division it should be placed over the divisor:—Thus, if we divide 17 by 4 we find 17 contains 4, 4 times, and leaves a remainder of 1, which we place over the 4; thus, $\frac{1}{4}$.

$$\begin{array}{r} 4)17 \\ \hline 4\frac{1}{4} \end{array}$$

Such expressions as $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{1}{7}$, $\frac{3}{8}$, &c., are called *fractions*. The number above the line is called the *numerator* (to numerate, means to tell how many.) The number below the line is called the *denominator,* to denominate, means to give a name.

NOTE.—It is important that the learner should fix permanently in the mind the fact, that the *denominator* of a fraction *tells* the *number* of *parts* that *make* the *whole* of *any thing*, or of *any number*.

SHORT DIVISION, OR WHEN THE DIVISOR IS LESS THAN TWENTY.

RULE.

1st. Write down the divisor on the left of the dividend, draw a curved line between them, and a straight line *under* the *dividend*.

2d. Find how many times the divisor is contained in the left-hand figure or figures of the *dividend*, and place the figure denoting the number of times under the straight line, for the first figure of the quotient.

3d. If there be no remainder, divide the next figure of the dividend for the next figure of the quotient, but when a remainder occurs, suppose it placed to the left of the next figure of the dividend, and then proceed to find how many times the divisor is contained in these two, and thus proceed till all the figures of the dividend are used.

4th. After the *first* trial for each additional figure in the dividend, a *new* figure must be placed in the quotient, when one additional figure will not contain the divisor, 0 must be put in the quotient, and then another figure of the dividend must be brought in.

NOTE.—When there are ciphers on the right-hand of the *divisor*, cut them off and omit them in the operation; also cut off and omit the same number of figures from the right-hand of the *dividend*. The figures cut off from the dividend must be placed on the right of the remainder.

EXAMPLES.

Divide 697236 by 12.

$$\begin{array}{r} 12)\underline{697236} \\ 58103 \end{array}$$

Here we say, 69 contains 12, 5 times, and leaves a remainder of 9, which with the 7 to the right of it makes 97,—this contains 12, 8 times, and

leaves a remainder of 1, which we suppose placed to the left of the 2, making 12,—this contains 12, 1 time, leaving no remainder, we next bring in the 3, and find that it will not contain the 12, we therefore put down 0 and bring in the 6 to the right of the 3, making 36, which contains 12, 3 times.

	(1.)	(2.)	(3.)	(4.)
	Dividend			
Divisor	7)84632	9)67842	4)876232	8)6534576
Quotient	$12070\frac{2}{7}$			

(5.)	(6.)	(7.)	(8.)
5)76430	5)46735	3)24624	9)4776327

PROOF OF DIVISION.

Multiply the *quotient* by the *divisor*, and add in the remainder, or numerator, (if there be any,) the result will be equal to the dividend.

9. Divide 67428 by 9 and prove the operation.
10. Divide 368424 by 7 and prove the work.
11. Divide 474351 by 3 and prove the work.
12. Divide 86342 by 11 Ans. $7849\frac{3}{11}$
13. Divide 384798 by 12 Ans. $32066\frac{6}{12}$ $(\frac{6}{12})=\frac{1}{2}$
14. Divide 69832 by 13 Ans. $5371\frac{9}{13}$
15. Divide 3004616 by 8 Ans. 375577
16. Divide 3847625 by 4 Ans. $961906\frac{1}{4}$
17. Divide 253725 by 6 Ans. $42287\frac{3}{6}$ $(\frac{3}{6}=\frac{1}{2})$

WHEN THE DIVISOR CONTAINS SEVERAL FIGURES, WORK BY LONG DIVISION.

Suppose bank stock to the amount of $78376 is to be divided equally among 24 men, what will be the share of each man?

```
         Dividend. Quot.
Divisor 24)783768($32657
           72
           --
            63
            48
            ---
            157
            144
            ---
             136
             120
             ---
              168
              168
              ---
```

Here we first take the 78 as a partial dividend, and find that 24 is contained 3 times in the 78, we then place the 3 in the quotient, and say 3 times 24 make 72, which we place under the 78, and subtract therefrom, this leaves a remainder of 6, to the right of this 6 we place the 3, the figure to the right of the 8, we then find that 24 is contained in the 63, 2 times, we place the 2 in the quotient, to the right of the 3, and then multiply the 24 by it, this product is 48, which we subtract from the 63, this gives a remainder of 15, to the right of which we bring down the 7 in the dividend, making 157, and find it will contain 24, 6 times, place the 6 in the quotient and multiply the 24 by it, this gives 144, which subtracted from 157 leaves 13, to the right of which we bring down the 6, making 136, this contains 24, 5 times, place the 5 in the quotient, and multiply the 24 by it, this gives 120, which we place under the 136 and subtract it therefrom, this leaves 16, to the right of which we bring down the 8, (the last figure of the dividend,) making 168, this will contain 24 7 times, we place the 7 in the quotient and multiply the 24 by it, and find the product to be 168, which we place under the 168, this subtracted leaves nothing.

You may now work by the following

RULE.

1st. Write down the divisor on the left of the dividend, draw a curved line between them, and also a curved line on the right of the dividend.

2d. Note the fewest figures of the dividend, counted from the left-hand that will contain the divisor; find how often they contain it, and set the figure in the quotient.

3d. Multiply the whole divisor by this figure; set the

product under the first figures of the dividend, and subtract it from them.

4th. To the remainder annex the next figure of the dividend, then find how often the divisor is contained in this new number, and set the figure in the quotient.

5th. Multiply the whole divisor by the last figure of the quotient, and subtract the product from the last number containing the divisor. To the remainder annex the next figure of the dividend, and find the figures of the quotient in the same way, till all the figures of the dividend are brought down.

```
      (2.)                                     (3.)
19)46832 ( 2464 16/19                29)764532(22918 10/29
   38           19                      58
   --         -----                     --
    88        22176                      84
    76        2464                       58
   ---           16                     ---
    123       -----                     265
    114       46832 Proof               261
    ---                                 ---
      92                                  43
      76                                  19
      --                                  ---
      16                                  242
                                          232
                                          ---
                                           10
```

4.	Divide	946238	by	37	Ans.	25574
5.	Divide	16234918	by	46	Ans.	352933
6.	Divide	186712	by	74	Quotient	$2523\frac{5}{37}$
7.	Divide	138756	by	62	Quotient	2238
8.	Divide	296268	by	84	Quotient	3527
9.	Divide	6716394	by	94	Quotient	71451
10.	Divide	4586841	by	3467	Quotient	1323
11.	Divide	2303	by	49	Quotient	47
12.	Divide	40231	by	75	Quotient	$536\frac{31}{75}$
13.	Divide	1264384	by	346	Quotient	$3654\frac{100}{346}$

NOTE.—In such examples as the following, omit the ciphers on the right of the divisor, and cut off the same number of figures from the right of the dividend.

EXAMPLES.

(1.)

```
19|00)402|20(21
   38
   --
    22
    19
    ---
    320 Remainder.
```

(2.)

```
386|0)16404|30(42 quotient.
      1544
      ----
       964
       772
       ----
       1923 Remainder.
```

(3.)

```
365|00)17841|23(48
       1460
       ----
        3241
        2920
        -----
         32123 Remainder.
```

(4.)

```
46|000)2462|312(53
       230
       ---
       162
       138
       -----
        24312 Remainder.
```

5. Divide	632410	by	19600	Quot. 32	Rem.	5210
6. Divide	374123	by	271000	Quot. 1	Rem.	103123
7. Divide	1962473	by	31200	Quot. 62	Rem.	28073
8. Divide	1739621	by	61000	Quot. 28	Rem.	31621

1. 16 ounces make a pound, then how many pounds will 192 ounces make? Ans. 12 pounds. Why? Since 16 ounces make one pound, as many times 16 as there are in 192 so many pounds 192 ounces will make.

2. Suppose a man has an income of $8760 a year, how much is that a day, there being 365 days in a year? Ans. 24 dollars. Why? If his yearly income were 365 dollars, it would evidently be one dollar a day, then as many times 365 as there are in 8760, so many dollars his income must be for each day.

3. Suppose a man earns 35 dollars a month, how many months will it take him to earn 490 dollars? Ans. 14 months. Why? Since it takes him one month to earn 35 dollars, it will take him as many months to earn 490 as there are times 35 in 490.

4. If 63 gallons of water will fill a hogshead, how many hogsheads will 5166 gallons fill? Ans. 82. Why? Since

63 gallons will fill one hogshead, as many times 63 as there are in 5166 so many hogsheads will 5166 gallons fill.

5. Suppose a man can travel 36 miles in a day, how many days will he require at that rate, to travel 2592 miles? Ans. 72 days. Why? Since he travels 36 miles in one day, it will require as many days to travel 2592 miles as there are times 36 in 2592.

6. A farmer has $756, with which he wishes to purchase cows, the price being 28 dollars a head, how many cows can he buy? Ans. 27. Why?

7. Suppose a man has a journey of 364 miles to travel, and can only go 14 miles a day, how many days will he require? Ans. 26. Why?

8. If 12 oranges cost 72 cents, what will one cost? Why? If 12 oranges cost 72 cents, one orange will cost one-twelfth ($\frac{1}{12}$) of 72 cents. What is the meaning of $\frac{1}{12}$? Ans. If any thing, or any number be divided into 12 equal parts, one of these parts is called one-twelfth ($\frac{1}{12}$).

9. How do you find $\frac{1}{12}$ of any number? Ans. Divide it by 12. How many twelfths make the whole?

10. How do you find one-thirteenth ($\frac{1}{13}$), $\frac{1}{14}$, $\frac{1}{15}$, $\frac{1}{16}$, $\frac{1}{17}$, $\frac{1}{18}$, $\frac{1}{19}$, &c., of any number; what is to be understood by the expressions $\frac{1}{13}$, $\frac{1}{14}$, $\frac{1}{15}$, $\frac{1}{16}$, $\frac{1}{17}$, $\frac{1}{18}$, $\frac{1}{19}$?

NOTE.—It is highly important that the learner should fully comprehend the relation which addition and subtraction, as well as that which multiplication and division have to each other: the following questions will serve to show the utility of observing this relation. Let the pupil understand that wherever a number has been multiplied by another, by dividing the *result* by the *multiplier*, the effect produced by the multiplication is destroyed, and *vice versa*, and that the same is also true in reference to addition and subtraction, and he will readily solve, not only the questions here given, but the solution of many others, which would otherwise appear quite difficult, will become obvious to any one who has been *taught* to *reason*, even in a slight degree.

11. A school teacher being asked how many scholars he had, said, if you add 24 to the number, and multiply this sum by 18, and then subtract 126 from the product, and divide the remainder by 12, the quotient will be 84; how many scholars had he? Ans. 39 scholars.

Now in order to solve the above question we commence with the 84, which is the result of division, we will destroy that result by performing multiplication (the converse of division) with the same number, which has been used as a *divisor*, that is, we will change the divisor (12) into a multiplier, thus, $84\times12=1008$; we again look at the question, and perceive that the next preceding operation in its frame-work is subtraction, the result of which we will destroy by *addition*, thus, $1008+126=1134$. The operation preceding subtraction, you will observe, is multiplication, the effect of which must be destroyed by *division*, thus, $\frac{1134}{18}=63$. Look once more at the question and you perceive that addition precedes the multiplication, subtraction of course will destroy this, thus, $63-24=39$. Having now destroyed the effect of every operation in the frame-work of the *question*, the number which remains must be that on which the first operation was supposed to be performed, viz., the number of scholars, which is the answer to the question.

12. John had a certain number of marbles, another boy gave him 16 more, then William said *he* had 24 times as many as John, and that if he were to give 48 of them to Thomas, and divide the remainder of *his* marbles among 36 boys, each of them would have 32; how many had John?

Ans. 34.

Explanation.—Here we will commence with 32, which you perceive is the result of division; this result multiplication will destroy, hence we change the divisor 36 into a multiplier, thus, $32\times36=1152$. Look again at the question and you perceive that the next preceding operation, viz. (that of giving 48 to Thomas) is subtraction, addition will destroy this, thus, $1152+48=1200$; again look at the question and you perceive that the expression, "William said he had 24 times as many," implies multiplication, division will destroy the result of this, thus, $\frac{1200}{24}=50$. Now look once more at the question, and you perceive the expression "Another boy gave him 16," implies addition, this subtraction will destroy, thus, $50-16=34$ marbles, the number John had.

13. A man driving his geese to market was met by another, who asked him how many he had; he replied, If I should sell 64 and multiply the remaining number by 14, and then

add 98 to the product and divide the sum by 16, the quotient will be 91; how many had he? Ans. 161 geese.

14. A quantity of dry goods were shipped to California, and sold for 16 times as much as they cost, after paying 496 dollars for freight, &c., the money received for them was divided equally among a company of 12 men, when each man received 3640 dollars; what was the first cost of the goods? Ans. 2761 dollars.

15. A man being asked how many sheep he had, said, If you add 124 to the number and divide the sum by 38, and then multiply the quotient by 26, the product will be 234; how many had he? Ans. 218.

16. Suppose I borrow from A., as much money as I have in my purse, (multiply by 2,) and then spend 56 cents, after this I borrow of B. as much as I have remaining, and then spend 72 cents, again I borrow as much as I have remaining and then spend 84 cents, and find I have 28 cents left; how much had I at first? Ans. 60 cents.

17. A man being asked how old he was, answered, if the years I have lived be multiplied by 12, and 84 be added to the product, and this sum be divided by 14, the quotient will be 36; what was his age? Ans. 35 years.

18. A gentleman bought a horse, which he sold for $24 less than he paid for him, (subtraction,) he then took the money which he received for the horse, and invested it in a speculation, by which he doubled (multiply by 2) the money invested, to this sum he added $60, and invested the amount in a like speculation, and again doubled his money, out of this sum he reserved $160, (subtraction,) and divided the remainder equally among his 4 daughters, the share of each being $196; what was the price he paid for the horse? Ans. $230.

19. 95 is $\frac{5}{7}$ of what number?

20. If $\frac{5}{7}$ of a certain number be multiplied by 28, and 164 be subtracted from the product, and the remainder be divided by 13, the quotient will be 192; what is the number? Ans. 133.

21. A boy engaged in planting corn, was asked by another boy, how many grains he put in each row of hills across the field, answered thus, If you multiply the number

of grains in a row by 32, and add 276 to the product, and then divide the sum by 54, and subtract 174 from the quotient, the remainder will be 96; can *you* tell how many grains this would make to each row? Ans. 447.

FRACTIONS.

A fraction signifies one or more of the equal parts into which a unit, or some number considered as an integer or whole is divided.

In one sense *any* number may be regarded as a *fraction*, and in another as an *integer* or *whole*.

Thus, as a measure of length, when we speak of an inch, we regard it as an *integer*, but if we make use of the *foot*, as the measuring unit, the *inch* being one of the 12 equal parts into which the foot is divided, becomes a fraction, and 5 inches being 5 of the equal parts, is also a fraction, &c. If we take the *yard* as the measuring unit, then the foot (being one of the 3 equal parts into which the yard is divided) becomes a fraction. Again, if we take the *furlong* as the measuring unit, then the yard (being one of the 220 equal parts into which the *furlong* is divided) becomes a fraction, while at the same time if the *mile* be taken as the measuring unit, then the furlong (being one of the 8 equal parts into which the mile is divided) becomes a fraction, and if the league be regarded as the unit, then the mile in turn becomes a fraction, &c., &c.

If any thing or any number be divided into 2 equal parts, one of these parts is called one-half of that thing or number. If into 3 equal parts, one of the parts is called one-third. If into 4 equal parts, one-fourth. If into 5 equal parts, one is called one-fifth. If into 12 equal parts, one of these parts is called one-twelfth; and for any number of parts we have a similar expression.

To express fractions by figures, we must use two numbers, with a line drawn between them, one to show into *how many*

parts the *integer* or *whole*, is to be divided, and the other to show how many of those parts are used.

One-half is written thus,			$\frac{1}{2}$	Two-fifths is written thus,			$\frac{2}{5}$
One-third	"	"	$\frac{1}{3}$	One-sixth	"	"	$\frac{1}{6}$
Two-thirds	"	"	$\frac{2}{3}$	Three-sevenths	"	"	$\frac{3}{7}$
One-fourth	"	"	$\frac{1}{4}$	Seven-eighths	"	"	$\frac{7}{8}$
Three-fourths	"	"	$\frac{3}{4}$	Seven-twelfths	"	"	$\frac{7}{12}$
One-fifth	"	"	$\frac{1}{5}$	Nine-sixteenths	"	"	$\frac{9}{16}$

The number below the line is called the *denominator*, because it gives name to the fraction, or *shows into how many parts* the integer is divided.

The number above the line is called the *numerator*, because it shows how many parts are used.

Thus, $\left\{\begin{array}{ll} \text{numerator} & 7 \\ \text{denominator} & \overline{16} \end{array}\right.$

A proper fraction is one whose numerator is less than the denominator; as, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{9}$, &c.

An improper fraction is one whose numerator is greater *than*, or equal *to*, the denominator; as, $\frac{7}{4}$, $\frac{4}{4}$.

A mixed number consists of a whole number and a fraction; as, $2\frac{1}{2}$, $3\frac{1}{4}$, $7\frac{3}{4}$, $6\frac{2}{3}$, &c.

A compound fraction is a fraction of a fraction, connected by the word of; as, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{1}{2}$.

Mixed numbers may be expressed in the form of improper fractions, thus, suppose you have 2 apples, and half of another apple, and you divide the 2 whole apples into halves; how many half apples would you then have? If you had three whole apples and one half apple, how many half apples would they all make? Ans. 7 half apples. Why? Since one whole apple will make 2 half apples, 3 whole apples will make 3 times 2 or 6 half apples, and one half added to 6 halves, will make 7 halves; expressed in figures $\frac{7}{2}$ apples.

Or you may say, since one whole apple makes 2 half apples, there must be twice as many half apples as there are whole ones.

How many halves in $4\frac{1}{2}$, in $8\frac{1}{2}$, in $7\frac{1}{2}$, in $9\frac{1}{2}$, in $12\frac{1}{2}$? Why?

How many thirds of an apple would you have if you had 4 whole apples, and one-third of another apple? Ans. 13 thirds. Why? Since one whole apple will make three-thirds, there must be three times as many thirds as whole apples,—3 times 4 make 12, and one-third added to 12 makes 13 thirds; thus, $4\times3+1=\frac{13}{3}$.

How do you change a mixed number to an improper fraction? Ans. multiply the whole number by the denominator of the fraction and add the numerator to the product, which place over the denominator.

EXAMPLES.

Change $8\frac{3}{4}$ to an improper fraction.

(1.)

$$\begin{array}{lc}
\text{Thus,} & 8\tfrac{3}{4} \\
 & 4 \\
\text{numerator} & \overline{35} \\
\text{denominator} & \overline{\ 4\ }
\end{array}\Bigg\}\ \text{improper fraction.}$$

2. Change $17\frac{2}{3}$ to thirds. Ans. $\frac{53}{3}$.
3. Change $47\frac{4}{5}$ to an improper fraction. Ans. $\frac{239}{5}$.
4. Change $19\frac{3}{8}$ to an improper fraction. Ans. $\frac{155}{8}$.
5. Change $29\frac{2}{3}$ to an improper fraction. Ans. $\frac{89}{3}$.
6. Change $37\frac{2}{9}$ to ninths. Ans. $\frac{335}{9}$.
7. Change $11\frac{1}{2}$ to halves. Ans. $\frac{23}{2}$.
8. Change $46\frac{5}{12}$ to twelfths. Ans. $\frac{557}{12}$.
9. Change $87\frac{1}{2}$ to halves.
10. Change $17\frac{3}{13}$ to thirteenths. Ans. $\frac{224}{13}$.
11. Change $8\frac{7}{16}$ to sixteenths. Ans. $\frac{135}{16}$.
12. Change $25\frac{7}{11}$ to an improper fraction. Ans. $\frac{282}{11}$.
13. Change $87\frac{2}{3}$ to an improper fraction.
14. Change $143\frac{5}{6}$ to an improper fraction. Ans. $\frac{863}{6}$.
15. Change $9\frac{6}{7}$ to an improper fraction.
16. Change $15\frac{1}{2}$ to an improper fraction.
17. Change $76\frac{11}{17}$ to an improper fraction. Ans. $\frac{1303}{17}$.
18. Change $24\frac{7}{8}$ to an improper fraction.
19. Change $128\frac{3}{5}$ to fifths.

20. How many dollars will 12 half dollars make? Ans. 6. Why? Since it takes 2 half dollars to make one whole dollar, as many times 2 as there are in 12, so many dollars 12 half dollars will make.

21. In $\frac{28}{3}$ how many units? Ans. $9\frac{1}{3}$. Why? Since it takes 3 thirds to make a unit, as many times three as there are in 28, so many units $\frac{28}{3}$ will make.

22. How do you change improper fractions to whole or mixed numbers? Ans. Divide the numerator by the denominator, and if there be a remainder place it over the denominator, at the right-hand of the integer or whole number.

23. In $\frac{53}{3}$ how many units?	Ans.	$17\frac{2}{3}$.
24. Change $\frac{35}{4}$ to a mixed number.	Ans.	$8\frac{3}{4}$.
25. Change $\frac{239}{5}$ to a mixed number.	Ans.	$47\frac{4}{5}$.
26. Change $\frac{155}{8}$ to a mixed number.	Ans.	$19\frac{3}{8}$.
27. Change $\frac{89}{3}$ to a mixed number.	Ans.	$29\frac{2}{3}$.
28. Change $\frac{335}{9}$ to a mixed number.	Ans.	$37\frac{2}{9}$.
29. In $\frac{63}{7}$ how many units?	Ans.	9.
30. How many units in $\frac{557}{12}$?	Ans.	$46\frac{5}{12}$.
31. Change $\frac{224}{13}$ to a mixed number.	Ans.	$17\frac{3}{13}$.
32. Change $\frac{23}{2}$ to a mixed number.	Ans.	$11\frac{1}{2}$.
33. Change $\frac{135}{16}$ to a mixed number.	Ans.	$8\frac{7}{16}$.
34. Change $\frac{282}{11}$ to a mixed number.	Ans.	$25\frac{7}{11}$.
35. Change $\frac{863}{6}$ to a mixed number.	Ans.	$143\frac{5}{6}$.
36. Change $\frac{1303}{17}$ to a mixed number.	Ans.	$76\frac{11}{17}$.
37. Change $\frac{69}{7}$ to a mixed number.	Ans.	$9\frac{6}{7}$.
38. Change $\frac{199}{8}$ to a mixed number.	Ans.	$24\frac{7}{8}$.
39. Change $\frac{31}{2}$ to a mixed number.	Ans.	$15\frac{1}{2}$.

40. If $2\frac{1}{3}$ yards of cloth will make a coat, how many yards will be required to make 7 coats? Ans. $16\frac{1}{3}$ yards. Why? Since it takes $2\frac{1}{3}$ yards to make one coat, it will take 7 times $2\frac{1}{3}$ yards to make 7 coats, 7 times 2 yards make 14 yards, and 7 times $\frac{1}{3}$ makes $\frac{7}{3}=2\frac{1}{3}$ yards, which added to 14 yards, make $16\frac{1}{3}$ yards.

A better method is to change the mixed number to an improper fraction, and place the numerator on the right of a vertical line, thus, $2\frac{1}{3}=\frac{7}{3}$.

Here we say, 7 times 7 parts make 49 parts, that is, $\frac{49}{3}$, and as three thirds make a unit, we say 3 in 49, $16\frac{1}{3}$ times.

$$\begin{array}{r|r} 3 & 7 \\ & 7 \\ \hline & 3)49 \\ \hline & 16\frac{1}{3} \text{ yards.} \end{array}$$

41. How many are 12 times $7\frac{1}{2}$, thus, $7\frac{1}{2}=\frac{15}{2}$.

Here we say, $7\frac{1}{2}=\frac{15}{2}$, then we place the numerator on the right of the line, and the denominator on the left, then instead of multiplying 15 by 12, and dividing the product by 2, we divide the 12 by 2, and multiply the 15 by 6.

$$\begin{array}{r|r} \cancel{2} & 15 \\ & \cancel{12} \\ & 6 \\ \hline & 90 \end{array}$$

42. How many are 16 times $29\frac{3}{4}$? $29\frac{3}{4}=\frac{119}{4}$.

43. Multiply $47\frac{2}{3}$ by 18.

Then $\begin{array}{r|r} \cancel{4} & 119 \\ & \cancel{16} \\ & 4 \\ \hline & 476 \end{array}$ $47\frac{2}{3}=\frac{143}{3}$ Then $\begin{array}{r|r} \cancel{3} & 143 \\ & \cancel{18} \\ & 6 \\ \hline \text{Ans.} & 858 \end{array}$

44. Multiply $17\frac{4}{5}$ by 35.

$17\frac{4}{5}=\frac{89}{5}$ Then $\begin{array}{r|r} \cancel{5} & 89 \\ & \cancel{35} \\ & 7 \\ \hline \text{Ans.} & 623 \end{array}$

45. How many are 24 times $\frac{7}{8}$?

46. How much is $\frac{7}{8}$ of 24?

Here 24 is supposed to be divided into 8 equal parts, and 3 must be one of those parts, and 7 times 3, 7 of those parts, $=21$, or we may say, 24 times 7 parts make 168, that is, $\frac{168}{8}=21$.

$$\begin{array}{r|r} \cancel{8} & 7 \\ & \cancel{24} \\ & 3 \\ \hline \text{Ans.} & 21 \end{array}$$ Why?

How do you multiply a fraction by a whole number? Ans. Place the numerator of the fraction on the right of a vertical line, and the denominator on the left; place the multiplier also on the right of the line, and divide the product of the numbers on the right, by the number on the left, or

when the number on the left will measure either of those on the right of the line divide first.

NOTE.—When mixed numbers are to be multiplied they must first be changed to improper fractions.

47. Multiply $7\frac{3}{4}$ by 12.

Here we say, $7\frac{3}{4}=31$ being numerator, we place it on the right of the vertical line, and the 4, (denominator,) on the left, the 12, (multiplier,) also on the right, in the first process we say, 4 into 12, 3 times, then 3 times 31 make 93, in the other we say, 12 times 31 make 372, then 4 in 372 goes 93 times, the result being the same in both cases.

$\frac{31}{4}$ numerator. / denominator.

$$\begin{array}{r|l} \cancel{4} & 31 \\ & \cancel{12} \\ & 3 \\ \hline & 93 \end{array} \qquad \text{Or,} \quad \begin{array}{r|r} \cancel{4} & 31 \\ & 12 \\ \hline & 372 \\ \hline & 93 \end{array}$$

48. 27 times $8\frac{7}{9}$ are how many?

49. 46 times $4\frac{3}{5}$ are how many?

What is to be understood by one number measuring another? Ans. One number is said to measure another when it will divide into it without leaving any remainder.

Here it is best to multiply first, as the 5 will not measure either of the numbers on the right of the line.

$$\begin{array}{r|r} \cancel{9} & 79 \\ & \cancel{27} \\ & 3 \\ \hline \text{Ans.} & 237 \end{array}$$

$$\begin{array}{r|r} 5 & 23 \\ & 46 \\ \hline \text{Ans.} & 1058 \\ \hline & 211\frac{3}{5} \end{array}$$

50. Multiply $23\frac{3}{4}$ by 28 — Ans. 665.

51. Multiply $16\frac{2}{3}$ by 15 — Ans. 250.

52. Multiply $17\frac{7}{8}$ by 48 — Ans. 858.

53. Multiply $6\frac{6}{7}$ by 34 — Ans. $233\frac{1}{7}$.

54. Multiply $13\frac{7}{12}$ by 72 — Ans. 978.

55. Multiply $29\frac{7}{8}$ by 56 — Ans. 1673.

56. Multiply $19\frac{4}{13}$ by 26 — Ans. 502.

57. Multiply $37\frac{7}{9}$ by 27 — Ans. 1020.

58. Multiply $23\frac{4}{5}$ by $3\frac{4}{7}$ — Ans. 85.

Here we say, $29\frac{4}{5}=\frac{119}{5}$, and $3\frac{4}{7}=\frac{25}{7}$, then place the numerators on the right of the line, and denominators on the left.

Thus,
$$\begin{array}{r|rr} \cancel{5} & \cancel{119} & 17 \\ \cancel{7} & \cancel{25} & 5 \\ \hline \text{Ans.} & & 85 \end{array}$$

How do you multiply one fraction by another? Ans. Place the *numerators* on the *right*, and the *denominators* on the *left* of a *vertical line*, and divide the product of the numbers on the right, by the product of those on the left, or when a number on either side of the line will measure one on the other side, divide first.

59. Multiply $32\frac{3}{5}$ by $4\frac{3}{8}$

$$\begin{array}{r|r} \cancel{5} & 163 \\ 8 & \cancel{35} \\ & 7 \\ \hline & 1141 \\ \hline \text{Ans.} & 142\frac{5}{8} \end{array}$$

60. Multiply $7\frac{2}{9}$ by $3\frac{1}{4}$

$$\begin{array}{r|l} 9 & 65 \\ 4 & 13. \\ \hline 36 & 845(23\frac{17}{36}=\text{Ans.} \\ & 72 \\ \hline & 125 \\ & 108 \\ \hline & 17 \end{array}$$

61. Multiply $6\frac{2}{3}$ by $9\frac{3}{4}$

$$\begin{array}{r|rr} \cancel{3} & \cancel{20} & 5 \\ \cancel{4} & \cancel{39} & 13 \\ \hline \text{Ans.} & & 65 \end{array}$$

62. Multiply $4\frac{4}{9}$ by $5\frac{2}{5}$ Ans. 24.
63. Multiply $18\frac{2}{3}$ by $5\frac{1}{7}$ Ans. 96.
64. Multiply $14\frac{5}{6}$ by $8\frac{2}{5}$ Ans. $124\frac{3}{5}$.
65. Multiply $47\frac{1}{2}$ by $7\frac{3}{5}$ Ans. 361.
66. Multiply $11\frac{2}{3}$ by $6\frac{3}{7}$ Ans. 75.
67. Multiply $65\frac{2}{3}$ by $27\frac{7}{12}$ Ans. $1811\frac{11}{36}$.
68. Multiply $37\frac{1}{2}$ by $26\frac{4}{15}$ Ans. 985.
69. Multiply $47\frac{6}{7}$ by $16\frac{4}{5}$ Ans. 804.

$811\frac{1}{5}$

70. Multiply $19\frac{1}{6}$ by $9\frac{3}{7}$ Ans. $180\frac{5}{7}$.
71. What is the product of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{8}{9}$? Ans. $\frac{14}{45}$.

Here observe we place all the numerators, viz., 3, 4, 2, 7, and 8, on the right of the line, and all the denominators, viz., 4, 5, 3, 8, and 9, on the left, there being a 4, 3, and an 8, on each side of the line, we cancel these, and have 2 and 7 to multiply together, for a numerator, and 5 and 9, on the left of the line for a *denominator*.

$$\begin{array}{r|l}
\cancel{4} & \cancel{3} \\
5 & \cancel{4} \\
\cancel{3} & 2 \\
\cancel{8} & 7 \\
9 & \cancel{8} \\
\hline
 & 14 \text{ numerator.} \\
\text{Ans} & \overline{45} \text{ denominator.}
\end{array}$$

72. What is the product of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $3\frac{1}{2}$?
Ans. $\frac{6}{5}=1\frac{1}{5}$.
73. What is the product of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{8}{9}$ of $\frac{3}{7}$ of $4\frac{1}{2}$?
Ans. $\frac{3}{7}$.
74. What is the product of $\frac{7}{12}$ of $\frac{4}{7}$ of $\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{6}{11}$ of $3\frac{2}{3}$?
Ans. $\frac{4}{9}$.
75. Multiply $6\frac{2}{3}$ by $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{5}{6}$ of $7\frac{1}{2}$.
Ans. $\frac{105}{4}=26\frac{1}{4}$.
76. What will $9\frac{3}{4}$ yards of muslin cost at $9\frac{1}{3}$ cents a yard?
Ans. 91 cents.
77. What will $7\frac{1}{7}$ cords of wood cost at $4\frac{1}{5}$ dollars a cord?
Ans. \$30.
78. What will $7\frac{1}{5}$ pounds of sugar cost at $8\frac{3}{4}$ cts. a pound?
Ans. 63 cents.

DIVISION OF FRACTIONS.

We will now presume the learner to understand division to be the *converse* of multiplication, and he will need but little explanation to enable him to divide numbers involving fractions, with the same facility as to perform the like operation on numbers in their most simple form.

Suppose you be required to multiply $23\frac{4}{5}$ by 13, you will proceed thus,

$$\begin{array}{r|l} 5 & 119 \\ & 13 \\ \hline & 1547 \\ \text{Ans.} & 309\frac{2}{5} \end{array}$$

That is, you change the $23\frac{4}{5}$ to an improper fraction $=\frac{119}{5}$, and place the numerator (119) on the right of a vertical line, and the denominator (5) on the left, the multiplier (13) you also place on the right of this line, &c.

Now suppose you are required to divide $309\frac{4}{5}$ by 13, here as in *multiplication* the $309\frac{4}{5}$ must be changed to an improper fraction $=\frac{1547}{5}$, the numerator of which we place on the right of a vertical line, and the denominator on the left, as in multiplication, but the *divisor* (13) we place on the left of this line, and then proceed as in multiplication of fractions. Compare this with the preceding question, and you will perceive that the operation has destroyed the result of multiplication; this then is division.

$$\begin{array}{r|l} \text{Thus, } 5 & \cancel{1547} \\ \cancel{13} & 119 \\ \hline \text{Result} & 23\frac{4}{5} \end{array}$$

How do you *divide* a fraction? Ans. Place the *numerator* of the *dividend* on the right of a vertical line, and its *denominator* on the left, then place the divisor on the left of this line, and proceed as in multiplication. If the divisor be a fraction, place its *numerator* on the left side of the line.

NOTE.—Observe that the *denominator* must never occupy the same side of the line as its *numerator*, hence, if you recollect on which side of the line to *place* the numerator, you will readily dispose of the denominator, as it must occupy the opposite side.

1. Divide $18\frac{3}{4}$ by $6\frac{1}{4}$

Here we say, $18\frac{3}{4}=\frac{75}{4}$, this being the dividend, we place its numerator, 75, on the right of the line, (and of course the denominator (4) on the opposite side,) the $6\frac{1}{4}=\frac{25}{4}$ being the divisor, its *numerator* must be placed on the left of this line.

$$\begin{array}{r|l} \cancel{4} & \cancel{75}\ 3 \text{ Ans.} \\ \cancel{25} & \cancel{4} \end{array}$$

2. Divide $17\frac{2}{5}$ by 29.

$$\text{Thus, } 17\frac{2}{5}=\frac{87}{5}\text{ (num.), then,}\quad \begin{array}{r|l} 5 & \cancel{87} \\ \text{divisor } \cancel{29} & 3 \text{ numerator.} \\ \hline \text{Ans.} & 5 \text{ denominator.} \end{array}$$

3. Divide $37\frac{2}{3}$ by 4.

Here we have 3 and 4 on the left of the line, and as neither of them will measure 113, we multiply them together, and then say, 12 will go into 113, $9\frac{5}{12}$ times.

$$\begin{array}{r|l} 3 & 113 \\ 4 & \overline{9\frac{5}{12}} \text{ quotient.} \\ \hline 12 & \end{array}$$

4. Divide $13\frac{1}{3}$ by $4\frac{1}{5}$ $\left\{\begin{array}{l} 13\frac{1}{3}=\frac{40}{3} \\ 4\frac{1}{5}=\frac{21}{5} \end{array}\right\}$

$$\text{Then } \begin{array}{r|l} 3 & 40 \\ 21 & 5 \text{ Quot.} \\ \hline 63 & 200(3\frac{11}{63} \\ & 189 \\ \hline & 11 \end{array}$$

5. Divide $28\frac{1}{3}$ by $3\frac{2}{5}$ $\left\{\begin{array}{l} 28\frac{1}{3}=\frac{85}{3} \\ 3\frac{2}{5}=\frac{17}{5} \end{array}\right\}$

$$\text{Then } \begin{array}{r|l} 3 & \cancel{85} \\ \cancel{17} & 5 \\ & 5 \\ \hline & 25 \\ \hline & 8\frac{1}{3} \end{array}$$

6. Divide $47\frac{1}{2}$ by $3\frac{1}{6}$ $\left\{\begin{array}{l} 47\frac{1}{2}=\frac{95}{2} \\ 3\frac{1}{6}=\frac{19}{6} \end{array}\right\}$

$$\text{Then } \begin{array}{r|l} \cancel{2} & \cancel{95} \\ \cancel{19} & \quad 5 \\ & \cancel{6}\ 3 \\ \hline & 15 \text{ Quotient.} \end{array}$$

7. Divide $29\frac{4}{7}$ by 9

$$\begin{array}{r|l} 7 & \cancel{207} \\ \cancel{9} & 23 \\ \hline & 3\frac{2}{7} \text{ Quotient.} \end{array}$$

8. Divide $14\frac{2}{3}$ by 11. Ans. $1\frac{1}{3}$.
9. Divide $35\frac{3}{4}$ by 3. Ans. $11\frac{11}{12}$.
10. Divide $23\frac{3}{4}$ by 5. Ans. $4\frac{3}{4}$.
11. Divide $43\frac{5}{6}$ by $2\frac{2}{3}$. Ans. $16\frac{7}{16}$.
12. Divide $29\frac{1}{4}$ by $4\frac{1}{2}$. Ans. $6\frac{1}{2}$.
13. Divide $79\frac{1}{5}$ by $3\frac{2}{3}$. Ans. $21\frac{3}{5}$.
14. Divide $16\frac{1}{9}$ by $6\frac{2}{3}$. Ans. $2\frac{5}{12}$.
15. Divide $79\frac{7}{8}$ by $53\frac{1}{4}$. Ans. $1\frac{1}{2}$.
16. Divide $\frac{7}{11}$ of $\frac{4}{5}$ by $\frac{3}{5}$ of $\frac{8}{11}$.

$$\text{Thus, } \begin{array}{r|l} \cancel{11} & 7 \\ \cancel{5} & \cancel{4} \\ 3 & \cancel{5} \\ \cancel{8} & \cancel{11} \\ 2 & \\ \hline & 1\frac{1}{6} \end{array}$$

17. Divide $11\frac{3}{7}$ by $\frac{4}{7}$ Ans. 20
18. Divide $9\frac{1}{2}$ by $3\frac{1}{6}$ Ans. 3
19. Divide $\frac{2}{3}$ of $\frac{3}{4}$ of $5\frac{1}{3}$ by $\frac{2}{5}$ of $\frac{4}{7}$ Ans. $11\frac{2}{3}$
20. Divide $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{7}{8}$ by $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{5}{8}$ Ans. $2\frac{2}{5}$

Observe that we place all the numerators of the dividend on the right of the line, and all those of the divisor on the left.

Solution.

$$\begin{array}{r|ll} \cancel{5} & 4 & \\ \cancel{7} & \cancel{5} & \\ \cancel{8} & \cancel{7} & \\ 1 & \cancel{2} & \\ \cancel{3} & \cancel{4} & \\ \cancel{8} & \cancel{9} & 3 \\ 5 & \cancel{8} & \\ \hline & & 12 \end{array}$$

$$2\tfrac{2}{5}$$

21. Divide $\frac{3}{4}$ of $\frac{8}{9}$ of 37 by $\frac{2}{3}$ of $\frac{3}{4}$ of $13\frac{7}{8}$. Ans. $3\frac{5}{9}$.

22. Divide $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of 18 by $\frac{1}{2}$ of $\frac{7}{8}$ of $\frac{6}{7}$ of $4\frac{1}{2}$. Ans. $4\frac{20}{27}$.

23. Divide $\frac{6}{7}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $3\frac{1}{2}$ by $\frac{1}{3}$ of $\frac{7}{8}$ of $2\frac{2}{3}$. Ans. $1\frac{17}{28}$.

24. Divide $\frac{2}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{11}{12}$ of $\frac{8}{11}$ of $22\frac{1}{2}$ by $\frac{5}{6}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $7\frac{1}{2}$. Ans. $\frac{24}{25}$.

25. If $4\frac{1}{2}$ yards of cotton cloth cost 63 cents, what will one yard cost? Ans. 14 cents. Why? $4\frac{1}{2}$=9 half yards, and since 9 half yards cost 63 cents, 1 half yard will cost one-ninth of 63 cents, and 2 half yards will cost $\frac{2}{9}$ of 63 cents.

Here $4\frac{1}{2}=\frac{9}{2}$, being the divisor, we place its numerator on the left of the line.

$$\begin{array}{r|l} \cancel{9} & \cancel{63} \\ & 7 \\ & 2 \\ \hline & 14 \text{ cents.} \end{array}$$

26. If $6\frac{1}{2}$ pounds of beef cost 39 cents, how much is that a pound? Ans. 6 cts. Why?—$6\frac{1}{2}$ lbs.=13 half pounds, and since 13 half pounds cost 39 cents, 2 half pounds or one pound will cost 2 thirteenths of 39 cents,

Thus, $\begin{array}{r|l} \cancel{13} & \cancel{39} \\ & 3 \\ & 2 \\ \hline & 6 \text{ cents.} \end{array}$

27. If $8\frac{1}{3}$ yards of cloth cost $37\frac{1}{2}$ dollars, how much is that a yard? Ans. $4\frac{1}{2}$ dollars, or 4 dollars and 50 cents. Why?—$8\frac{1}{3}=\frac{25}{3}$, and since 25 parts cost $37\frac{1}{2}$ dollars, one part will cost one twenty-fifth of $37\frac{1}{2}$ dollars, and 3 parts will cost 3 twenty-fifths of $37\frac{1}{2}$, or $\frac{75}{2}$ dollars,

Thus, $\begin{array}{r|l} 2 & \cancel{75} \text{ dollars.} \\ \cancel{25} & 3 \\ & 3 \\ \hline & 9 \\ \hline & \$4\frac{1}{2} \end{array}$

28. If $4\frac{3}{5}$ pounds of pork cost $17\frac{1}{4}$ cents, how much is that a pound?

$$\begin{array}{r|l} 4 & \cancel{69} \\ & 3 \\ \cancel{23} & 5 \\ \hline & 15 \\ \hline \text{Ans.} & 3\frac{3}{4} \end{array}$$

A GENERAL RULE FOR THE SOLUTION OF ALL QUESTIONS WHERE THREE NUMBERS ARE GIVEN, AND A FOURTH IS REQUIRED.

1st. Select from the different *numbers* given in the question the one which corresponds in *name* with the required answer, and place it on the right of a vertical line, (if it be a fraction, place its numerator on the right of the line,) then consider the relation which *this number* bears to the other *numbers* given in the question, in order to ascertain whether the required answer will be greater or less *than this number*, and place the greater or less of the *other* numbers on the right of the line accordingly, that is, the greater number on the right of the line, if the answer must be *greater*, and the *less* if the *answer* must be *less* than the first mentioned number, and of course the other number on the opposite side of the line.

NOTE.—In stating fractions let the numerators be placed according to the rule, and consequently the denominators will be placed on the opposite side; then proceed to multiply the numbers together, on the right of the line for a dividend, and those on the left for a divisor, or when a number on either side of the line will measure one on the opposite side use the quotient instead.

29. If 72 tons of hemp cost 13680 dollars, how many dollars will 28 tons cost?

Here we select 13680 as the number which corresponds in *name* with the required answer, and place it on the right of a vertical line; then we observe that this 13680 dollars is the price of 72 tons, and that the *price* of 28 tons which is required as the answer, must be *less*

$$\begin{array}{r|r} & \$ \\ & \cancel{13680} \\ & 190 \\ \cancel{72} & 28 \\ \hline \text{Ans.} & \$5320 \end{array}$$

than the 13680; therefore we place the 28, the *less* of those two numbers, on the right of the line, and 72 on the opposite side.

30. If a man can build 256 rods of fence in 32 days, how many rods can he build in 56 days?

In stating this question, we select 256 as the number which corresponds in name with the required answer, and place it on the right of the line; now this being the number of rods which can be built in 32 days, a greater number can be built in 56 days; therefore we place the 56, the greater of those two numbers, on the right of the line.

	Rods.
	~~256~~
~~32~~	56
	8
Ans.	448 Rods.

31. If a man travel 210 miles in 6 days, how far can he travel in 45 days? Ans. 1575 miles.

32. If $4\frac{1}{2}$ yards of cloth will make one cloak, how many yards will make 13 cloaks. Ans. $58\frac{1}{2}$ yards.

33. If 24 men can build a certain wall in 63 days, how many men will be required to build another such wall in $10\frac{1}{2}$ days?

Operation.

	24 men.
~~21~~	~~63~~
	2
Ans.	144 men.

34. If a man can perform a journey of 126 miles in 7 days, how many miles can he travel in $17\frac{1}{2}$ days? Ans. 315 miles.

35. If $17\frac{4}{7}$ bushels of wheat will sow $7\frac{4}{17}$ acres, how many bushels will be required to sow $24\frac{1}{2}$ acres? Ans. $59\frac{1}{2}$ bushels.

36. If a man travel 396 miles in $87\frac{3}{4}$ hours, how many miles can he travel in $4\frac{7}{8}$ hours? Ans. 22 miles.

37. If 27 horses will eat $47\frac{1}{2}$ bushels of oats in a week, how many bushels will 81 horses eat in the same time? Ans. $142\frac{1}{2}$ bushels.

38. If 17 men perform a piece of work in 25 days, how many men will be required to do the same in 5 days? Ans. 85 men.

39. If a ship sail $47\frac{1}{2}$ miles in $9\frac{1}{2}$ hours, in how many hours will she sail 75 miles? Ans. 15 hours.

40. If 16 cords of wood be worth 40 dollars, what is the value of 96 cords? Ans. 240 dollars.

41. If 64 soldiers eat 448 pounds of beef in a week, how many pounds will 265 soldiers eat in a week?

Ans. 1855 pounds.

42. If 33 dollars will pay for 198 yards of cloth, how many yards can be bought for 17 dollars? Ans. 102 yards.

TO FIND THE LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS.

The least common multiple of two or more numbers is the least number that may be divided by them without a remainder.

43. What is the least common multiple of 4, 6, 16, and 24? Here we begin with 24, the largest number, and as 4 and 6 are both factors of 24, we omit them, and as 16 has a common factor with 24, viz., 8, we divide it by 8, and substitute the quotient 2, for 16, then we multiply the 24 by 2, and find 48 to be the least multiple of the given numbers,

$$\text{Thus, } \left\{ \frac{\cancel{4},\ \cancel{6},\ \cancel{16},\ 24.}{2\times24=48.} \right.$$

In order to find the least common multiple of several numbers, retain the largest number and compare each of the others in succession with it. If any of the other numbers contain a factor in common with the greatest, divide that other number by this factor and use the quotient in its stead. When any number will measure another, it may be cancelled or thrown out of the question.

44. What is the least common multiple of 5, 15, 30, 7, 14, and 28?

$$\frac{\cancel{5},\ \cancel{15},\ 30,\ \cancel{7},\ \cancel{14},\ \cancel{28}}{30\times14=420.\quad \text{Ans.}}$$

In this question we find 5 and 15 are both factors of 30, and 7 and 14 of 28, we therefore cancel them,—28 has a common factor with 30, viz., 2, therefore we divide the 28 by 2, and use the quotient 14.

45. What is the least common multiple of 6, 8, 9, 12, 16, and 18?

$$\cancel{6},\ \cancel{8},\ \cancel{9},\ \cancel{12},\ \cancel{16},\ 18$$
$$\cancel{2}\times 8\times 18=144. \quad \text{Ans.}$$

46. What is the least common multiple of 5, 6, 7, 8, 9, 10, and 24?

$$\cancel{5},\ \cancel{6},\ 7,\ \cancel{8},\ \cancel{9},\ \cancel{10},\ 24$$
$$7\times 3\times 5\times 24=2520. \quad \text{Ans.}$$

47. What is the least common multiple of 2, 3, 4, 5, 6, 7, 8, 14, 15, and 16?

$$\cancel{2},\ \cancel{3},\ \cancel{4},\ \cancel{5},\ \cancel{6},\ \cancel{7},\ \cancel{8},\ \cancel{14},\ 15,\ 16$$
$$2\times 7\times 15\times 16=3360.$$

48. What is the least common multiple of 8, 12, 16, 18, 24?

$$\cancel{8},\ \cancel{12},\ \cancel{16},\ \cancel{18},\ 24$$
$$2\times 3\times 24=144. \quad \text{Ans.}$$

49. What is the least common multiple of 8, 9, 12, 16, 18, 24, 36, 72?

$$\cancel{8},\ \cancel{9},\ \cancel{12},\ \cancel{16},\ \cancel{18},\ \cancel{24},\ \cancel{36},\ 72$$
$$2\times 72=144. \quad \text{Ans.}$$

50. What is the least common multiple of 6, 14, 12, 18, 20, 24, 28, 32, and 48?

$$\cancel{6},\ \cancel{14},\ \cancel{12},\ \cancel{18},\ \cancel{20},\ \cancel{24},\ \cancel{28},\ \cancel{32},\ 48$$
$$3\times 5\times 7\times 4\times 48=20160. \quad \text{Ans.}$$

51. What is the least common multiple of 4, 5, 6, 8, 10, and 12? Ans. 120.

52. What is the least common multiple of 2, 3, 4, 7, 8, 14, and 18? Ans. 504.

53. What is the least common multiple of 5, 6, 8, 10, and 12? Ans. 120.

54. What is the least common multiple of 3, 4, 6, 7, and 24? Ans. 168.

55. What is the least common multiple of 12, 25, 30, and 45?

$$\cancel{12},\ \cancel{25},\ \cancel{30},\ 45$$
$$4\times 5\times \cancel{2}\times 45=900. \quad \text{Ans.}$$

56. What is the least common multiple 30, 45, 55?

$$\frac{\cancel{30},\ \cancel{45},\ 55}{2\times9\times55=990.}\quad \text{Ans.}$$

57. What is the least common multiple of 6, 8, 10, 18, 20, and 24? Ans. 360.

58. What is the least common multiple of 14, 19, 38, and 57? Ans. 798.

59. What is the least common multiple of 20, 36, 48, and 50? Ans. 3600.

60. What is the least common multiple of 3, 4, 5, 6, 7 12, 14, and 21? Ans. 420.

61. What is the least common multiple of 3, 4, 5, 8, 20, and 30? Ans. 120.

62. What is the least common multiple of 9, 10, 15, 18, 19, 38, and 45? Ans. 1710.

63. What is the least common multiple of 6, 7, 8, 12, 14, 16, and 20? Ans. 1680.

64. What is the least common multiple of 9, 11, 16, 18, 22, and 36? Ans. 1584.

65. What is the least common multiple of 25, 35, 60, 72, and 96? Ans. 50,400.

66. What is the least common multiple of 8, 10, 12, 13, 16, 24, and 26? Ans. 3120.

67. What is the least common multiple of 32, 42, and 63? Ans. 2016.

68. What is the least common multiple of 8, 9, 15, and 16? Ans. 720.

69. If you divide an apple into 3 equal parts, and then divide each of these parts into 4 pieces, into how many parts will the whole apple be divided?

70. If you divide an apple into 3 parts, what is one of those parts called? $\frac{1}{3}$ is equal to how many twelfths? Ans. $\frac{1}{3}\times4=\frac{4}{12}$; hence we see that if the numerator and denominator of a fraction be both multiplied by the same number, the value of the fraction will not be altered.

ADDITION OF FRACTIONS.

1. Add $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$. Ans $\frac{16}{8}=2$.

In this question we say, 1 part, and 3 parts, and 5 parts, and 7 parts, make 16 parts, but the *denominator*, which always shows into *how many parts* a unit is divided, being 8, we perceive that the 16 parts, that is, $\frac{16}{8}$, is equal to 2 units, if each of these were a part or parts of a dollar, then their sum would be equal to 2 dollars.

2. What is the sum of $\frac{1}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{9}{12}$, and $\frac{11}{12}$? Ans. $\frac{33}{12}=2\frac{9}{12}=2\frac{3}{4}$.

3. What is the sum of $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, and $\frac{13}{16}$? Ans. $\frac{48}{16}=3$.

In such questions as the above, we have only to find the sum of all the numerators, and then see how many times the common denominator is contained in it. But if we wish to add together $\frac{3}{4}$, $\frac{7}{8}$, $\frac{7}{12}$, and $\frac{2}{3}$, &c., we must first find the least common denominator,

$$\text{Thus, } \underline{\cancel{4},\ \cancel{8},\ 12,\ \cancel{3}}$$
$$2\times12=\text{ least com. denominator.}$$

Now since the value of a fraction is not altered by multiplying both its numerator and denominator by the same number, we multiply $\frac{3}{4}$ by 6, $\frac{7}{8}$ by 3, $\frac{7}{12}$ by 2, and $\frac{2}{3}$ by 8, we have $\frac{18}{24}$, $\frac{21}{24}$, $\frac{14}{24}$, and $\frac{16}{24}$, the sum of which is $\frac{69}{24}=2\frac{21}{24}=2\frac{7}{8}$.

It is not necessary to multiply the denominator, if we only know by what number to multiply the numerator, this we can determine by dividing each denominator into the least common denominator, after that is found; thus, the denominators 4, 8, 12, and 3 will go 6, 3, 2, and 8 times successively into 24.

$$\begin{array}{ll} \text{Then} & 3\times6=18 \\ & 7\times3=21 \\ & 7\times2=14 \\ & 2\times8=\underline{16} \\ & \text{sum}\quad 69 \\ & \quad\frac{69}{24}=2\frac{7}{8}.\quad \text{Ans.} \end{array}$$

NOTE.—When a number of fractions are to be added together change them to a common denominator, and then add their numerators, and find how many times it contains the common denominator.

4. What is the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{12}$?

denominators. ~~2~~ ~~4~~ ~~6~~ ~~8~~ ~~12~~

~~24~~ — ~~2~~, ~~4~~, ~~6~~, ~~8~~, 12

quotients. numerators.

12×1= 12 — 2×12=24 least common denominator.
6×3= 18
4×5= 20
3×7= 21
2×5= 10

81

$\frac{81}{24}=3\frac{3}{8}$.

5. What is the sum of $\frac{7}{8}$, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{5}{6}$, and $\frac{11}{12}$?

denominators. ~~8~~ ~~3~~ ~~9~~ ~~6~~ ~~12~~

~~72~~ — ~~8~~, ~~3~~, ~~9~~, ~~6~~, 12

numerators.

9× 7 =63 — 2×3×12=72 common denominator.
24× 2 =48
8× 2 =16
12× 5 =60
6×11 =66

253

$\frac{253}{72}=3\frac{37}{72}$.

6. What is the sum of $\frac{7}{8}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{2}$, and $\frac{11}{12}$?

~~8~~ ~~3~~ ~~4~~ ~~6~~ ~~2~~ ~~12~~

24 — 8, ~~3~~, ~~4~~, ~~6~~, ~~2~~, 12

quotients. numerators.

3× 7 =21 — 2×12= least common denominator.
8× 2 =16
6× 3 =18
4× 5 =20
12× 1 =12
2×11 =22

109

$\frac{109}{24}=4\frac{13}{24}$.

RULE.

Find the least common multiple of all the denominators, which will be the least common denominator, divide this

common multiple by each of the given denominators, and multiply the quotient by the respective numerators of the fractions, and the sum of the products, divided by the least common denominator, will be the required sum of the given fractions.

7. What is the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{8}{9}$, $\frac{11}{12}$, $\frac{13}{16}$, and $\frac{11}{18}$?

$\cancel{4}, \cancel{8}, \cancel{6}, \cancel{3}, \cancel{9}, \cancel{12}, \cancel{16}, 18$

$\cancel{2}\times8\times18=144$, least common denominator.

given denominators.	quotients. × given numerators.	products.
	144	
$\cancel{4}$	36×3	$=108$
$\cancel{8}$	18×7	$=126$
$\cancel{6}$	24×5	$=120$
$\cancel{3}$	48×2	$=96$
$\cancel{9}$	16×8	$=128$
$\cancel{12}$	12×11	$=132$
$\cancel{16}$	9×13	$=117$
$\cancel{18}$	8×11	$=88$

$\frac{915}{144}=6\frac{51}{144}$. 915 = sum of the products. Ans. $=6\frac{17}{48}$.

8. What is the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, and $\frac{1}{2}$?
Ans. $3\frac{15}{24}=3\frac{5}{8}$.

9. What is the sum of $\frac{8}{9}$, $\frac{1}{6}$, $\frac{11}{12}$, $\frac{3}{4}$, and $\frac{2}{3}$?
Ans. $3\frac{14}{36}=3\frac{7}{18}$.

10. What is the sum of $\frac{6}{7}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{11}{14}$, and $\frac{1}{2}$? Ans. $3\frac{43}{56}$.

11. What is the sum of $\frac{4}{5}$, $\frac{3}{8}$, $\frac{9}{10}$, $\frac{7}{16}$, and $\frac{13}{20}$? Ans. $3\frac{13}{80}$.

12. What is the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{9}{10}$?
Ans. $6\frac{153}{840}=6\frac{51}{280}$.

13. What is the sum of $\frac{5}{12}$, $\frac{7}{9}$, $\frac{1}{6}$, $\frac{17}{18}$, and $\frac{19}{24}$? Ans. $3\frac{7}{72}$.

14. What is the sum of $\frac{2}{3}$, $\frac{3}{8}$, $\frac{5}{9}$, $\frac{7}{20}$, $\frac{4}{5}$, $\frac{3}{10}$, and $\frac{37}{40}$?
Ans. $3\frac{350}{360}=3\frac{35}{36}$.

15. What is the sum of $\frac{8}{15}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{8}{9}$, and $\frac{5}{12}$?
Ans. $4\frac{335}{360}=4\frac{67}{72}$.

16. What is the sum of $\frac{19}{24}$, $\frac{25}{36}$, $\frac{5}{8}$, $\frac{11}{12}$, $\frac{4}{9}$, $\frac{5}{6}$, and $\frac{17}{18}$?
Ans. $5\frac{18}{72}=5\frac{1}{4}$.

17. What is the sum of $7\frac{3}{4}$, $6\frac{2}{3}$, $8\frac{4}{5}$, $2\frac{7}{12}$, and $5\frac{13}{20}$?

Here we first find the sum of the fractions $\frac{3}{4}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{12}$, and $\frac{13}{20}$.

Thus, $\cancel{4}, \cancel{3}, \cancel{5}, \cancel{12}, 20$

$3\times20=60$ least common denominator.

given denominators	quotients × given nums.		given integers.
4			
3	60		
5	15×3	$=45$	7
12	20×2	$=40$	6
20	12×4	$=48$	8
	5×7	$=35$	2
	3×13	$=39$	5
		$\frac{207}{60}=3\frac{9}{20}$	
		$=31\frac{9}{20}$ Ans.	

NOTE.—When mixed numbers are given, first find the sum of the fractions, which add to the sum of the integers, and you will then have the required sum.

18. What is the sum of $8\frac{7}{9}$, $7\frac{3}{4}$, $5\frac{5}{8}$, $12\frac{1}{2}$, $13\frac{2}{3}$, and $17\frac{5}{6}$? Ans. $66\frac{11}{12}$.
19. What is the sum of $6\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{12}$, and $8\frac{3}{4}$? Ans. $17\frac{3}{10}$
20. What is the sum of $19\frac{3}{8}$, $\frac{4}{9}$, $\frac{11}{12}$, $\frac{5}{6}$, $\frac{3}{4}$, and $37\frac{1}{2}$? Ans. $59\frac{59}{72}$.

SUBTRACTION OF FRACTIONS.

A boy having $\frac{7}{8}$ of a dollar, gave $\frac{4}{8}$ of a dollar to his mother; how much had he left? Ans. If we take 4 parts from 7 parts, the remainder will be 3 parts, that is, $\frac{3}{8}$.

1. If you take $\frac{5}{12}$ from $\frac{11}{12}$, how much will be left? Ans. $\frac{6}{12}=\frac{1}{2}$; that is, if you take 5 parts from 11 parts, the remainder will be 6 parts, or $\frac{6}{12}$; but as the denominator shows into how many parts a unit is divided, and according to this denominator it is divided into 12 parts, 6 parts being the half of 12 parts, $\frac{6}{12}$ must be equal to $\frac{1}{2}$.
2. $\frac{13}{16}$ less $\frac{8}{16}$=how much? That is, if you take 8 parts from 13 parts, how many parts will be left?
3. $\frac{17}{20}$ less $\frac{8}{20}$=how much?
4. $\frac{19}{24}$ — $\frac{8}{24}$=how much? (This — signifies subtraction.)
5. $\frac{27}{40}$ — $\frac{18}{40}$ is how much?

6. Subtract $\frac{23}{47}$ from $\frac{32}{47}$. That is, take 23 parts from 32 parts.

7. Subtract $\frac{29}{68}$ from $\frac{56}{68}$. Ans. $\frac{27}{68}$.
8. Subtract $\frac{47}{89}$ from $\frac{75}{89}$. Ans. $\frac{28}{89}$.
9. Subtract $\frac{78}{97}$ from $\frac{86}{97}$. Ans. $\frac{8}{97}$.
10. Subtract $\frac{14}{31}$ from $\frac{29}{31}$. Ans. $\frac{15}{31}$.
11. Subtract $\frac{19}{49}$ from $\frac{38}{49}$. Ans.
12. Subtract $\frac{79}{100}$ from $\frac{92}{100}$. Ans. $\frac{13}{100}$.
13. Subtract $\frac{45}{137}$ from $\frac{114}{137}$. Ans.
14. Subtract $\frac{149}{237}$ from $\frac{216}{237}$. Ans. $\frac{67}{237}$.
15. Subtract $\frac{163}{400}$ from $\frac{320}{400}$. Ans. $\frac{157}{400}$.
16. Subtract $\frac{7}{12}$ from $\frac{7}{8}$. Here the fractions not having a common denominator, as in the preceding examples, we must find the least common denominator;

Thus, $\cancel{8}$ 12
8|24 $2\times12=24$ least common denominator.
$3\times7=21$ parts.
12| $2\times7=14$ parts.
Ans. $\frac{7}{24}$

TO SUBTRACT FRACTIONS WHOSE DENOMINATORS ARE UNLIKE.

RULE.

Change the fractions to a common denominator, then subtract the less numerator from the greater, and the remainder placed over the common denominator will give the answer required.

17. Subtract $\frac{4}{7}$ from $\frac{13}{16}$. Here the denominators 7 and 16 having no common factor the least common denominator will be their product. That is $16\times7=112$,

Then, | $13\times\ 7=91$=greater numerator.
| $4\times16\quad 64$=less numerator.
Ans. $\frac{17}{112}$

18. Subtract $\frac{5}{9}$ from $\frac{19}{24}$.

$\cancel{9}$ 24
24|72 3×24=least common denominator.
$3\times19=57$
9| $8\times\ 5=40$
Ans. $\frac{17}{72}$

19. Subtract $\frac{7}{12}$ from $\frac{11}{16}$.

$$\begin{array}{l} \cancel{12}\quad 16 \\ \hline 3\times16=48 \end{array} \qquad \begin{array}{r|lr} 16 & 48 & \\ 12 & 3\times11 & 33 \\ & 4\times\ 7 & 28 \\ \hline & \text{Ans.} & \frac{5}{48} \end{array}$$

20. Subtract $\frac{9}{16}$ from $\frac{29}{36}$. Ans. $\frac{35}{144}$.
21. Subtract $\frac{7}{16}$ from $\frac{2}{3}$. Ans. $\frac{11}{48}$.
22. Subtract $\frac{5}{18}$ from $\frac{3}{4}$. Ans. $\frac{17}{36}$.
23. Subtract $\frac{4}{15}$ from $\frac{11}{12}$. Ans. $\frac{39}{60}=\frac{13}{20}$.
24. Subtract $\frac{9}{20}$ from $\frac{7}{15}$. Ans. $\frac{1}{60}$.
25. Subtract $\frac{3}{8}$ from $\frac{13}{20}$. Ans. $\frac{11}{40}$.
26. Subtract $\frac{4}{9}$ from $\frac{17}{18}$. Ans. $\frac{9}{18}=\frac{1}{2}$.
27. Subtract $\frac{2}{3}$ from $\frac{3}{4}$. Ans. $\frac{1}{12}$.
28. Subtract $\frac{9}{16}$ from $\frac{29}{44}$. Ans. $\frac{17}{176}$.
29. Subtract $4\frac{2}{3}$ from $12\frac{7}{8}$. Here the least common denominator of the fractions is 24,

$$\begin{array}{l|l} \text{Then,} & 12\frac{7}{8}=12\frac{21}{24} \\ \text{And} & \ \ 4\frac{2}{3}=\ \ 4\frac{16}{24} \\ \hline & \text{Ans. } 8\frac{5}{24}. \end{array}$$

16 parts from 21 parts leaves 5 parts, that is $\frac{5}{24}$, and 4 from 12 leaves 8.

30. Subtract $7\frac{3}{4}$ from $24\frac{2}{3}$.

$$\begin{array}{rl} 24\frac{8}{12} & \text{numerator.} \\ & \text{denominator} \\ 7\frac{9}{12} & \\ \hline \text{Ans. } 16\frac{11}{12} & \end{array}$$

In this question the $\frac{2}{3}$ is equal to 8 parts, that is $\frac{8}{12}$ and the $\frac{3}{4}$ to $\frac{9}{12}$, and as we cannot take 9 parts from 8 parts, we borrow one from the 24, and as the denominator shows in this case, that a unit is divided into 12 parts, the one we borrow from the 24 units will make 12 parts, which, together with the 8 parts, will make 20 parts; then we take the 9 parts from the 20 parts, and have 11 parts, that is $\frac{11}{12}$ left; now since we borrowed one from the 24, we must either call it 23, and then take the 7 therefrom, or call the 7, 8, and subtract 8 from the 24, which in either case will leave a remainder of 16; the latter is the more common method; that is, call the lower number one more than it is, instead of calling the upper one less.

31. Subtract $27\frac{3}{4}$ from $62\frac{7}{8}$. Ans. $35\frac{1}{8}$.
32. Subtract $34\frac{7}{8}$ from $51\frac{11}{12}$. Ans. $17\frac{1}{24}$
33. Subtract $28\frac{2}{3}$ from $45\frac{1}{2}$. Ans. $16\frac{5}{6}$.
34. Subtract $12\frac{3}{8}$ from $20\frac{7}{12}$. Ans. $8\frac{5}{24}$.

35. Subtract $14\frac{7}{8}$ from $18\frac{2}{3}$. Ans. $3\frac{19}{24}$.
36. Subtract $17\frac{9}{16}$ from $35\frac{5}{12}$. Ans. $17\frac{41}{48}$.
37. Subtract $19\frac{4}{5}$ from $46\frac{8}{15}$. Ans. $26\frac{11}{15}$.
38. Subtract $24\frac{5}{6}$ from $73\frac{17}{18}$. Ans. $49\frac{2}{18}=49\frac{1}{9}$.
39. Subtract $37\frac{7}{9}$ from $84\frac{2}{3}$. Ans. $46\frac{8}{9}$.
40. Subtract $13\frac{3}{16}$ from 22.

Here there being no fraction attached to the greater number, we borrow a unit from the 22, which, as the denominator shows, will contain 16 parts, then we take 3 parts from 16 parts, and the remainder is 13 parts, that is $\frac{13}{16}$, and calling the 13 one more, or 14, to make up the 1 we borrowed, we say, 14 from 22 leaves 8.

$$\begin{array}{rr} & 22 \\ & 13\frac{3}{16} \\ \hline \text{Ans.} & 8\frac{13}{16} \end{array}$$

The above process is the same in effect as the following

RULE.

Subtract the numerator from the denominator of the fraction, and under the remainder write the denominator, and carry one to the whole number of the subtrahend, or number to be subtracted.

EXAMPLES.

41. Subtract $17\frac{2}{3}$ from 28. Ans. $10\frac{1}{3}$.
42. Subtract $24\frac{7}{16}$ from 43. Ans. $18\frac{9}{16}$.
43. Subtract $32\frac{12}{19}$ from 87. Ans. $54\frac{7}{19}$.
44. Subtract $137\frac{29}{41}$ from 225. Ans. $87\frac{12}{41}$
45. Subtract $229\frac{3}{8}$ from 517. Ans. $287\frac{5}{8}$.
46. Subtract $3462\frac{8}{11}$ from 7648. Ans. $4185\frac{3}{11}$.
47. Subtract $206\frac{1}{2}$ from 7387. Ans. $7180\frac{1}{2}$.
48. Subtract $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{7}{8}$ from $\frac{1}{2}$ of $\frac{9}{16}$ of $\frac{4}{5}$ of $3\frac{1}{3}$.

Here we are to understand that the fractions are first to be multiplied. The product of those to be subtracted gives $\frac{7}{16}$, that of the others, $\frac{3}{4}$, which is equal to $\frac{12}{16}$; then we take 7 parts from 12, and the remainder is 5 parts, that is, $\frac{5}{16}$.

$$\begin{array}{r|l} 4 & \cancel{3} \\ \cancel{3} & \cancel{2} \\ 4\cancel{8} & 7 \\ \hline & 16 \end{array} \quad \frac{7}{16} \qquad\qquad \begin{array}{r|l} \cancel{2} & 1 \\ 1\cancel{6} & \cancel{9}3 \\ \cancel{5} & \cancel{4} \\ 4\cancel{3} & 1\cancel{0} \\ \hline & \end{array} \quad \frac{3}{4}=\frac{12}{16}$$

NOTE.—Compound fractions must be changed to single ones by multiplication, and those single fractions to such as have a common denominator, and then subtracted.

49. Take $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{9}$ from $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{4}{5}$ of $6\frac{2}{3}$.

$$\begin{array}{r|l} \cancel{2} & 1 \\ \cancel{3} & \cancel{2} \\ \cancel{4} & \cancel{3} \\ 9 & \cancel{8}\ 2 \end{array} \ \frac{2}{9} \qquad \begin{array}{r|l} \cancel{4} & \cancel{3} \\ 3\ \cancel{6} & \cancel{5} \\ \cancel{5} & \cancel{4} \\ \cancel{3} & \cancel{20} \end{array} \quad \frac{10}{3}=\frac{30}{9}-\frac{2}{9}=\frac{28}{9}=3\frac{1}{9}.$$

Here the fractions when simplified by multiplication, are $\frac{2}{9}$ and $\frac{10}{3}$; this $\frac{10}{3}$ is equal to $\frac{30}{9}$, then we take 2 parts from 30 parts, and the remainder is 28 parts, that is, $\frac{28}{9}$ which is equal to $3\frac{1}{9}$

50. Take $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{4}{5}$ from $\frac{7}{9}$ of $\frac{9}{16}$ of $\frac{8}{11}$ of $8\frac{1}{4}$.

$$\begin{array}{r|l} \cancel{4} & \cancel{3} \\ \cancel{3} & \cancel{2} \\ 3\ \cancel{6} & \cancel{5} \\ \cancel{5} & \cancel{4} \\ \hline \text{Product} & \frac{1}{3} \end{array} \qquad \begin{array}{r|l} \cancel{9} & 7 \\ 2\ \cancel{16} & \cancel{9} \\ \cancel{11} & \cancel{8} \\ 4 & \cancel{33}\ 3 \\ \hline \text{Product} & \frac{21}{8} \end{array}$$

Now we must take $\frac{1}{3}$ from $\frac{21}{8}$; here 24 is the least common denominator, $\frac{21}{8}=\frac{63}{24}$ and $\frac{1}{3}=\frac{8}{24}$, then we must take 8 parts from 63 parts, and the remainder is 55 parts; that is $\frac{55}{24}$.

Ans. $2\frac{7}{24}$.

51. From $\frac{7}{8}$ of $\frac{6}{11}$ of $\frac{19}{33}$ of $13\frac{1}{5}$, take $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $4\frac{1}{2}$.

$$\begin{array}{r|l} 4\ \cancel{8} & 7 \\ 11 & \cancel{6}\ 3 \\ 2\ \cancel{33} & 19 \\ 5 & \cancel{66}\ \cancel{2} \\ \hline 110 & 399 \end{array} \qquad \left.\begin{array}{r|l} 2 & 1 \\ 4 & 3 \\ \cancel{8} & 7 \\ \cancel{9} & \cancel{8} \\ 2 & \cancel{9} \end{array}\right\} \text{Ans. } \frac{21}{16}.$$

Here the least common denominator is 880.

Then, 110)880

399 × 8 = 3192=parts

And 16)880(55×21=1155=parts

2037=parts, that is, $\frac{2037}{880}=2\frac{277}{880}$ Ans.

52. From $\frac{2}{3}$ of $\frac{3}{4}$ of $3\frac{2}{3}$ take $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $2\frac{1}{4}$. Ans. $1\frac{1}{12}$.
53. From $\frac{7}{8}$ of $\frac{4}{9}$ of $3\frac{3}{5}$ take $\frac{3}{4}$ of $\frac{8}{11}$ of $\frac{11}{12}$ of $2\frac{7}{10}$. Ans. $\frac{1}{20}$.
54. From $\frac{8}{9}$ of $3\frac{3}{4}$ take $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $2\frac{1}{4}$. Ans. $\frac{81}{112}$.
55. From $\frac{7}{9}$ of 7^{1} take $\frac{3}{4}$ of $5\frac{3}{5}$. Ans. $\frac{7}{5}=1\frac{2}{5}$.

TABLES OF MONEY, WEIGHTS, AND MEASURES.

UNITED STATES MONEY.

10 mills make	1 cent, *ct.*
10 cents	1 dime,
10 dimes, or 100 cents	1 dollar, $.
10 dollars	1 eagle.

In business, dollars and cents alone are counted, no mention being made of mills, dimes, and eagles. Ten cent pieces are dimes.

ENGLISH MONEY.

4 farthings, *fa.* make	1 penny, *p.*
12 pence	1 shilling, *s.*
20 shillings	1 pound, £.
21 shillings sterling	1 guinea, *gui.*
28 shillings, N. E.	1 guinea.

NOTE.—One pound sterling is equal to $\$4.44\frac{4}{9} = \$\frac{40}{9}$.

TROY WEIGHT.

24 grains, *grs.* make	1 pennyweight, *dwt*
20 pennyweights	1 ounce, *oz.*
12 ounces	1 pound, *lb.*

By this weight are weighed gold, silver, and jewels.

AVOIRDUPOIS WEIGHT.

16 drams, *drs.* make	1 ounce, *oz.*
16 ounces	1 pound, *lb.*
28 pounds	1 quarter, *qr.*
4 quarters	1 hundred weight, *cwt.*
20 hundred weight	1 ton, *ton.*

By this weight are weighed things of a coarse, drossy nature; as flour, cheese, butter, hay, &c.; also, all metals except gold and silver.

NOTE 1.—By a law of Massachusetts, and also of Ohio, the cwt. contains 100 lbs., instead of 112 lbs. But a ton is reckoned at the custom-houses of the United States at 2240 lbs.

NOTE 2.—A barrel of flour weighs 196 lbs., a bushel of wheat is estimated at 60 lbs., of rye or Indian corn at 56 lbs., of barley at 48 lbs., of oats at 33 lbs., and of clover seed at 60 lbs.

APOTHECARIES' WEIGHT.

20 grains *grs.* make	1 scruple ℈.
3 scruples	1 drachm ʒ.
8 drachms	1 ounce ℥.
12 ounces	1 pound ℔.

By this weight apothecaries mix their medicines; but they buy and sell by Avoirdupois.

The pound and ounce of this weight are similar to those of Troy weight.

LONG MEASURE.

3 barley-corns *b.c.* make	1 inch, *in.*
12 inches	1 foot, *ft.*
3 feet	1 yard, *yd.*
5½ yards or 16½ feet	1 rod, pole, or perch, *p.*
40 poles	1 furlong, *fur.*
8 furlongs or 1760 yards	1 mile, *m.*
3 miles	1 league, *l.*
60 geographic or 69½ common miles	1 degree, °.

360 of these degrees will go round the earth; hence the earth is 360 times 69½ miles, that is 25020 miles in circumference.

A hand is 4 inches, and is applied in measuring the height of horses.

A fathom is 6 feet, and is used to express the depth of water.

This is used for measuring distances; as roads, lines, length of timber, &c.

CLOTH MEASURE.

2¼ inches *ins.* make	1 nail, *na.*
4 nails	1 quarter of a yard, *qr.*
4 quarters	1 yard, *yd.*
3 quarters	1 Ell Flemish, *E.Fl.*
5 quarters	1 Ell English, *E.E.*
6 quarters	1 Ell French, *E.F.*

By this measure, which is only a branch of long measure, cloth, tapes, ribbons, &c., are measured ; the name expresses the use.

SQUARE OR SUPERFICIAL MEASURE.

144 square inches make	1 square foot.
9 square feet	1 square yard.
100 square feet, or 10 feet sq.	1 sq. of mechanics' work.
$272\frac{1}{4}$ square feet, or $30\frac{1}{4}$ square yards	1 sq. rod, pole, or perch.
40 rods	1 rood.
4 roods	1 acre.
640 acres	1 square mile.

All surfaces, as land, flooring, &c., are measured by this measure.

The measure of boards, floors, &c., is generally expressed in square feet and inches; that of land in acres, roods, and poles.

DRY MEASURE.

4 gills, *gls.* make	1 pint, *pt.*
2 pints	1 quart, *qt.*
4 quarts	1 gallon, *gal.*
2 gallons, or 8 quarts	1 peck, *pc.*
4 pecks	1 bushel, *bush.*

Dry measure is used for grain, fruit, salt, oysters, &c.

NOTE.—A Winchester bushel is $18\frac{1}{2}$ inches in diameter, and 8 inches deep. The standard gallon, dry measure, contains $268\frac{4}{5}$ cubic inches.

WINE MEASURE.

4 gills, *gls.* make	1 pint, *pt.*
2 pints	1 quart, *qt.*
4 quarts	1 gallon, *gal.*
42 gallons	1 tierce, *tier.*
63 gallons	1 hogshead, *hhd.**
84 gallons	1 puncheon, *pun.*
2 hogsheads, or 126 gallons	1 pipe, or butt, *pe.*
2 pipes, or 4 hogsheads	1 tun, *tun.*

NOTE.—The wine gallon contains 231 cubic inches: water, wine, and spirits are measured and sold by this measure.

* A barrel is = to $\frac{1}{2}$ a hogshead.

ALE AND BEER MEASURE.

2 pints, *pts.* make	1 quart, *qt.*
4 quarts	1 gallon, *gal.*
32 gallons	1 barrel, *bar.*
54 gallons	1 hogshead *hhd.*
2 hogsheads	1 butt, *butt.*
2 butts	1 tun, *tun.*

NOTE.—The ale gallon contains 282 cubic inches: milk is sold by the beer gallon.

TIME.

Time is thus divided:

60 seconds, *sec.* make	1 minute, *min.*
60 minutes	1 hour, *h.*
24 hours	1 day, *d.*
7 days	1 week, *w.*
4 weeks	1 lunar month
13 lunar months, 1 d. 6 h. or 365 days and 6 hours	1 Julian year.
12 calendar months	1 year, *y.*
100 years	1 century.

NOTE.—The six hours in each year are not reckoned till they amount to one day; hence, a common year consists of 365 days, and every fourth year, called leap year, of 366 days.

The following is a statement of the number of days in each of the twelve months, as they stand in the calendar or almanac:

The fourth, eleventh, ninth, and sixth,
Have thirty days to each affix'd;
And every other thirty-one,
Except the second month alone,
Which has but twenty-eight in fine,
Till leap year gives it twenty-nine.

Or,

Thirty days are in September,
In April, June, and November;
All the rest have thirty-one,
Except February alone,
Which has but twenty-eight in fine,
Till leap year gives it twenty-nine.

SOLID MEASURE.

1728 inches, *in.* make	1 foot, *ft.*
27 feet	1 yard, *yd.*

40 feet of round timber, or 50 feet of hewn timber, make a ton, 16 cubic feet make a foot of wood, and 8 feet of wood, or 128 cubic feet make a cord.

The cubic contents of any thing which has 6 sides—its opposite sides being equal—is found in cubes, by multiplying together, the length, breadth, and depth.

CIRCULAR MOTION.

60 seconds, ″ make	1 minute ′.
60 minutes	1 degree °.
30 degrees	1 sign, *s.*
12 signs, or 360 degrees	1 circle, *c.*

REDUCTION, OR TRANSFORMATION.

Reduction is the operation of changing the denomination of numbers, without altering their value.

EXAMPLES.

1. In 15 lbs. Troy weight, how many ounces? Ans. 180 ounces. Why? Since 1 pound is equal to 12 ounces, there must be 12 times as many ounces as there are pounds.

2. In 350 lbs. Troy, how many ounces? Ans. 4200 oz. Why

3. In 76 pounds Avoirdupois, how many ounces? Ans. 1216 ounces. Why? Since 1 pound is equal to 16 ounces, there must be 16 times as many ounces as there are pounds.

4. In 56 pounds Avoirdupois, how many drams? Ans. 14336 drams. Why? Since 1 pound is equal to 16 ounces, there must be 16 times as many ounces as there are pounds, 16 times 56 is 896, and since 1 ounce is equal to 16 drams, there will be 16 times as many drams as there are ounces, and 56 lbs. is equal to 896 oz., and 16 times 896= 14336 drs.

Note.—The learner should be required to give mental solutions to as great a number of such questions as the following, as possible, and in all cases to give the reason for the operation.

EXAMPLES IN TROY WEIGHT.

5. In 95 pounds, how many ounces? Ans. 1140 oz. Why?

6. In 237 ounces, how many pennyweights? Ans. 4740 pennyweights. Why? Since 1 ounce is equal to 20 pennyweights, there will be 20 times as many pennyweights as there are ounces.

7. How many pennyweights in 174 ounces? Why?

8. In 145 pennyweights, how many grains? Ans. 3480. Why? Since 1 pennyweight is equal to 24 grains, there will be 24 times as many grains as there are pennyweights.

9. In 15 pounds, how many pennyweights? Ans. 3600 pennyweights. Why?

10. In 75 pounds, how many grains? Ans. 432000 grs. Why? First change the pounds to ounces, then the ounces to pennyweights, and the pennyweights to grains, and give the reason for each operation.

11. In 672 ounces, how many pounds? Ans. 56 pounds. Why? Since it takes 12 ounces to make one pound, as many times 12 as there are in 672, so many pounds 672 ounces will make.

12. In 6912 ounces how many pounds? Why?

13. In 34800 pennyweights, how many ounces? Ans. 1740 ounces. Why?

14. In 3480 grains how many pennyweights? Ans. 145 pennyweights. Why?

15. In 3600 pennyweights, how many pounds? Ans. 15 pounds? Why?

16. In 432000 grains how many pounds? Ans. 75 lbs. Why?

EXAMPLES IN AVOIRDUPOIS WEIGHT.

17. In 376 pounds, how many ounces? Ans. 6016 ounces. Why?

18. In 37 quarters, how many pounds? Ans. 1036 pounds. Why? Since 1 quarter is equal to 28 pounds,

there will be 28 times as many pounds as there are quarters, the number of quarters in this question being 37, the number of pounds therefore will be 28 times 37.

19. In 3 cwt., how many lbs.? Why?

20. Change 7 cwt. to ounces. Ans. 12544 oz.

21. Change 2 tons to drams. Ans. 1146880 drs.

The following rule may serve as a guide to the pupil, though it were better that he should arrive at the proper result by a regular and simple process of reasoning.

RULE FOR REDUCTION.

When a greater denomination is to be changed to a smaller, multiply the greater denomination by that number which is required of the smaller, to make *one* of the greater; adding to the product so many of the smaller denomination as are expressed in the given sum. Perform a like operation on this product, and on each succeeding product.

When a smaller denomination is to be changed to a greater, divide the smaller denomination by that number which is required of the smaller to make *one* of the next greater; the quotient will be of the greater denomination, and the remainder will be of the same denomination with the dividend. Perform a like operation on this quotient, and on each succeeding quotient.

22. Change 17 cwt. to drams.

Here we reason thus,—since 1 cwt. is equal to 4 quarters, there will be 4 times as many quarters as there are hundred weight.

Secondly, since 1 quarter is equal to 28 pounds, there will be 28 times as many pounds as quarters, then since 1 pound is equal to 16 ounces, there will be 16 times as many ounces as pounds,—and lastly, since 1 ounce is equal to 16 drams, there will be 16 times as many drams as ounces.

$$\begin{array}{rl} 17 & \text{cwt.} \\ 4 & \\ \hline 68 & \text{qrs.} \\ 28 & \\ \hline 544 & \\ 136 & \\ \hline 1904 & \text{lbs.} \\ 16 & \\ \hline 30464 & \text{oz.} \\ 16 & \\ \hline 487424 & \text{drams.} \end{array}$$

23. Change 487424 drams to cwt.

16)487424 drams.	
16)30464 oz.	
28)1904 lbs	4)68 qrs.
168	17 cwt.
224	
224	

Here we say, since 16 drams are required to make one ounce, as many times 16 as there are in 487424, so many ounces will 487424 drs. make, that is, 30460 ounces.

Secondly, since it requires 16 ounces to make one pound, as many times 16 as there are in 30464, so many pounds will 30464 ounces make, 1904 pounds.

Thirdly, since it requires 28 pounds to make 1 quarter, as many times 28 as there are in 1904, so many quarters will 1904 pounds make, viz., 68 quarters, and lastly, since 4 quarters are required to make 1 cwt., as many times 4 as there are in 68 so many cwt. will 68 quarters make, viz., 17 cwt.

24. Charge 8 tons to pounds. Ans. 17920 lbs.

25. Change 17920 lbs. to tons, and write on your slate an explanation of the process, giving the reason why you proceed as you do.

26. Change 6 tons to drams. Ans. 3440640 drs. Why?

27. Change 3440640 drams to tons. Ans. 6 tons. Give the reason.

APOTHECARIES' WEIGHT.

28. Change 392 pounds to ounces.
Ans. 4704 oz. Why?

29. Change 1278 ounces to drams.
Ans. 10224 drs. Why?

30. Change 1008 drams to scruples.
Ans. 3024 ℈. Why?

31. Change 256 scruples to grains.
Ans. 5120 grs. Why?

32. Change 15360 grains to scruples.
Ans. 768 ℈. Why?

33. Change 192 ounces to grains.
Ans. 92160 grs. Why?

34. Change 276480 grains to ounces.
Ans. 576 oz. Why?

35. Change 84 pounds to grains.
Ans. 483840 grs. Why?

36. Change 483840 grains to pounds.
Ans. 84 lbs. Why?

LONG MEASURE.

37. Change 7463 miles to furlongs.
Ans. 59704 fur. Why?

38. Change 179112 furlongs to miles.
Ans. 22389 miles. Why?

39. Change 546 furlongs to poles. Ans. 21840 poles.

40. Change 196560 poles to furlongs. Ans. 4914 fur.

41. Change 27648 poles to yards. Ans. 152064 yds.

42. Change 152064 yards to poles. Ans. 27648 poles.

43. Change 343 yards to inches. Ans. 12348 inches.

44. Change 12348 inches to yards. Ans. 343 yds. Why?

45. Change 23 miles to inches. Ans. 1457280 in.

46. Change 4371840 inches to miles.
Ans. 69 miles. Why?

CLOTH MEASURE.

47. Change 3747 yards to nails. Ans. 59952 na. Why?

48. Change 419664 nails to yards. Ans. 26229 yds.

49. Change 736 English ells to nails. Ans. 14720 na.

50. Change 103040 nails to English ells.
Ans. 5152 E.E. Why?

51. Change 3645 Flemish ells to inches. Ans. 98415 in.

NOTE.—A Flemish ell is 27 inches, a yard 36 inches, an English ell 45 inches, and a French ell 54 inches.

LAND OR SQUARE MEASURE.

52. In 7698 square feet, how many square inches?
Ans. 1108512 sq. in. Why?

53. In 2217024 square inches, how many square feet?
Ans. 15396 sq. ft. Why?

54. In 479 square yards, how many square inches?
Ans. 620784 sq. in. Why?

55. In 3103920 square inches, how many square yards?
Ans. 2395 sq. yds.
56. In 192 square perches, how many square feet?
Ans. 52272 sq. ft.

CUBIC OR SOLID MEASURE.

57. In 12 solid yards, how many solid inches?
Ans. 559872 solid in. Why?
58. In 7 tons of round timber, how many cubic inches?
Ans. 483840 cubic in. Why?
59. In 2419200 cubic inches, how many tons of round timber? Ans. 35 tons. Why?
60. In 34 cords of wood, how many cubic or solid feet?
Ans. 4352 solid feet. Why?
61. In 17408 solid feet of wood, how many cords of wood? Ans. 136 cords. Why?
62. In 131328 solid inches, how many solid feet?
Ans. 76 solid feet. Why?
63. In 884736 solid inches, how many cords?
Ans. 4 cords. Why?

WINE MEASURE.

64. In 76 hogsheads of wine, how many gills?
Ans. 153216 gills. Why?
65. In 459648 gills, how many hogsheads?
Ans. 228 hhds. Why?
66. In 63 tons of wine, how many quarts?
Ans. 63504 quarts. Why?
67. In 190512 quarts, how many tons?
Ans. 189 tons. Why?

DRY MEASURE.

68. In 153 bushels, how many pecks
Ans. 612 pecks. Why?
69. In 4284 pecks, how many bushels?
Ans. 1071 bushels. Why?

70. In 1536 pints, how many bushels? Ans. 24 bu.
71. In 264 bushels, how many pints? Ans. 16896 pts.

TIME.

72. Change 152 years to calendar months.
Ans. 1824 months. Give the reason.
73. In 12768 calendar months, how many years?
Ans. 1064 years.
74 In 19 years, how many days? (Supposing 365 days to make a year.) Ans. 6935 days.
75. In 20805 days, how many years? (of 365 days each.)
Ans. 57 years. Why?
76. In 37 weeks, how many minutes?
Ans. 372960 minutes. Why?
77. In 745920 minutes, how many weeks?
Ans. 74 weeks. Why?
78. In 36 hours, how many seconds?
Ans. 129600 seconds. Why?
79. In 907200 seconds, how many hours?
Ans. 252 hours. Why?

ENGLISH MONEY.

80. In £378, how many shillings?
Ans. 7560 shillings. Why?
81. In 68040 shillings, how many pounds?
Ans. £3402. Why?
82. In 763 shillings, how many farthings?
Ans. 36624 farthings.
83. In 18312 farthings, how many shillings?
Ans. $381\frac{1}{2}$ shillings.
84. In 36 pounds, how many pence? Ans. 8640 pence.
85. In 60480 pence, how many pounds?
Ans. £252. Why?
86. In 96 pounds, how many farthings?
Ans. 92160 farthings.
87. In 645120 farthings, how many pounds sterling?
Ans. £672.

88. In 57 pounds, 7 shillings, how many shillings?

Here we say, since 1 pound is equal to 20 shillings, there will be 20 times as many shillings in any number of pounds, as there are pounds; now 20 times 57 is 1140: but we have 7 shillings more to add to this 1140 shillings, which makes 1147 shillings.

```
 £   s.
 57  7
 20
1147 shillings.
```

89. In 76 pounds, 13 shillings, and 8 pence, how many pence?

```
              £    s.   d.
Solution.    76   13    8
             20
           1533 s.
             12
     Ans. 18404 d.
```

90. In 18404 pence, how many pounds?

Here we may say, since it requires 12 pence to make 1 shilling, as many times 12 as there are in 18404, so many shillings will 18404 pence make, viz., 1533 shillings and 8 pence of a remainder. Again, since it requires 20 shillings to make 1 pound, as many times 20 as there are in 1533, so many pounds will 1533 shillings make, viz., 76 pounds and a remainder of 13 shillings; hence, we find 18404 pence is equal to £76, 13 shillings, and 8 pence.

```
                    d.
                12)18404
Solution.     2|0)153|3 s.
Ans.           £76 13s. 8d.
```

NOTE TO THE LEARNER.—Observe that the remainder will always be of the same name as the *dividend*, or the number divided. That you may the better understand this fact, we will suppose you are required to divide a basket of apples among as many persons as you can give 12 apples a-piece, and to have what remains for yourself. Now, if the number remaining be 7, it cannot be 7 potatoes, nor seven slate-pencils, nor 7 persons; but 7 apples, because apples were divided.

PROMISCUOUS QUESTIONS.

91. Change 47 pounds, 10 ounces, 15 pennyweights to pennyweights. Ans. 11495 dwts. Why?

92. Change 11495 pennyweights to pounds.
Ans. 47 lbs. 10 oz. 15 dwts.

93. Change 5 pounds, 6 ounces, 4 pennyweights, 20 grains, to grains. Ans. 31796 grs.

94. Change 31796 grains to pounds. Ans.

95. Change 8 tons, 6 cwt. 2 quarters, to qrs. Ans. 666 qrs.

96. Change 666 quarters to tons. Ans. Why?

97. Change 3 tons, 25 pounds, to pounds. Ans. 6745 lbs.

98. Change 6745 pounds to tons. Ans.

99. In 17 yards, 2 quarters, 2 nails, how many nails?
Ans. 282 nas.

100. Change 282 nails to yards. Ans.

101. In 15 yards, 2 feet, how many inches?
Ans. 564 in. Why?

102. In 1128 inches, how many yards? Ans. 31 yds. 1ft.

103. In 53 acres, 1 rood, how many perches?
Ans. 8520 p.

104. In 8540 perches, how many acres? Ans.

105. In 34 acres, 3 roods, 17 perches, how many perches?
Ans. 5577 p.

106. In 5577 perches, how many acres?
Ans. Why?

107. How many pounds, Troy, and how many grains, in 5760 ounces? Ans. 480 lbs. 2764800 grs.

108. How many ounces in 480 pounds, Troy?
Ans. 5760 oz.

109. How many nails and yards are in 46 quarters? Ans. 184 nas. 11 yds. 2 qr.

110. How many yards and inches in 100 feet? Ans. 33 yds. 1 ft. 1200 in.

111. How many miles are in 567 furlongs?
Ans. 70 m. 7 fur.

112. How many poles are in 567 furlongs?
Ans. 22680 p.

113. In 567 furlongs, how many leagues?
Ans. 23 l. 1 m. 7 ft.

114. How many yards are in 567 furlongs?
Ans. 124740 yds.

115. How many furlongs are in 124740 yards?
Ans. 567 fur.

116. How many feet are in 38 inches? Ans. 3 ft. 2 in.
117. How many yards are in 38 feet? Ans. 12 yds. 2 ft.
118. How many inches are in 38 feet? Ans. 456 in.
119. How many feet are in 456 inches? Ans. 38 ft.
120. How many inches are in 162 yards? Ans. 5832 in.
121. How many gallons are in 360 pints? Ans. 45 pts.
122. In 1760 rods, how many acres? Ans. 11 a.
123. How many square inches are in 2 square yards?
Ans. 2592 sq. in.
124. What will 3 bushels of beans cost at 5 cents a pint?
Ans. $9.60 cts.

NOTE.—There are 64 pints in a bushel, and in 3 bushels there will be 3 times 64 pints, or 192 pints; now, since 1 pint is worth 5 cents, the whole price will be 5 times as many cents as there are pints; that is, 192 × 5 = 960 cents: now, if we point off the two right hand figures for cents, all to the left of that point will be dollars; then 960 cents is equal to $9.60 cents—read 9 dollars and 60 cents.

125. What will 7 bushels, 2 pecks of peaches cost, at 45 cents a peck? Ans. $13.50 cts.

126. What is the value of 14 yards, 3 quarters of cloth, at 75 cents for a quarter of a yard? Ans. $44.25.

127. What is the value of a piece of gold, weighing 1 pound, 3 pennyweights, at 12½ cents per grain? Ans. $729.

128. How many bushels of apples may be bought for 4 dollars and 75 cents, at 12½ cents per peck?

Here we say, since 12½ cents will pay for 1 peck, as many times 12½ as there are in 475, so many pecks will 475 cents buy, that is, 38 pecks; and as it takes 4 pecks to make a bushel, as many times 4 as there are in 38, so many bushels will 38 pecks make.

$$\begin{array}{r|l} & \text{Cents.} \\ & \cancel{475} \\ \cancel{25} & \quad 2 \\ & \ \ 19 \\ \end{array}$$

$$4\overline{)38}\ \text{pc.}$$

Ans. 9 bush. 2 pc.

129. How many boxes, each to hold 12 pounds, may be filled out of a hogshead of tobacco weighing 6 cwt. and 3 quarters. Ans. 63 boxes.

130. A grocer has 34 cwt. 2 qrs. 12 lbs. of sugar, which he wishes to put into boxes, each of which will hold 68 lbs., how many boxes will he require? Ans. 57 boxes.

131. How many spoons, each to weigh 2 oz. 5 dwt., can be made out of a tankard weighing 2 lbs. 5 oz. 5 dwt.?

Ans. 13 spoons.

132. How many steps of 2 feet 8 inches, will a man take in walking 2 miles? Ans. 3960 steps.

133. How many casks, each to contain 84 lbs, may be filled out of a hogshead of sugar weighing 17 cwt. 1 qr.?

Ans. 23 casks.

134. A tract of land containing 1299600 square perches, is to be divided into 25 farms of equal size, how many acres will there be in each? Ans. 324 A. 3 r. 24 p.

135. How many English ells in 395 yards?

Ans. 316 E. ells.

136. How many yards in 84 Flemish ells?

Ans. 63 yards.

137. In 285 English ells, how many Flemish ells?

Ans. 475 Fl. ells.

138. Purchased a cargo of molasses consisting of 87 hhd., at 33 cents a gallon, what did it amount to? Ans. $1808.73.

139. A merchant bought 29 bales of cotton cloth, each bale containing 57 yards, the price was 15 cents a yard, what did it amount to? Ans. $247.95.

140. Reduce 86 solid feet to yards.

Ans. 3 solid yards, 5 solid feet.

141. Reduce 191934 solid inches to feet.

Ans. 111 solid feet, 126 solid inches.

142. Reduce 876 solid feet to yards.

Ans. 32 solid yards, 12 solid feet.

143. In 92 quarters of cloth, how many English ells?

Ans. 18 E. ells, 2 qrs.

144. Reduce 467 nails to yards.

Ans. 29 yds. 0 qrs. 3 nas.

145. Reduce 3741 inches to English ells.

Ans. 83 E. ells, 6 ins.

146. Reduce 467 yards to English ells.

Ans. 373 E. ells, 3 qrs.

147. In 27 acres, 2 roods, 16 rods, how many lots of 32 rods each? Ans. 138 lots.

148. How many times does a carriage wheel, 22 feet in circumference, go round in one mile? Ans. 240 times.

149. In 35 tons weight, how many wagon loads of 22 cwt. each? Ans. $31\frac{9}{11}$ loads.

150. In £186 4s., how many guineas of 21 shillings each? Ans. $177\frac{1}{3}$ guineas.

151. How many cannon balls, at 24 lbs. each will it take to weigh 1 ton gross weight? Ans. 93 balls, 8 lbs.

152. How many times must you apply a pole 12 feet long to the ground, to measure 1 mile? Ans. 440 times.

153. How many 9 gallon kegs may be filled from 17 hhd. wine measure? Ans. 119 kegs.

154. How many steps of $2\frac{3}{4}$ feet each, will a man take in walking 25 miles? Ans. 48000 steps.

155. How many sixpences in £95.3s.?

Ans. 3806 sixpences.

156. How many fourpences in £84?

Ans. 5040 fourpences.

COMPOUND ADDITION.

When the numbers to be added are all of the same *name* or *denomination*, the operation is called *simple addition*, but when the numbers to be added are of *different denominations*, the operation is called *compound addition*.

A butcher killed an ox, one of the hind quarters of which weighed 1 cwt. 3 qrs. 17 lbs., the other 2 cwt. 1 qr. 9 lbs., one of the fore quarters weighed 1 cwt. 2 qrs. 16 lbs., the other 1 cwt. 2 qrs. 14 lbs., the hide weighed 3 qrs. 7 lbs., the tallow 2 qrs. 19 lbs., what was the weight of the whole?

(1.)

	cwt.	qrs.	lbs.
	1	3	17
	2	1	9
	1	2	16
	1	2	14
		3	7
		2	19
Ans.	8	3	26

Here we write the numbers of the same name under each other, that is, pounds under pounds, quarters under quarters, and cwt. under cwt., then we commence with the right hand column, which we add up, and find it amounts to 82 lbs., now as 28 lbs. make 1 qr., by dividing the 82 by 28 we find 82 is equal to 2 qrs. and a remainder of 26 lbs., this 26 lbs. we write under the

column of lbs., and count the 2 qrs. with the column of qrs., the whole amounting to 15 qrs., which we divide by 4, and find the 15 qrs. equal to 3 cwt., and a remainder of 3 qrs., which we write under the column of qrs., and count the 3 cwt. with the column of cwt., the whole amounting to 8 cwt.

RULE FOR THE ADDITION OF COMPOUND NUMBERS.

1st. Write the numbers to be added so that units of the same order may stand directly under each other.

2d. Begin with the lowest order, add the numbers together and divide their sum by the number of units of this denomination which is required to make a unit of the next higher; write the remainder under the column added and carry the quotient to the next column. Proceed thus with every denomination.

PROOF THE SAME AS IN SIMPLE ADDITION.

NOTE.—In writing the numbers down if any intermediate denomination is wanting, supply its place with a cipher.

(2.)

	lbs.	oz.	dwt.	grs.
Add	75	8	12	13
	13	6	7	7
	25	8	10	20
	28	5	18	18
Amount	143	5	9	10

(3.)

	lbs.	oz.	dwt.	grs.
Add	67	2	17	21
	78	10	13	15
	26	8	8	3
	14	7	18	12

(4.)

	l.	m.	fur.	p.
Add	117	2	7	39
	132	1	3	29
	999	2	7	39
	137	0	5	16

(5.)

	m.	fur.	p.	yds.	ft.	in.
Add	275	1	25	1	1	6
	137	7	13	2	2	10
	159	2	15	0	0	11
	137	6	37	1	2	10

DRY MEASURE.

(6.)

bu.	pk.	qt.	pt.
3	2	0	1
4	0	3	0
1	3	7	1
1	2	6	1

(7.)

bu.	pk.	qt.	pt.
7	3	7	1
6	0	0	1
9	2	4	1
9	1	6	1

TROY WEIGHT.

(8.)

lbs.	oz.	dwt.	grs.
13	0	17	19
42	11	19	23
31	9	0	4
24	1	13	13

(9.)

lbs.	oz.	dwt.	grs.
90	10	10	10
80	2	2	22
30	3	3	3
20	10	6	11

APOTHECARIES' WEIGHT.

(10.)

℔	℥	ʒ	℈	grs.
7	2	7	2	14
4	6	5	0	17
8	1	6	1	10
2	2	7	0	1

(11.)

℔	℥	ʒ	℈	grs.
21	4	7	2	0
42	11	4	1	15
23	9	3	0	14
11	10	0	1	10

AVOIRDUPOIS WEIGHT.

(12.)

tons.	cwt.	qrs.	lbs.
8	14	1	14
7	11	3	17
9	10	0	10
7	7	1	18

(13.)

cwt.	qrs.	lbs.	oz.	drs.
9	3	27	15	15
41	2	14	7	9
33	1	20	12	14
14	3	10	11	9

CLOTH MEASURE.

(14.)

yds.	qrs.	na.
79	2	1
25	1	3
14	3	2
46	2	1

(15.)

E. e.	qrs.	na.
86	4	2
44	3	0
21	0	2
5	0	3

(16.)

Fl. e.	qrs.	na.
14	2	1
25	1	0
14	0	3
25	0	0

17. Add 15 yards, 3 quarters, 2 nails; 45 yards, 2 quarters; 1 yard, 3 nails; and 125 yards.

18. Add 14 English ells, 3 quarters; 25 English ells, 2 quarters, 3 nails; and 3 quarters, 1 nail.

WINE MEASURE.

(19.)

hhd.	gal.	qts.	pts.
51	53	1	1
27	39	3	0
9	13	0	1

(20.)

tun.	hhd.	gal.	qts.
37	2	37	2
19	1	59	1
28	2	0	0

21. A merchant bought two casks of brandy, containing as follows, viz., 70 gals. 3 qts.; 67 gals. 1 qt., how many hogsheads, of 63 gallons each, in the whole?

LAND MEASURE.

(22.)

acre.	r.	p.
75	3	2
24	0	0
98	1	0
75	3	0

(23.)

acre.	r.	p.
150	3	39
265	2	11
284	1	9
326	0	0

(24.)

yd.	ft.	in.
8	2	12
10	1	95
12	1	115
20	0	46

25. Add 125 acres, 2 roods; 400 acres, 3 roods, 28 perches; 56 acres, 20 perches; and 500 acres.

26. Add 15 yards, 2 feet; 2 yards, 1 foot; 14 yards, 2 feet; and 25 yards.

27. There are 3 fields, which measure as follows, viz., 17 acres, 3 roods, 16 perches; 28 acres, 5 roods, 18 perches; 11 acres, 0 rooks, 25 perches; how much land in the three fields?

SOLID OR CUBIC MEASURE.

(28.)

ton.	ft.	in.
29	36	1229
12	19	64
8	11	917

(29.)

yd.	ft.	in.
75	22	1412
9	26	195
3	19	1091

(30.)

cord.	ft.
37	119
9	110
48	127

TIME.

(31.)

year.	mo.
75	5
16	10
14	11
25	2

(32.)

week.	d.	h.	m.
2	1	10	40
1	6	9	20
3	5	20	12
2	3	7	56

(33.)

day.	h.	m.	s.
4	20	56	54
3	19	25	22
2	8	0	3
	6	0	0

34. Add 10 years, 3 months; 45 years, 6 months; 75 years, 11 months; 15 years; and 96 years.

35. Add 7 weeks, 1 day, 5 hours, 45 minutes; 2 weeks, 4 days, 22 hours; 6 days, 15 hours, 10 minutes; and 5 hours.

ENGLISH MONEY.

(36.)

£	s.	d.	qr.
8	12	9	2
31	6	11	0
42	18	3	1
2	3	8	3
85	1	8	2

(37.)

£	s.	d.	qr.
57	11	11	1
27	13	2	3
48	9	6	1
73	10	9	2

(38.)

£	s.	d.
67	18	10
150	19	6
175	16	8
37	14	7

39. Add £1 1s. 1d.; £10 10s. 10d.; £100 0s. 7d.; £73 4s. 9d.; £43 8s. 11d. Ans. £228 6s. 2d.

40. Add £54 7s. 6d.; 19s. 11d.; £144 3s. 10d.; £132 18s.; £43 6s. 8d. Ans. £375 16s. 2d.

41. Add £444 4s. 11d. 3qrs.; £26 16s. 4d. 1qr.; £372 10d. 2qrs.; £1780 14s. 6d. 2qrs.; £200 10s. 10d. 3qrs. Ans. £2824 7s. 7d. 3qrs.

COMPOUND SUBTRACTION.

Suppose a farmer has 234 bushels, 2 pecks, 3 quarts, of oats, and should sell 176 bushels, 3 pecks, and 2 quarts, how much would he have left?

Here we write the less number under the greater, placing those of the same denomination under each other; that is, bushels under bushels, pecks under pecks, and quarts under quarts; then, commencing with the lowest denomination, we take 2 quarts from 3 quarts, and set down 1 (the remainder) under the quarts; next we take the pecks: here as we cannot take 3 pecks from 2 pecks, we borrow 1 from the bushels in the upper number—this 1 bushel, being equal to 4 pecks, when added to the 2 pecks, make 6 pecks—then we say, 3 pecks from 6 pecks leaves 3 pecks—and as we borrowed 1 from the upper number of bushels, instead of calling the upper number 1 less, we call the lower 1 more, and then proceed as in simple subtraction.

(1.)

	bu.	pc.	qt.
	234	2	3
	176	3	2
Ans.	57	3	1

Work by the following

RULE.

1st. Set down the lesser number under the greater, placing the same denominations directly under each other.

2d. Begin with the lowest denomination, and if the number expressing that denomination be less than the number directly over it, make the subtraction as in simple numbers. But if it be greater, subtract it from the upper number increased by so many units as make one unit of the next higher denomination, and carry this one which has been borrowed to the next higher denomination, as in subtraction of simple numbers.

3d. Do the same for all the denominations, and set down the several remainders, and they will form the true remainder.

PROOF.

Add the remainder to the subtrahend—their sums should be equal to the minuend.

NOTE.—In writing compound numbers, if any intermediate order is vacant, the place must be filled with a cypher.

TROY WEIGHT.

(2.)

	lb.	oz.	dwt.	gr.
From	18	6	16	13
Take	9	6	18	6

(3.)

lb.	oz.	dwt.	gr.
24	0	16	44
4	0	16	16

APOTHECARIES' WEIGHT.

(4.)

	℔	℥	ʒ	gr.
From	9	1	2	12
Take	5	10	1	13

(5.)

℔	℥	ʒ	℈	gr.
34	11	7	2	18
24	11	7	2	19

AVOIRDUPOIS WEIGHT.

(6.)

	ton.	cwt.	qr.	lb.
From	35	11	1	24
Take	14	3	1	16

(7.)

cwt.	qr.	lb.	oz.	dr.
3	2	24	12	13
1	3	21	11	14

(8.)

	ton.	cwt.	qr.	lb.
From	24	19	3	20
Take	15	14	3	22

(9.)

cwt.	qr.	lb.	oz.	dr.
12	2	20	11	14
9	3	14	12	11

LONG MEASURE.

(10.)

	mile.	ft.	rd.	yd.
From	18	5	36	4
Take	11	4	38	1

(11.)

rd.	yd.	ft.	in.
38	2	1	10
16	3	1	11

LAND, OR SQUARE MEASURE.

(12.)

	acre.	rd.	p.
From	327	3	28
Take	77	2	30

(13.)

acre.	rd.	p.	s. yd.	s. ft.	s. in.
2	1	17	19	6	72
1	0	30	16	6	112

CUBIC, OR SOLID MEASURE.

(14.)

	c.	s. ft.	s. in.
From	400	64	1004
Take	163	68	832

(15.)

c.	s. ft.	s. in.
1300	30	146
300	58	875

WINE MEASURE.

(16.)

	tun.	hhd.	gal.
From	25	3	45
Take	17	2	62
Rem.			

(17.)

hhd.	gal.	qt.
45	13	2
25	2	3

(18.)

gal.	qt.	pt
75	3	1
22	1	0

19. Subtract 14 tuns, 2 hogsheads, 10 gallons, from 24 tuns, 1 hogshead, 9 gallons.

20. Take 22 hogsheads, 2 quarts, from 95 hogsheads, 10 gallons, 3 quarts, 1 pint.

DRY MEASURE.

	(21.)			(22.)			(23.)		
	bu.	pc.	qt.	bu.	pc.	qt.	pc.	qt.	pt.
From	95	3	2	84	2	1	3	7	0
Take	22	0	1	36	3	2	2	3	1
Rem.									

24. Subtract 125 bushels, 3 pecks, 2 quarts, from 195 bushels.

25. Subtract 450 bushels from 500 bushels, 3 pecks.

TIME.

	(26.)		(27.)			(28.)			
	yr.	mo.	w.	d.	h.	d.	h.	m.	s.
From	75	3	32	6	20	36	14	30	25
Take	25	4	12	4	22	15	12	25	32
Rem.									

CLOTH MEASURE.

	(29.)			(30.)			(31.)			(32.)		
	yd.	qr.	na.	yd.	qr.	na.	yd.	qr.	na.	yd.	qr.	na.
From	87	1	3	176	3	2	426	1	1	400	0	0
Take	48	2	1	84	1	3	148	2	1	90	0	1
Rem.	38	3	2									

	(33.)			(34.)	(35.)	(36.)			(37.)		
	E.F.	qr.	na.	E.F.	E.F.	E.E.	qr.	na.	E.E.	qr.	na.
From	142	1	1	18	$216\frac{1}{3}$	26	3	2	62	1	0
Take	46	1	2	$4\frac{2}{3}$	$73\frac{2}{3}$	6	3	2	26	3	3

38. From 81 yards, 2 quarters, take 18 yards, 2 quarters, 2 nails. Ans. 62 yds. 3 qrs. 2 nas.

39. Take 45 yards, 2 nails, from 90 yards.
Ans. 44 yds. 3 qrs. 2 nas.

40. Bought 27 yards, 1 quarter of linen, of which 13 yards, 2 quarters, 1 nail were sold—how much remains unsold?

Ans. 13 yds. 2 qrs. 3 nas.

ENGLISH MONEY.

	(41.)			(42.)			(43.)		
	£	s.	d.	£	s.	d.	£	s.	d.
From	5	0	6	10	6	3	145	18	$9\frac{1}{2}$
Take	2	9	3	5	7	6	104	12	$10\frac{1}{4}$
Rem.	2	11	3	4	18	9	41	5	$11\frac{1}{4}$

	(44.)			(45.)			(46.)			(47.)		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
From	5	10	3	37	12	6	25	4	9	46	2	3
Take	4	6	2	27	18	9	14	5	6	25	1	9

	(48.)			(49.)			(50.)		
	£	s.	d.	£	s.	d.	£	s.	d.
From	45	6	$3\frac{3}{4}$	142	10	$3\frac{1}{2}$	2640	18	$11\frac{1}{4}$
Take	22	4	$6\frac{1}{2}$	45	9	$2\frac{3}{4}$	1221	19	$6\frac{1}{2}$

51. Subtract 24 pounds, 10 shillings, and 6 pence, from 36 pounds, 9 shillings, and 3 pence.

52. Subtract 26 pounds, from 120 pounds, 15 shillings, and 9 pence.

53. Subtract 9000 pounds, from 9672 pounds, 18 shillings, and $11\frac{1}{2}$ pence.

54. Subtract 45 pounds, 14 shillings, and $3\frac{1}{2}$ pence, from 500 pounds.

NOTE.—Subtraction and also addition of Federal Money is the same as that of simple numbers, only observing to point off 2 figures on the right hand for cents, and all to the left of this point will be dollars. If there be mills in the given number, point off the first figure on the right for mills, the next two for cents, then all to the left of the cents will be dollars.

FEDERAL MONEY.

	(55.)		(56.)		(57.)	
	$	cts.	$	cts.	$	cts.
From	162	37	3647	32	7345	13
Take	78	43	1478	85	6471	85
Rem.	83	94				

	(58.)		(59.)			(60.)		
	$	cts.	$	cts.	m.	$	cts.	m.
From	12624	04	7642	36	7	9726	42	3
Take	6285	43	826	94	8	7842	67	5

APPLICATION.

61. If from a block of marble containing 8 cubic feet, there be sawed a piece containing 3 cubic feet, 349 cubic inches, how much will remain? Ans. 4 ft. 1379 c. in.

62. If from a stick of round timber containing 2 tons, 18 cubic feet, 1410 cubic inches, there be taken 38 cubic feet, 1720 cubic inches, how much will remain?

Ans. 1 ton, 19 c. ft. 1418 c. in.

63. I bought 11 yards of cloth, and cut off a piece containing 3 yards, 2 quarters, 2 nails; how much remains?

Ans. 7 yds. 1 qr. 2 na.

64. Take 34 yards, 2 nails, from 47 yards, 2 quarters.

Ans. 13 yds. 1 qr. 2 na.

65. Take 14 Flemish ells, 1 quarter, 3 nails, from 25 Fl. ells. Ans. 10 Fl. ells, 1 qr. 1 na.

66. A merchant bought a hogshead of wine, which, after standing in the cellar some time, was found to have leaked so much as to contain only 51 gallons, 1 quart, 2 gills; how much was lost by leakage?

Ans. 11 gals. 2 qts. 1 pt. 2 gills.

67. A vintner filled a vessel containing 1 hogsheads, 17 gallons, 1 pint, out of a tun of wine; how much remains?

Ans. 2 hhd. 45 gals. 3 qts. 1 pt.

68. From 5 days, 10 hours, 27 minutes, 15 seconds, take 2 days, 4 hours, 13 minutes, 29 seconds.

Ans. 3 d. 6 h. 13 min. 46 sec.

69. From 83 days, 17 hours, 3 minutes, take 59 days, 7 hours, 12 minutes, 30 seconds.

Ans. 24 d. 9 h. 50 min. 30 sec.

60. Borrowed $473.67, and have since paid $348.75; how much remains to be paid? Ans. $124.92.

71. Deposited in bank $863.37½; I have since drawn checks to the amount of $587.75; how much have I yet remaining in the bank? Ans. $275.62½.

72. If out of 6 pounds, 10 ounces, 6 drams, 2 scruples, of medicine be taken, 4 pounds, 5 ounces, 4 drams, 1 *scruple*, 17 grains; what quantity will remain?

Ans. 2 ℔ 5 ℥ 2 ʒ 0 ℈ 3 grs.

73. A certain rope is 365 yards, 1 foot, 6 inches long. If 84 yards, 2 feet, 4 inches, be cut off from it, how long will the remainder be? Ans. 280 yds. 2 ft. 2 in.

74. The distance from Philadelphia to Trenton is about 30 miles, 3 furlongs, 16 poles. A person going from one place to the other, stopped at an inn, when he had travelled 18 miles, 3 furlongs, 26 poles; how much further had he still to go? Ans. 11 miles 7 fur. 30 ps.

75. Bought 145 yards, 3 quarters, of cloth, and sold thereof 95 yards, 2 quarters, 3 nails; how much remains?

Ans. 50 yds. 1 na.

76. If from a piece of cambric, containing 25 yards, 3 quarters, 3 nails, there be taken 16 yards, 2 quarters, how much will be left? Ans. 9 yds. 1 qr. 3 na.

77. A farmer had 450 acres, 3 roods of land, but gave his son 150 acres, 3 roods, 25 perches; how much had he remaining? Ans. 299 acres, 3 roods, 15 prs.

78. A merchant bought a quantity of goods in England, for £346 13s. 7d., and sold the same for £435 12s. 9d.; how much did he gain? Ans. £88 19s. 2d.

79. What is the difference between £37 12s. 4d., and £46 9s. 3d.? Ans. £8 16s. 11d.

80. A merchant sold goods to the amount of £136 7s. 6½d., and received in payment £50 10s. 4¾d.; how much remained due? Ans. £85 17s. 1¾d.

81. A man bought a farm for £1256 10s., and in selling it, lost £87 10s. 6d.; how much did he sell it for?

Ans. £1168 19s. 6d.

82. A man bought a horse for £27, and a pair of oxen for £19 12s. 8½d.; how much was the horse valued more than the oxen?

83. A merchant drew from a hogshead of molasses, at one time, 13 gallons, 3 quarts; and another time, 5 gallons, 2 quarts, 1 pint; what quantity was there left?

Ans. 43 gals. 2 qts. 1 pt.

84. A pipe of brandy, containing 118 gallons, sprang a

leak, when it was found only 97 gallons, 3 quarts, 1 pint, remained in the cask; how much was the leakage?

85. There was a silver tankard which weighed 3 pounds, 4 ounces; the lid alone weighed 5 ounces, 7 pennyweights, 13 grains; how much did the tankard weigh without the lid?

COMPOUND MULTIPLICATION.

A merchant bought 3 pieces of muslin, each containing 41 yards, 3 quarters, and 3 nails; how many yards, quarters, and nails, in the three pieces?

(1.)

yd.	qr.	na.
41	3	3
		3
125	3	1

Here we say, 3 times 3 nails make 9 nails, equal to 2 quarters and 1 nail, we write the 1 nail under the nails, and carry the 2 quarters, thus, 3 times 3 quarters make 9 quarters, and 2 quarters to carry, make 11 quarters, equal to 2 yards, and 3 quarters, we write the 3 quarters under the quarters and carry the two yards, thus, 3 times 1 make 3, and 2 to carry, make 5, put down 5, then 3 times 4 make 12.

RULE FOR COMPOUND MULTIPLICATION.

Begin with the lowest denomination, and multiply each denomination separately; divide each product by the number which is required of its own denomination to make 1 of the next higher; write the remainder under the denomination multiplied, and carry the quotient to the product of the next higher denomination.

EXAMPLES.

(2.)

£	s.	d.
5	4	2
		2
10	8	4

(3.)

£	s.	d.
10	15	6
		3
32	6	6

(4.)

£	s.	d.
21	9	$2\frac{1}{4}$
		4
85	16	9

(5.)

£	s.	d.
15	0	$9\frac{3}{4}$
		5
75	4	$0\frac{3}{4}$

(6.)

£	s.	d.
4	2	1
		3

(7.)

£	s.	d.
12	3	9
		6

(8.)

£	s.	d.
25	4	$1\frac{1}{2}$
		7

(9.)

£	s.	d.
96	4	$9\frac{1}{4}$
		8

(10.)

lb.	oz.	dwt.	gr.
14	6	8	9
			10

(11.)

℔	℥	ʒ	℈	gr.
4	3	7	2	15
				11

(12.)

£	s.	d.
17	17	$6\frac{1}{4}$
		12

(13.)

w.	d.	h.	m.	sec.
4	4	10	28	15
				8

(14.)

bu.	pk.	qt.
416	2	6
		2

(15.)

bu.	pk.	qt.
64	3	6
		3

(16.)

gal.	qt.	pt.
63	1	1
		4

(17.)

hhd.	gal.	qt.	pt.
4	60	2	1
			5

(18.)

lb.	oz.	dwt.	gr.
17	5	12	6
			3

(19.)

ton.	cwt.	qr.	lb.	oz.	dr.
6	17	3	13	2	15
					4

(20.)

℔	℥	ʒ	℈	gr.
4	10	7	2	1
				3

(21.)

lea.	m.	fur.	p.
15	2	7	30
			6

(22.)

yd.	ft.	in.
14	2	11
		7

(23.)

yd.	qr.	na.
16	3	3
		8

(24.)

E.ell.	qr.	na.
42	4	1
		9

(25.)

acre.	r.	p.
47	3	15
		2

(26.)

tun.	hhd.	gal.	qt.	pt.
2	3	40	3	1
				10

(27.)

bu.	pk.	qt.
6	3	7
		5

(28.)

bu.	pk.	qt.
14	3	2
		6

(29.)

week.	d.	h.	min.	sec.
4	5	20	32	10
				7

30. How many yards are in 3 pieces, each 26 yards, 2 quarters, 2 nails? Ans. 79 yds. 3 qrs. 2 nas.

31. How much is in 5 pieces, each measuring 34 English ells, 4 quarters, 3 nails? Ans. 174 E. ells, 3 qrs. 3 nas.

32. How many Flemish ells are in 6 pieces, allowing 36 Flemish ells, 2 quarters, 1 nail to each piece?

Ans. 220 F. ells, 1 qr. 2 nas.

33. Multiply 29 yards, 1 quarter, 2 nails, by 8.

Ans. 235 yds.

34 How much grain is in 10 bags, each 2 bushel, 2 pecks, 2 quarts? Ans. 25 bus. 2 pks. 4 qts.

35. How much is in 12 casks, each g ging 31 gallons, 2 quarts, 1 pint? Ans. 379 gals. 2 qts.

36. How much is in 10 bars of silver, each weighing 10 ounces, 10 pennyweights, 10 grains?

Ans. 8 lbs. 9 oz. 4 dwts. 4 grs.

37. What will 9 yards come to, at £1 16s. 6¼d. per yard?

Ans. £16 8s. 8¼d.

38. Tell what 6 yards will come to, at 16 shillings 3¼ pence. Ans. £4 17s. 7½d.

39. If I get 2 bushels, 2 pecks, 4 quarts of oats, for 1 bushel of wheat, how much oats should I have for 10 bushels of wheat? Ans. 26 bus. 1 pk.

40. If it take 4 bushels, 3 pecks, 4 quarts of wheat to make 1 barrel of flour, how much will it take to make 12 barrels? Ans. 58 bus. 2 pks.

41. If 1 case-bottle holds 1 quart, 1 pint, 2 gills, how much will 9 hold? Ans. 3 gals. 3 qts. 1 pt. 2 gills.

42. A goldsmith bought 11 ingots of silver, each of which weighed 4 pounds, 1 ounce, 15 pennyweights, 22 grains. What is the weight of the whole?

Ans. 45 lbs. 7 oz. 15 dwts. 2 grs.

43. A grocer bought 5 hogsheads of sugar, weighing each

12 cwt. 1 quarter, 27 pounds. How much did the whole weigh? Ans. 62 cwt. 1 qr. 23 lbs.

44. Sold 10 pieces of cloth, measuring each 17 yards, 3 quarters, 2 nails. How many yards were there in all?
Ans. 178 yds. 3 qrs.

45. There are 5 bags of apples, each of which contains 2 bushels, 3 pecks. How many bushels are there in the whole? Ans. 13 bus. 3 pks.

46. In 15 loads of hay, each weighing 1 ton, 3 cwt. 2 quarters, how many tons?

47. What is the weight of 24 hogsheads of sugar, each weighing 3 cwt., 2 quarters, 17 pounds?

cwt.	qr.	lb.	
3	2	17	weight of 1 hogshead.
		8	8 times the weight of 1 hogshead.
29	1	16	weight of 8 hogsheads.
		3	3 times the weight of 8 hogsheads.
88	0	20	the weight of 24 hogsheads.

48. How much is 56 times 14 bushels, 3 pecks, 3 quarts 1 pint, dry measure? ($56=8\times7$)
Ans. 832 bus. 0 pk. 4 qts.

49. In 45 ingots of silver, each weighing 3 pounds, 7 ounces, 14 pennyweights, 13 grains, how many pounds?
Ans. 163 lbs. 11 oz. 14 dwts. 9 grs.

50. 36 boxes contain each 18 pounds, 7 ounces, 11 pennyweights, 12 grains of specie, how much do they all weigh?
Ans. 670 lbs. 8 oz. 14 dwts.

COMPOUND DIVISION.

1. Divide 7 cwt. 2 quarters, 12 pounds, 14 ounces, by 4.

	cwt.	qr.	lb.	oz.	dr.
4)	7	2	12	14	0
	1	3	17	3	8

Here we say 4 into 7, 1 time and 3 over; that is, 3 cwt.= 12 quarters, which added to the 2 quarters, make 14 quarters, 4 will go into this, 3 times and 2 over; this 2 quarters is equal to 56 pounds, which, added

to the 12 pounds, make 68 pounds, into which 4 will go 17 times, leaving no remainder; next 4 will go into 14, 3 times and 2 over; this 2 ounces is equal to 32 drams, into which 4 goes 8 times.

RULE FOR COMPOUND DIVISION.

Divide each denomination separately, beginning with the highest. Whenever a remainder occurs, reduce it to the next lower denomination, add it to the number expressed in the lower denomination, and divide it therewith.

2. If £28, 13 shillings, 6 pence be divided equally among 3 men, how much would each man receive?
Ans. £9 11s. $2\frac{2}{3}$d.

3. Divide £46, 15 shillings, 6 pence by 8.
Ans. £5 16s. $11\frac{1}{4}$d.

4. Divide 126 tuns, 3 hogsheads, 40 gallons, 2 quarts, 1 pint, by 9. Ans. 14 tuns, 0 hhds. 25 gals. 2 qts. $0\frac{1}{9}$ pt.

5. Divide 58 miles, 2 furlongs, 32 rods, by 8.
Ans. 7 m. 2 fur. 14 rds.

6. If 15 cwt. 3 quasters, 18 pounds of flour be packed equally in 9 barrels, how much will each barrel contain?
Ans. 1 cwt. 3 qrs. 2 lbs.

7. Paid 17 shillings 6 pence, for 4 bushels of salt, how much was it per bushel? Ans. 4s. $4\frac{1}{2}$d.

8. If 78 cwt. 3 quarters, 10 pounds of sugar be divided equally among 5 men, what will be each one's share?
Ans. 15 cwt. 3 qrs. 2 lbs.

9. A farmer wishes to divide 139 acres, 3 roods, 16 perches, into 8 equal fields, how much will there be in each field?
Ans. 17 a. 1 rd. 37 prs.

10. Suppose a man divides a farm of 214 acres, 3 roods, 12 perches, equally among his 9 sons, how much does each receive? Ans. 23 a. 3 rds. $19\frac{1}{9}$ prs.

11. Divide 158 pounds, 9 ounces, 1 pennyweight, 21 grains, by 9. Ans. 17 lbs. 7 oz. 13 dwts. 13 grs.

12. I man had 2824 gallons, 2 quarts, 1 pint of wine, which he put into 59 casks of equal size, how many gallons did each cask contain?

Here we work by long division. After dividing the gallons, a remainder of 51 gallons occurs; this we multiply by 4, to change it to quarts; adding the 2 quarts to the product, making 206 quarts; into this, 59 goes 3 times—the quotient of course is 3 quarts; in dividing the quarts, a remainder of 29 quarts occurs, this we multiply by 2, to change it to pints, adding the 1 pint to the product, making 59 pints; into this, 59 will go 1 time, the quotient 1 pint.

```
   gals. qt. pt.  gal. qt. pt.
59)2824   2   1  (47    3   1
   236
   ---
    464
    413
    ---
     51 gals.
      4
    ---
 59)206
    177
    ---
     29
      2
     --
     59
     59
```

13. Divide 264 cwt. 3 quarters, 7 pounds, by 19.
Ans. 13 cwt. 3 qrs. 21 lbs.

14. Divide 1987 bushels, 3 pecks, 4 quarts, by 38.
Ans. 52 bus. 1 pk. 2 qts.

15. Divide £3981, 17 shilling, 7 pence, by 47.
Ans. £84 14s. 5d.

16. Divide 748 yards, 3 quarters, 3 nails, by 23.
Ans. 32 yds. 2 qrs. 1 na.

17. Divide 473 pounds, 7 ounces, 6 pennyweights, 6 grains, gross weight, by 34. Ans. 13 lbs. 11 oz. 3 dwts. 3 grs.

18. Divide 632 acres, 3 roods, 28 perches, by 29.
Ans. 21 a. 3 rds. 12 prs.

19. Divide 832 miles, 7 furlongs, 37 yards, by 31.
Ans. 26 m. 6 fur. 207 yds.

APPLICATION.

20. A person purchased a silver dish weighing 2 pounds, 10 ounces, 15 pennyweights, 21 grains; a bowl weighing 1 pound, 1 ounce, 16 pennyweights, 14 grains; and a tankard weighing 2 pounds, 8 ounces, 5 pennyweights, 12 grains, what is the weight of the whole?
Ans. 6 lbs. 8 oz. 17 dwts. 23 grs.

21. Add together 7 pounds, 5 ounces, 6 pennyweights, 18 grains; 9 pounds, 4 ounces, 17 grains; 11 pounds, 13 penny-

weights; 14 pounds, 5 grains; 18 pounds, 1 ounce, 19 pennyweights, 8 grains. Ans. 60 lbs.

22. A druggist mixed four simple articles together, the first weighed 3 ℥, 4 ʒ, 1 ℈; the second, 4 ℥, 3 ʒ, 2 ℈; the third, 4 ʒ, 18 grains; and the fourth, 6℥, 5 ʒ, 2 ℈, 18 grains, what is the weight of the whole? Ans. 1℔, 3 ℥, 2 ʒ, 16 grs.

23. A grocer sold 5 hogsheads of sugar, weighing as follows; the first 8 cwt. 1 quarter, 11 pounds; the second 4 cwt. 2 quarters, 20 pounds; the third 5 cwt. 19 pounds; the fourth 7 cwt. 3 quarters; the fifth 7 cwt. 3 quarters, 9 pounds; what did the whole weigh?

Ans. 33 cwt. 3 qrs. 8 lbs.

24. Add together 13 pounds, 11 ounces, 15 drams; 17 pounds, 13 ounces, 11 drams; 14 pounds, 14 ounces; 16 pounds, 10 drams; 19 pounds, 7 ounces, 12 drams; and 17 pounds, 9 ounces, 9 drams. Ans. 99 lbs. 9 oz. 9 dr.

25. Two men start from the same place, one travels 104 miles, 1 furlong, 10 rods, due east; and the other 95 miles, 6 furlongs, 30 rods, due west; how far are they apart?

Ans. 200 ms.

26. What is the amount of £23, 11 pence; £13, 17 shillings, 3 quarters; 16 shillings, 8 pence; and 11 pence, 3 quarters?

Ans. £37 15s. 7d. 2qrs.

27. Bought the following quantities of oil, viz., 12 gallons, 3 quarts; 2 hogsheads, 42 gallons, 2 quarts, 1 pint; and 13 hogsheads, 56 gallons; what was the whole amount?

Ans. 16 hhds. 48 gal. 1 qt. 1 pt.

28. Add together £250, 18 shillings, 9 pence, 3 quarters; £16, 7 shillings, 2 quarters; £21, 19 shillings, 3 pence; 18 shillings, 6 pence; and £36. Ans. £326 3s. 7d. 1 qr.

29. What is the amount of 5 cwt. 3 quarters, 27 pounds; 2 quarters, 29 pounds; 12 cwt. 1 quarter, 17 pounds; and 36 cwt. 16 pounds? Ans. 55 cwt. 1 qr. 5 lbs.

30. Bought, at one time, 7 bushels, 3 pecks of wheat; at another, 9 bushels, 1 peck; and had, previously, in each of two bins, 6 bushels, 3 pecks; what was the whole amount?

Ans. 30 bus. 2 pks.

31. Sold one cow, for £10, 15 shillings, 6 pence; another, for £6, 19 shillings, 11 pence; and a colt, for £12, 6 shillings, 4 pence; how much did they all amount to? Ans. £30 1s. 9d.

32. Bought four casks of wine, of which the first contained 42 gallons, 2 quarts, 1 pint; the second, 65 gallons, 1 pint; the third, 50 gallons, 3 quarts; and the fourth, 55 gallons, 1 quart, 1 pint; how many gallons did I purchase?
Ans. 213 gals. 3 qts. 1 pt.

33. Purchased three pieces of land; the first contained 17 acres, 1 rood, 35 rods; the second, 36 acres, 2 roods, 21 rods; and the third, 46 acres, 37 rods; how much land did I purchase? Ans. 100 as. 1 rood, 13 rds.

34. Bought three pieces of lace, containing as follows: No. 1, 17 yards, 3 quarters, 2 nails; No. 2, 25 yards, 2 quarters, 1 nail; No. 3, 32 yards, 3 quarters, 2 nails; how many yards were bought? Ans. 76 yds. 1 qr. 1 na.

35. A person has three farms—the first contains 120 acres, 3 roods; the second, 256 acres, 1 rood; the third, 300 acres; how many acres are in all? Ans. 677 as.

36. Sold two casks of cider—one of which contained 31 gallons, 3 quarts, and the other 36 gallons, 2 quarts, 1 pint; how much was in the two casks? Ans. 68 gals. 1 qt. 1 pt.

37. There are three bags of wheat—the first contains 2 bushels, 3 pecks, 7 quarts; the second, 3 bushels, 3 pecks, 4 quarts; the third, 4 bushels; how much is in the three bags? Ans. 10 bus. 3 pks. 3 qts.

38. In 6 hogsheads of tobacco, weighing as follows: No. 1, 13 cwt. 2 quarters, 12 pounds; No. 2, 15 cwt. 3 quarters, 19 pounds; No. 3, 14 cwt. 1 quarter, 23 pounds; No. 4, 17 cwt. 3 quarters, 10 pounds; No. 5, 16 cwt. 1 quarter, 18 pounds; No. 6, 10 cwt. 2 quarters, 25 pounds; how many parcels, each 41 pounds? Ans. 243.

39. Bought 5 bales of cloth, containing, No. 1, 42 yards, 3 quarters; No. 2, 35 yards, 2 quarters; No. 3, 39 yards, 3 quarters; No. 4, 45 yards, 3 quarters; No. 5, 27 yards, 2 quarters; how many yards are there in the whole? And admitting each quarter to cost $1.25 cents, what is their value?
Ans. 191 yds. 1 qr. Value, $956.25.

40. In one tract of land there are 235 acres, 2 roods, 16 perches; in another, 346 acres, 1 rood, 29 perches; in a third, 198 acres, 3 roods, 35 perches, and in each of two others, 225 acres, 2 roods, 28 perches; how many acres in all? Ans. 1232 as. 1 rd. 16 prs.

41. The great bell at Moscow, the largest in the world, weighs 198 tons, 2 cwt. 1 quarter; the bell at Oxford, the largest in England, 7 tons, 11 cwt. 3 quarters, 4 pounds; St. Paul's bell, at London, 5 tons, 2 cwt. 1 quarter, 22 pounds; and the Tom of Lincoln, 4 tons, 16 cwt. 3 quarters, 18 pounds; what is the sum of their weights?

Ans. 215 ts. 13 cwt. 1 qr. 16 lbs.

42. If one cistern contains 25 hogsheads, 27 gallons, 3 quarts; a second, 37 hogsheads, 26 gallons, 2 quarts; a third, 35 hogsheads, 54 gallons, 1 quart; and a fourth, 45 hogsheads, 15 gallons, 3 quarts; what quantity will they jointly contain? Ans. 143 hhds. 61 gals. 1 qt.

PROMISCUOUS QUESTIONS.

43. Bought a barrel of flour for 6 dollars 75 cents, a cord of wood for 4 dollars 25 cents, a hat for 3 dollars 62 cents, three yards of cloth for 12 dollars 68 cents, and two bushels of potatoes for 96 cents; what did the whole amount to?

Ans. \$28.26.

Note.—Observe to place dollars under dollars, and cents under cents, the cents must occupy two places of figures, in all cases; thus, suppose you are required to write in figures, twenty-three dollars and seven cents, it must be written thus, \$23.07, that is, if the number of cents is less than 10, a cipher must be placed on the left of it. This character \$, signifies that the number it is placed before, is dollars, from which we separate cents by a point.

44. Add the following, \$24.65, \$37.25, \$67.25, \$32.47, \$9.07, \$4.02, \$2.65, \$39.06. Ans. \$216.42.

45. A man bought a horse for \$67.75, and sold him for \$92.87; how much did he gain? Ans. \$25.12.

46. A lady went shopping, and bought 3 yards of silk, at \$2.45 a yard, 7 yards of lace, at 25 cents a yard, 12 yards of linen, at 56 cents a yard, and a shawl for \$9.40; what did the whole cost? Ans. \$25.22.

47. What cost 28 cords of wood, at \$3.87 a cord?

Note.—Multiply the \$3.87, as you would the number 387 by 28, and point off two figures to the right, for cents, the figures to the left of this point will be dollars.

48. Bought 43 tons of hay, at $13.75 a ton; what did the whole amount to?

$$\begin{array}{r} \$13.75 \\ 43 \\ \hline 4125 \\ 5500 \\ \hline \$591.25 \end{array}$$

49. What is the value of a hogshead of wine containing 63 gallons, at $1.63 per gallon? Ans. $102.69. Why?

50. Sold 19 cords of wood, at $5.75; what did it amount to? Ans. $109.25. Why?

51. What will 369 tons of potash amount to, at $132.55 a ton? Ans. $48910.95. Why?

52. Bought 47 hogsheads of salt, each containing 7 bushels, at $1.12 a bushel; what was the cost of the whole? Ans. $368.48. Why?

53. A farmer sold 347 bushels of wheat, at $1.25 a bushel; what did it amount to? Ans. $433.75.

54. What will 17½ tons of coal cost, at $9.62 cents per ton? Ans. $168.35. Why?

55. What will 148 tons of plaster of Paris cost, at $2.28 a ton? Ans. $337.44.

56. If 13 bushels of apples cost $3.38, how much is that a bushel? Ans. 26 cents. Why? Since 13 bushels cost 338 cents, 1 bushel will cost 1 thirteenth of 338 cents. How do you get one-thirteenth of any number?

Ans. Divide the number by 13.

57. Bought 38 barrels of cider for $52.25 cents; what was the price of 1 barrel, at that rate?

Ans. $1.37½. Why?

58. Suppose a man earns $8.75 a week, and spends $5.87½ a week, how much will he save at this rate, in 42 weeks? Ans. $120.75.

59. A tailor bought 27⅘ yards of broadcloth, at $4.75 a yard, how much did he pay for the whole?

Ans. $132.05.

60. Bought 17 bushels of potatoes at 37½ cents a bushel, and 45 bushels of oats at 35 cents a bushel, what is the amount of the whole? Ans. $22.12½.

61. If 78 barrels of fish cost \$303.42, what will one barrel cost? Ans. \$3.89.

62. A dealer in flour bought 342 barrels of Pennsylvania brand, for \$1768.14, how much was that a barrel?
Ans. \$5.17.

63. If I pay \$468.75 for 750 pounds of wool, what is the value of 1 pound? Ans. \$0.625 or thus, \$0.62½.

64. If a piece of cloth, measuring 125 yards, cost \$181.25, what is that a yard? Ans. \$1.45.

65. If 536 quintals of fish cost \$1913.52, how much is that a quintal? Ans. \$3.57.

66. Bought a farm, containing 84 acres, for \$3213; what did it cost me per acre? Ans. \$38.25.

67. At \$954 for 3816 yards of flannel, what is that a yard? Ans. \$0.25.

68. Bought 23 firkins of butter, each containing 42 pounds, for 16½ cents a pound; what would that be a firkin, and how much for the whole? Ans. \$159.39 for the whole.

69. A man killed a beef, which he sold as follows, viz., the hind quarters, weighing 129 pounds each, for 5 cents a pound; for the fore quarters, one weighing 123 pounds, and the other 125 pounds for 4½ cents a pound; the hide and tallow weighing 163 pounds, for 7 cents a pound, to what did the whole amount? Ans. \$35.47.

70. A merchant bought \$37,520 worth of flour, and after selling it all, found he had gained \$4,281; what did he sell it for? Ans. \$41,801.

71. A man bought a farm for \$3,230, paying with a farm worth \$1,240, a yoke of oxen worth \$75, a wagon worth \$83, three cows worth \$54, and the rest in money; how much money did he pay? Ans. \$1,778.

72. A barrel of pork contains 200 pounds; now if you buy 3 barrels, each of which lacks 4 pounds of being full, at 16 cents a pound, what must you give for the whole?
Ans. \$94.08.

73. A merchant bought a ship for \$11,200, and after expending \$1,248 in repairs, sold her for \$14,000; what did he gain? Ans. \$1,553.

74. 10 children inherited \$4,535 a-piece; what did they all inherit? Ans. \$45,350.

75. If I give $141 for a piece of cloth containing 48 yards, how much must I sell it for to gain $1 a yard?

Ans. $189.

76. A man who owned the following quantities of land, 500 acres, 17 acres, 121 acres, and 98 acres, sold 325 acres; how much did he have left? Ans. 411 acres.

77. What number subtracted from 2,521 leaves 178?

Ans. 2,343.

78. How long will it take a steamboat to go 345 miles, if she moves 15 miles an hour? Ans. 23 hours.

79. A man sold a cargo of salt for $2,300, and made on it $625; what did it cost him? Ans. $1,675.

80. How many 4 gallon kegs, 5 gallon kegs, 7 gallon kegs, and 8 gallon kegs, and of each an equal number, can be filled from 8 casks of wine, each containing 36 gallons?

Ans. 12 of each.

DECIMAL FRACTIONS.

A decimal fraction is a fraction whose denominator is 10, 100, 1000, or some other power of 10. A decimal fraction is usually expressed by writing the numerator only, with a point prefixed to it; thus, $\frac{3}{10}$, $\frac{17}{100}$, $\frac{436}{1000}$, &c., are decimal fractions, and are expressed thus, .3, .17, .436, &c.; when a whole number and decimal are written together, the decimal point is placed between them thus, 37.4, is $37\frac{4}{10}$, 6.37, is $6\frac{37}{100}$.

Whole numbers increase in value, in a tenfold proportion, from right to left, and decimals decrease in the same proportion, from left to right.

Thousands.	Hundreds.	Tens.	Units.	.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.
4	3	2	1	.	1	2	3	4

Ciphers annexed to decimals, neither increase nor decrease their value; thus, .7, .70, .700, are of the same value, being $\frac{7}{10}$, $\frac{70}{100}$, $\frac{700}{1000}$, but ciphers prefixed to decimals, decrease their value in a tenfold proportion; thus, .7, .07, .007, are of different values.

Hence it will be perceived that decimals may be operated upon in precisely the same manner as whole numbers are, only observing to write the decimal point in its proper place.

ADDITION OF DECIMALS.

RULE.

Place the numbers so that the decimal points may stand directly under each other, then add as in whole numbers, and write the point in the sum immediately below the other points.

EXAMPLES.

1. Add .3, .76, .43, .047, 4.36.

```
  .3
  .76
  .43
  .047
 4.36
 -----
 5.897
```

2. Add .4, .36, .004, 7.489, 3.306.

```
   .4
   .36
   .004
  7.489
  3.306
 ------
 11.559
```

3. Add 6.43, .0075, 19.6, 4.76, and 34.5. Ans. 65.2975.
4. Add 72.4, 36.147, 8.6245, 79.482, 96.37.
5. Add 364., 74.3, .047, 87.63478, 47.39.
6. Add 476.473, 9.765, 437., 225.75, 92.84563.
7. Add 37.94, 75.25, 360.42, 347.625, 1463.278.
8. Add 462.943, 123.404, 862.31, 9876.47842.
9. Add 4463., 47.32, 9465., 478.75, 932.4876.

SUBTRACTION OF DECIMALS.

RULE.

Place the less number under the greater, and subtract as in whole numbers, and point off as many figures for decimals as there are in that number, which contains the greatest number of decimals.

EXAMPLES.

1. From 764.375 subtract 79.6473.

$$\begin{array}{rr} & 764.375 \\ & 79.6473 \\ \hline \text{Ans.} & 684.7277 \end{array}$$

2. From 3762.7345 take 947.6432. Ans. 2815.0913.
3. From 406.37296 take 79.627. Ans. 326.74596.
4. From 2167. take 48.3624. Ans. 2118.6376.
5. From 3704.3678 take 21.96843.
6. From 6749.358702 take 4.70069.
7. From 24.0603 take 7.90485.
8. From seven hundred and thirty-eight, and seventeen-hundredths, take eighty-three, and twenty-nine thousanths.

Ans. 655.141.

9. A man owning a farm containing 325.13 acres, sold off 59.325 acres, how many acres had he left?

Ans. 265.805 acres.

10. From 37640. take .0462. Ans. 37639.9538.

MULTIPLICATION OF DECIMALS.

RULE.

Multiply as in whole numbers, and point off as many figures for decimals, in the product, as there are decimals in the multiplicand *and* multiplier, but if there be not so many figures in the product as there are decimal figures in both factors, prefix ciphers to supply the deficiency.

EXAMPLES.

1. Multiply 47.38 by 4.34.

```
      47.38
       4.34
      -----
      18952
     14214
    18952
   --------
Ans. 205.6292
```

2. Multiply .163 by .047.

```
   .163
   .047
   ----
   1141
   652
 -------
 .007661
```

3. Multiply 76.81 by 3.2. Ans. 245.792.
4. Multiply .612 by 4.12. Ans. 2.52144.
5. Multiply .1234 by .0046. Ans. .00056764.
6. Multiply 29.831 by .952. Ans. 28.399112.
7. Multiply 3.92 by 196. Ans. 768.32.
8. Multiply .154 by .136. Ans. .020944.
9. Multiply 7.45 by 8.36. Ans. 62.282.
10. Multiply 25.238 by 12.17. Ans. 307.14646.
11. Multiply .3759 by .945. Ans. .3552255.
12. Multiply 79000. by .079. Ans. 6241.
13. What will 26.58 cords of wood cost at $5.68 per cord? Ans. 150.9744.
14. What will 37.48 bushels of wheat cost at $1.175 per bushel? Ans. $44.039.

NOTE.—The learner will observe that the answer contains but three decimals, whereas there should be five, according to the rule, but the two figures to the right of the 39 being ciphers are of no account, and are therefore omitted.

DIVISION OF DECIMALS.

Divide as in whole numbers, and point off as many decimals in the quotient, as the number of decimals in the dividend exceeds those of the divisor.

If there be not so many figures in the quotient as the rule directs to be pointed off, prefix ciphers to supply the defect. If the number of decimal figures in the divisor be greater than that of the dividend, annex ciphers to the dividend.

EXAMPLES.

1. Divide 86.075 by 27.5. Ans. 3.13.
2. Divide 24.73704 by 3.44. Ans. 7.191.
3. Divide .863972 by .92. Ans. .9391.
4. Divide 4.13 by 572.4. Ans. .00721+
5. Divide 234.70525 by 64.25. Ans. 3.653.
6. Divide 186.9 by 7.476. Ans. 25.
7. Divide 315 by 124. Ans. 2.5403+
8. Divide 9811.0047 by 108649. Ans. 0903.
9. Divide 637.531 by 4.72. Ans. 135.070127+
10. Divide 267.15975 by 13.25. Ans. 20.163.
11. Divide 210 by 240. Ans. .875.
12. Divide 315 by 124. Ans. 2.5403+

REDUCTION OF DECIMALS.

To reduce a vulgar fraction to a decimal.

RULE.

Annex ciphers to the numerator for a dividend, and divide by the denominator.

EXAMPLES.

1. Reduce $\frac{5}{8}$ to a decimal.

$$8\overline{)5000}$$
$$.625$$

2. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{11}{12}$, $\frac{3}{16}$, $\frac{3}{25}$, $\frac{1}{8}$, to decimals.
Ans. .75, .833, .91666, .1875, .12, .125.
3. Reduce $\frac{11}{16}$, and $\frac{19}{24}$, to decimals.
Ans. .6875, and .791666.
4. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ to a decimal.

Thus,
$$\begin{array}{r|l} \cancel{3} & 20000 \\ \cancel{4} & \cancel{3} \\ \cancel{5} & \cancel{4} \\ 6 & \cancel{5} \\ \hline & .3333 \end{array}$$

5. Reduce $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{2}{3}$ to a decimal.

$$\begin{array}{r|l} 8 & \cancel{7} \\ \cancel{4} & \cancel{3} \\ \cancel{7} & \cancel{6}\ 3000 \\ \cancel{3} & \cancel{2} \\ \hline & .375 \end{array}$$

6. Reduce $\frac{3}{16}$ of $3\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{5}{8}$ of $\frac{1}{2}$ to a decimal.
Ans. .1875.

7. Reduce $\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{4}{5}$ to a decimal.
Ans. .45714+

8. Reduce $\frac{9}{16}$ of $8\frac{1}{2}$ of $\frac{11}{12}$ of $\frac{3}{17}$ of $\frac{1}{4}$ to a decimal.
Ans. .193359.+

To reduce the lower denominations of a compound number to the decimal of a higher denomination.

RULE.

Place the different denominations of the compound number on the right of a vertical line, directly under each other, proceeding in order, from the least to the greatest;—on the left of the line opposite to each of the given numbers, place that number which is required of the less to make one of the next greater, then proceed to divide the number on the right by that standing opposite to it, on the left of the line, and place the quotient of each division as decimal parts on the right of the number next below it.

EXAMPLES.

9. Reduce 14s. 6d. 3qrs. to the decimal of a pound.

$$\begin{array}{rr|l} & 4 & 3 \\ & 12 & 6.75 \\ & 20 & 14.5625 \\ \hline \text{Ans.} & £ & .728125 \end{array}$$

10. Reduce 13 pennyweights, 18 grains, to the decimal of a pound.

24	18
20	13.75
12	.6875
Ans.	.0572916 lb.

11. Reduce 3 quarters, 14 pounds, to the decimal of a hundred weight. Ans. .875 cwt.

12. Reduce 429 yards, 2 feet, 6 inches, to the decimal of a mile. Ans. .244223+m.

13. Reduce 2 roods, 16 perches, to the decimal of an acre. Ans. .6 acres.

14. Reduce 2 furlongs to the decimal of a league. Ans. .083333 league.

15. Reduce 10s. 9¼d. to the decimal of a pound. Ans. £.5385416+

16. Reduce 72 days to the decimal of a year, computing the year at 365 days? Ans. .1972602+

17. Reduce 2 quarters, 21 pounds, 8 ounces, 12 drams to the decimal of a cwt. Ans. .6923828+cwt.

18. Reduce 16 gallons, 3 quarts, 1 pint of wine to the decimal of a hogshead. Ans. .267857+

19. Reduce 3 quarts, 1 pint to the decimal of a bushel. Ans. .1090375.

20. Reduce 7 hours, 35 minutes, 48 seconds to the decimal of a day. Ans. .31652777+

To reduce the decimal of a higher denomination to its value in whole numbers of a lower denomination.

RULE.

Multiply the decimal by that number of the next lower denomination which makes a unit of the higher, and point off as many figures for decimals, as there are in the given decimal. Proceed in like manner with the decimal in each succeeding product.

EXAMPLES.

21. What is the value of .765 of a £.

```
    £
  .765
    20
  ------
  15.300 s.
    12
  ------
  3.600 d.
     4
  ------
  2.400 Ans. 15s. 3d. 2.4fr.
```

22. What is the value of .6725 of a cwt.?

```
    cwt.
   .6725
       4
   ------
  2.6900 qr.
      28
   ------
   55200
  13800
  -------
 19.3200 lbs.
      16
  -------
  5.1200 oz.
      16
  -------
  1.9200  Ans. 2 qrs. 19 lbs. 5 oz. 1.92 drs.
```

23. What is the value of .6875 of a yard?
Ans. 2 qrs. 3 nas.

24. What is the value of .4694 pounds Troy?
Ans. 5 oz. 12 dwt. 15.744 grs.

5. What is the value of .2083 hogsheads?
Ans. 13.1229 gals.

26. What is the value of .4765 of a bushel?
Ans. 1 pk. 7.248 qts.

27. What is the value of .785 of an acre?
Ans. 3 roods 5 rds. 18.15 sq. yds.

28. What is the value of .0653 of a mile in rods, yards, feet, and inches? Ans. 20 rods, 4 yds. 2 ft. 9.408 in.

29. What is the value of .761 of a day?
Ans. 18 h. 15 min. 50.4 sec.

30. What is the value of .95 of a pound Troy?
Ans. 11 oz. 8 dwt.

PERCENTAGE.

The term per cent. is an abbreviation of per centum, which signifies by the hundred.

1. What is 6 per cent. of $34.73?

In order to find the 100th part of any number, we need only point off 2 figures to the right as decimals.

$$\begin{array}{r} 34.73 \\ 6 \\ \hline \text{Ans. } \$208.38 \end{array}$$

2. What is 3 per cent. of $439?

The two figures which we here point off to the right of the dollars as decimals are cents, because a cent is the 100th part of a dollar.

$$\begin{array}{r} 4.39 \\ 3 \\ \hline \text{Ans. } \$13.17 \end{array}$$

3. What is 9 per cent. of $375? Ans. $33.75.
4. What is 8 per cent. of $2767? Ans. $221.36.
5. What is 17 per cent. of $362? Ans. $61.54.
6. What is 7 per cent. of $2745? Ans. $192.15.
7. What is 12 per cent. of $6748? Ans. $809.76.
8. What is 7½ per cent. of $6428?

Here we point off 2 figures, for decimals, and multiply by 7½ changed to an improper fraction.

$$\begin{array}{r|r} \cancel{2} & \cancel{64.28} \\ & 32.14 \\ & 15 \\ \hline & \$482.10 \end{array}$$

9. What is 7⅕ per cent. of $4365? Ans. $314.28.
10. What is 25 per cent. of $648?

25 being ¼ of 100, we simply take ¼ of the $648 for the answer.

$$\begin{array}{r} 4)\underline{648} \\ \text{Ans. } \$162 \end{array}$$

11. What is 25 per cent. of $376? Ans. $94.
12. What is 25 per cent. of $7363? Ans. $1843.25.
13. What is 12½ per cent. of $392? Ans. $49.

12½ is ⅛ of 100, hence, to get 12½ per cent. we divide by 8.

14. What is 12½ per cent. of $479? Ans. $59.87½.
15. What is 20 per cent. of $735? Ans. $147=⅕ of 735.
16. What is 20 per cent. of $9347? Ans. $1869.40.
17. What is 37½ per cent. of $432?

Ans. $162=⅜ of 432.

18. What is 37½ per cent. of $192? Ans. $72.
19. What is 5 per cent. of $6340?

Ans. $317=$\frac{1}{20}$ of $6340.

20. What is the $6\frac{1}{4}$ per cent. of \$192? Ans. \$12=$\frac{1}{16}$ of \$192.
21. What is $6\frac{1}{4}$ per cent. of \$348? Ans. \$21.75.
22. What is $33\frac{1}{3}$ per cent. of \$4575? Ans. \$1525=$\frac{1}{3}$ of 4575.
23. What is $33\frac{1}{3}$ per cent. \$148? Ans. \$$49.33\frac{1}{3}$.
24. What is $18\frac{3}{4}$ per cent. of \$384? Ans. \$72=$\frac{3}{16}$ of \$384.
25. What is $18\frac{3}{4}$ per cent. of \$474? Ans. \$$88.87\frac{1}{2}$.
26. What is $31\frac{1}{4}$ per cent. of \$576? Ans. \$180=$\frac{5}{16}$ of \$576.
27. What is $31\frac{1}{4}$ per cent. of \$346? Ans. $108.12\frac{1}{2}$.
28. What is $16\frac{2}{3}$ per cent. of \$8376? Ans. \$1396=$\frac{1}{6}$ of \$8376.
29. What is $16\frac{2}{3}$ per cent. of \$437? Ans. $72.83\frac{1}{3}$.
30. What is $8\frac{1}{3}$ per cent. of \$348? Ans. \$29=$\frac{1}{12}$ of \$348.
31. What is $8\frac{1}{3}$ per cent. of \$536? Ans. \$$44.66\frac{2}{3}$.
32. What is $9\frac{1}{11}$ per cent. of \$473? Ans. \$43=$\frac{1}{11}$ of \$473.
33. What is $9\frac{1}{11}$ per cent. of \$328? Ans. \$$29.81\frac{9}{11}$.
34. What is $11\frac{1}{9}$ per cent. of \$846? Ans. \$94=$\frac{1}{9}$ of \$846.
35. What is $11\frac{1}{9}$ per cent. of \$237? Ans. \$$26.33\frac{1}{3}$.
36. What is $14\frac{2}{7}$ per cent. of \$483? Ans. \$69=$\frac{1}{7}$ of \$483.
37. What is $14\frac{2}{7}$ per cent. of \$346? Ans. \$$49.42\frac{6}{7}$.

SQUARE NUMBERS.

The square of a number is its second power, or it is the product of a number multiplied by itself, thus the square of 4 is 16, because 4 multiplied by 4 is equal to 16. The square of a number may be represented thus, $4^2=16$.

The following table should be thoroughly learned by the pupils.

$1^2=1$	$8^2=64$	$15^2=225$
$2^2=4$	$9^2=81$	$16^2=256$
$3^2=9$	$10^2=100$	$17^2=289$
$4^2=16$	$11^2=121$	$18^2=324$
$5^2=25$	$12^2=144$	$19^2=361$
$6^2=36$	$13^2=169$	$20^2=400$
$7^2=49$	$14^2=196$	

The number itself, or first power, is called the root, to distinguish it from the 2d power, or square.

MENTAL OPERATIONS.

1. What cost 13 yards of muslin, at 13 cents a yard? Why?
2. What will 14 pounds of butter cost, at 14 cents a pound? Why?
3. What cost 18 pineapples, at 18 cents a-piece? Why?
4. What cost 15 dozen eggs, at 15 cents a dozen? Why?
5. What cost 17 pounds of coffee, at 17 cents a pound? Why?
6. What cost 19 books, at 19 cents a-piece? Why?
7. What cost 16 quires of paper, at 16 cents a quire? Why?
8. What cost 20 yards of linen, at 20 cts. a yard? Why?
9. What cost 15 bushels of lime, at 17 cents a bushel?

Thus, $16^2-1=\$2.55$ Ans.

NOTE.—The product of any two numbers is equal to the square of the mean, minus the square of half their difference.—The mean is a number as much greater than the less, as it is less than the greater.

10. What cost 14 spelling books, at 18 cents a piece?

Thus, 16^2-4 $2.52 Ans.

Remarks.—16 is as much greater than 14, as it is less than 18; 16 is therefore the arithmetical mean between 14 and 18, and half the difference of 18 and 14 is 2, and the square of 2 is 4, or we may say, the differenee between the mean and either of the given numbers is 2, the square of which is 4, hence to multiply any two numbers together,

RULE.

From the square of the mean, subtract the square of the difference between either of the given numbers and the mean.

11. What cost 17 quarts of oil, at 13 cents a quart?

Thus, $15^2-4=\$2.21$ Ans.

12. What cost 15 gallons of molasses, at 19 cents a gallon?
13. What cost 16 watermelons, at 18 cents a piece?
14. What cost 19 primers, at 13 cents a-piece?
Ans. $16^2-9=\$2.4$[illegible], 7
15. What cost 13 tons of hay, at $14 a ton?

Thus, $13^2+13=\$182$.

That is, to multiply any two numbers together, which differ by a unit, we add the less number to its square, for 13 added to 13 times 13, make 14 times 13.

16. What cost 16 ounces of gold dust, at 17 dollars per ounce? Ans $16^2+16=\$2.72$.
17. What cost 14 yards of ribbon, at 15 cents a yard?
18. What cost 17 dozen buttons, at 18 cents a dozen?
19. What cost 18 bushels of apples, at 19 cents a bushel?

TABLE OF SQUARE NUMBERS CONTINUED.

Here let the learner observe that the two right-hand figures, in the square of any number, as much less than 25 as another is greater, are the same in one case as in the other, for example, in the square of 23 and 27, the one as much less as the other is greater than 25, the two right-hand figures are the same.—This law holds true in all cases. The facilities in calculations which the learner will derive from a familiarity with the squares of all numbers up to 100, will abundantly repay him for all the labour he may bestow in learning them.

$21^2=441$
$22^2=484$
$23^2=529$
$24^2=576$
$25^2=625$
$26^2=676$
$27^2=729$
$28^2=784$
$29^2=841$
$30^2=900$

MENTAL OPERATIONS CONTINUED.

20. What cost 21 bushels of lime, at 21 cents a bushel?
21. What cost 22 quires of paper, at 22 cents a quire?
22. What cost 23 cows, at $23 a head? Why?
23. If a boy can solve 24 questions in one hour, how many can he solve at that rate in 24 hours? Why?
24. What will 25 pounds of butter cost, at 25 cents a pound? Why?
25. What will 26 gallons of wine cost, at 26 cts. a gallon?
26. What cost 27 yards of linen, at 27 cents a yard? Why?
27. What cost 28 bushels of oats, at 28 cents a bushel?
28. What cost 29 quarts of ice cream, at 29 cents a quart?
29. What cost 30 copies of Smith's Grammar, at 30 cents a copy?
30. What cost 23 shad, at 27 cents a piece?

Ans. $25^2-4=\$6.21$.

31. What cost 22 pecks of dried peaches, at 28 cents a peck?

Ans. $25^2-9=\$6.16$.

32. What cost 24 dozen herring, at 28 cents a dozen?
33. What cost 27 copybooks, at 29 cents a-piece?
34. What cost 23 melons, at 24 cents each?

Ans. $23^2+23=\$5.52$.

35. What cost 26 tons of hay, at $27 a ton?
36. What cost 18 acres of land, at $22 an acre?
37. What cost 16 hogsheads of tobacco, at $24 a hogshead?
38. What cost 17 pounds of tea, at 23 cents a pound?

TABLES OF SQUARE NUMBERS CONTINUED.

$31^2=961$	$19^2=361$
$32^2=1024$	$18^2=324$
$33^2=1089$	$17^2=289$
$34^2=1156$	$16^2=256$
$35^2=1225$	$15^2=225$
$36^2=1296$	$14^2=196$
$37^2=1369$	$13^2=169$
$38^2=1444$	$12^2=144$
$39^2=1321$	$11^2=121$
$40^2=1600$	$10^2=100$

The fourth column is placed here in order to afford the learner an opportunity of observing further, that the *two right-hand figures* in the square of any number which is as much less than 25 as another is greater, are the same in the former case as in the latter, or which is the same thing, the *two right-hand figures* in the square of any number as much *less* than 20 as another is *greater* that 30, are the same in one case as the other.

MENTAL OPERATIONS CONTINUED.

39. What cost 34 pounds of salmon, at 34 cents a pound? Why?
40. What cost 37 yards of cotton cloth, at 37 cents a yard? Why?
41. What cost 32 bushels of potatoes, at 32 cents per bushel?
42. What cost 39 pigeons, at 39 cents each? Why?
43. What cost 38 penknives, at 38 cents each?
44. What cost 36 yards of calico, at 36 cents a yard?
45. What cost 34 sheep-skins, at 38 cents each?

Ans. $36^2-4=$ \$12.92.

46. What cost 32 pocket maps, at 36 cents each?
47. What cost 33 head of cattle, at \$39 a head?
48. What can a man earn in 26 months, at \$34 a month?
49. What must you pay for 24 days' boarding, at 36 cents a day?
50. If a boy can earn 35 cents a day, how much can he earn in 25 days?
51. If one acre will produce 37 bushels of corn, how much will 23 acres produce at the same rate?
52. If a railroad car runs 36 miles an hour, how many miles will it run at the same rate in 24 hours?

TABLE OF SQUARE NUMBERS CONTINUED.

$41^2=1681$	9^2	$46^2=2116$	4^2
$42^2=1764$	8^2	$47^2=2209$	3^2
$43^2=1849$	7^2	$48^2=2304$	2^2
$44^2=1936$	6^2	$49^2=2401$	1^2
$45^2=2025$	5^2	$50^2=2500$	0^2

The numbers have now been extended as much beyond 25 as to be equal to all below that number.

The learner need have no difficulty in remembering the squares in this column; the law which has before been explained, should be kept constantly in mind, and as a further means of association, observe that the two right-hand figures in the square are in every instance the square of the difference between the units figure of the root, and 10. The two left-hand figures in the square will also be easily remembered, if we observe that the number is formed by adding 1 less than the right-hand or units figure of the root to the square of the left-hand, or tens figure, thus in the square of 47, the left-hand figures are 22, 1 less than 7, added the square of $4=16+6$.

The following column can also be remembered without the least difficulty, if we observe how the square is formed.

Here the two left-hand figures in every instance, may be produced, by adding the right-hand figure of the root to the square of the left-hand figure, and the number expressed by the two right-hand figures in the square is the square of the right-hand figure in the root.

In order that the learner may perceive the practical utility of committing these tables to memory, a few questions in Arithmetical Progression will here be given for mental solution.

$51^2=2601$
$52^2=2704$
$53^2=2809$
$54^2=2916$
$55^2=3025$
$56^2=3136$
$57^2=3249$
$58^2=3364$
$59^2=3481$
$60^2=3600$

53. What cost 15 yards of linen, at 2 cents for the first yard, 4 for the second, 6 for third, &c., increasing 2 cents every yard. Ans. $15^2+15=\$2.40$.

Hence when both the first term and common difference is 2,

RULE.

Add the number of terms to its square, and the sum will be the number sought.

54. Suppose a man should agree to work 24 months, at the rate of \$2 for the first month, \$4 for the second, \$6 for

the third, &c., increasing \$2 every month, how much would he receive for the 24 months? Ans. $24^2+24=\$600$.

55. A farmer agreed to sell 36 head of fat cattle, at the rate of \$2 for the first, \$4 for the second, \$6 for the third, &c., what did the whole amount to?

56. A man being asked how much he would sell his horse for, said he had 4 shoes on, and there were 8 nails in each shoe, and he would sell him at the rate of \$1 for the first nail, 2 for the second, 3 for the third, &c. How much would the horse come to at that rate?

$$\text{Ans. } \frac{32^2+32}{2}=\$528.$$

In this question, the first term and common difference each being 1, the result is but half as much as if it were 2.

57. How many times does a clock strike in 12 hours?

$$\text{Ans. } \frac{12^2+12}{2}=78.$$

58. What would 38 yards of silk cost, at 2 cents for the first yard, 4 for the second, 6 for the third, &c.?

59. What would 58 bushels of potatoes cost, at 2 cents for the first bushel, 4 for the second, &c.?

60. It is said that the clocks of Venice go to 24 o'clock; how many times will one of those clocks strike in 24 hours?

61. I am willing to sell 46 bushels of oats at the rate of 3 cents for the first bushel, 6 for the second, 9 for the third, &c. how much will the 46 bushels come to?

$$\text{Ans. } \frac{(46^2+46)\times 3}{2}=\$32.43.$$

In this question, the common difference figure being 3, the answer will be greater than when the common difference is 2 in the proportion of 3 to 2.

Hence, whatever may be the common difference, the answer found by the rule already given, will be to the true answer as 2 to the common difference, that is, if the first term and common difference are the same.

62. What cost 28 bushels of potatoes at 4 cents for the first bushel, 8 for the second, &c.?

Ans. $(28^2+28)\times 2=\$15.68$.

63. What will 56 bushels of corn cost at 2 cents for the first bushel, 4 for the second, &c.?

64. What will 18 acres of land cost at the rate of \$5 for the first acre, 10 for the second, &c.?

Ans. $\frac{(18^2+18)\times 5}{2}=\$855$.

65. What cost 27 bushels of salt at the rate of 3 cents for the first bushel, 6 for the second, 9 for the third, &c.

Ans. \$11.34.

66. What are 49 yards of cloth worth at the rate of 4 cents for the first yard, 8 for the second, 12 for the third, &c.

67. What will 29 yards of silk cost at the rate of 5 cents for the first yard, 10 for the second, &c., increasing 5 cents every yard.

68. What will 39 bushels of wheat cost at 6 cents for the first bushel, 12 for the second, 18 for the third, &c.

69. What will 59 sheep cost at 6 cents for the first, 12 for the second, &c.? Ans. \$106.20.

70. What cost 23 bushels of oats at the rate of 4 cents for the first bushel, 8 for the second, 12 for the third, &c.?

71. What cost 43 books at the rate of 2 cents for the first, 4 for the second, 6 for the third, &c.?

72. What cost 53 acres of land at the rate of \$4 for the first acre, 8 for the second, 12 for the third, &c.?

All or nearly all the preceding questions in arithmetical progression may be solved mentally, if the pupils have thoroughly committed to memory the square numbers given in the tables.

In a few cases, however, it will be more convenient to use the slate.

TABLE OF SQUARE NUMBERS CONTINUED.

$61^2=3721$	$11^2=121$	$64^2=4096$	$14^2=196$
$62^2=3844$	$12^2=144$	$65^2=4225$	$15^2=225$
$63^2=3969$	$13^2=169$	$66^2=4356$	$16^2=256$

$67^2=4489$	$17^2=289$	$74^2=5476$	$24^2=576$
$68^2=4624$	$18^2=324$	$75^2=5625$	$25^2=625$
$69^2=4761$	$19^2=361$	$76^2=5776$	$26^2=676$
$70^2=4900$	$20^2=400$	$77^2=5929$	$27^2=729$
$71^2=5041$	$21^2=441$	$78^2=6084$	$28^2=784$
$72^2=5184$	$22^2=488$	$79^2=6241$	$29^2=841$
$73^2=5329$	$23^2=529$	$80^2=6400$	$30^2=900$

In the above table, the column of squares on the right hand are thus placed with their roots, in order that the learner may associate them with the squares of the left hand column. He will observe, of course, that two figures (units and tens) are in regular succession, the same in one column as in the other.

In the following columns the same principle will be observed.

		*	
$81^2=6561$	$19^2=361$	$91^2=\ 8281$	$9^2=81$
$82^2=6724$	$18^2=324$	$92^2=\ 8464$	$8^2=64$
$83^2=6889$	$17^2=289$	$93^2=\ 8649$	$7^2=49$
$84^2=7056$	$16^2=256$	$94^2=\ 8836$	$6^2=36$
$85^2=7225$	$15^2=225$	$95^2=\ 9025$	$5^2=25$
$86^2=7396$	$14^2=196$	$96^2=\ 9216$	$4^2=16$
$87^2=7569$	$13^2=169$	$97^2=\ 9409$	$3^2=\ 9$
$88^2=7744$	$12^2=144$	$98^2=\ 9604$	$2^2=\ 4$
$89^2=7921$	$11^2=121$	$99^2=\ 9801$	$1^2=\ 1$
$90^2=8100$	$10^2=100$	$100^2=10000$	$0^2=\ 0$

In the column of squares, marked thus *, the two left hand figures may be obtained by adding twice the right hand figure of the root to 80.

NOTE TO TEACHERS.—The above table should be written with chalk upon the black board, and the pupils required to recite them in concert, proceeding first from the top to the bottom of the column, then from the bottom to the top, and finally the teacher should point promiscuously to the numbers in the roots, requiring the pupils to continue the concert, when the squares are rubbed out.

The square of any number is equal 4 times tne square of half that number, hence, if you wish to find the square, say

of 174, and you know the square of 87 to be 7569, you have only to multiply 7569 by 4 and you have the square of 174 =30276.

Again, the square of any number is equal 9 times the square of $\frac{1}{3}$ of that number, hence, if you wish to find the square, say of 288, and you know the square of 96, ($\frac{1}{3}$ of 288) you have only to multiply 9216 (the square of 96) by 9 and you have 82944 the square of 288.

The square of any number is equal 16 times the square of $\frac{1}{4}$ of the number, hence, if you wish to find the square, say of 276, and you know the square of 69 ($\frac{1}{4}$ of 276,) you have only to multiply 4761 (the square of 69) by 16, and you have the square of 276=76176.

The square of any number is equal 25 times the square of $\frac{1}{5}$ of that number, hence, if you wish to find the square, say of 495, you have only to square 99 and multiply the square by 25, that is, suppose 2 ciphers annexed and divide by 4.

The square of any number is equal 81 times the square of $\frac{1}{9}$ of the number, hence, to find, say the square of 783, if you know the square of 87, $\frac{1}{9}$ of 783, you have only to square 87, and multiply that square by 81,

Thus, 7569×81
60552

and you have the square of 783=613089.

MENTAL OPERATIONS.

75. What cost 57 cords of wood, at $5.70 a cord?
76. What cost 63 barrels of flour, at $6.30 a barrel?
77. What cost 84 tons of hay, at $8.40 a ton?
78. What cost 18 pair of shoes, at $1.80 a pair?
79. What cost 46 yards of cloth, at $4.60 a yard?
80. What cost 75 acres of land, at $7.50 per acre?
81. If a man can travel 37 miles in a day, how far can he travel at the same rate, in 37 days?
82. If one acre of land produce 68 bushels of corn, how many bushels will 68 acres produce at the same rate?
83. If a vessel sail 97 miles in a day, how far at that rate, will she sail in 97 days?

84. If a hat cost \$4.50, what would 45 hats cost at the same rate?

85. Bought 85 pieces of muslin, each piece containing 87 yards, how many yards in the whole?

Ans. $86^2-1=7395$ yards.

86. Bought 63 pounds of tea, at 65 cents a pound, what did it amount to?

87. What cost 74 cwt. of iron, at \$7.80 per cwt.?

88. Bought 93 cwt. of tobacco, at \$9.70 per cwt., what did it amount to?

89. What cost 24 acres of grass, at \$3.60 an acre?

90. What cost 32 pounds of cinnamon, at 38 cts. a pound?

91. What cost 46 tons of coal, at \$5.40 per ton?

92. What cost 33 yards of carpet, at 77 cents a yard?

93. What will a drove of 73 horses be worth at \$87 a head?

94. What cost 47 baskets of sweet potatoes, at 53 cents a basket?

All the above questions should be solved without the use of the slate, the ciphers should not be regarded in making the calculation, but must be annexed to the result.

MENTAL OPERATIONS IN FRACTIONS.

To square any number containing $\frac{1}{2}$, as $4\frac{1}{2}$, $7\frac{1}{2}$, &c.

RULE.

Add the whole number to its square, and annex the fraction $\frac{1}{4}$, to the sum, or multiply the whole number by the next higher whole number, and annex $\frac{1}{4}$ to the product.

95. What is the square of $3\frac{1}{2}$? Ans. $3^2+3=12$ to which annex $\frac{1}{4}$, and we have the square of $3\frac{1}{2}=12\frac{1}{4}$,

Or $3\times4+\frac{1}{4}=12\frac{1}{4}$.

96. What is the square of $7\frac{1}{2}$?

Ans. $7^2+7+\frac{1}{4}=56\frac{1}{4}$

Or, $7\times8+\frac{1}{4}=56\frac{1}{4}$.

97. What is the square of $6\frac{1}{2}$? Ans. $42\frac{1}{4}$.

98. What is the square of $8\frac{1}{2}$?

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99. What is the square of $12\frac{1}{2}$?
100. What is the square of $15\frac{1}{2}$?
101. What is the square of $17\frac{1}{2}$?
102. What is the square of $19\frac{1}{2}$?
103. What is the square of $27\frac{1}{2}$?
104. What is the square of $16\frac{1}{2}$?
105. What is the square of $14\frac{1}{2}$?
106. What is the square of $11\frac{1}{2}$?
107. What is the square of $32\frac{1}{2}$?
108. What is the square of $39\frac{1}{2}$?
109. What is the square of $23\frac{1}{2}$?
110. What is the square of $29\frac{1}{2}$?
111. What is the square of $24\frac{1}{2}$?
112. What is the square of $13\frac{1}{2}$?
113. What is the square of $21\frac{1}{2}$?
114. What is the square of $22\frac{1}{2}$?
115. What is the square of $25\frac{1}{2}$?
116. What is the square of $28\frac{1}{2}$?
117. What is the square of $36\frac{1}{2}$?
118. What is the square of $43\frac{1}{2}$?
119. What is the square of $49\frac{1}{2}$?
120. What cost $7\frac{1}{2}$ pounds of beef, at $7\frac{1}{2}$ cents a pound?
121. What cost $13\frac{1}{2}$ pounds of sugar, at $13\frac{1}{2}$ cts. a pound?
122. What cost $17\frac{1}{2}$ yards of linen, at $17\frac{1}{2}$ cents a yard?
123. What cost $12\frac{1}{2}$ quarts of honey, at $12\frac{1}{2}$ cts. a quart?
124. What are $24\frac{1}{2}$ hundred oysters worth, at $24\frac{1}{2}$ cents a hundred?
125. What will $37\frac{1}{2}$ bushels of oats cost, at $37\frac{1}{2}$ cents a bushel?
126. What cost $42\frac{1}{2}$ gallons of molasses, at $42\frac{1}{2}$ cents a gallon?
127. What are $34\frac{1}{2}$ bushels of apples worth, at $34\frac{1}{2}$ cents a bushel?

To multiply any two numbers together, each of which involves the fraction $\frac{1}{2}$, *as* $7\frac{1}{2}$ *by* $3\frac{1}{2}$, *or* $5\frac{1}{2}$ *by* $9\frac{1}{2}$, &c.

RULE.

To the product of the whole numbers add half their sum, plus $\frac{1}{4}$.

NOTE.—If the sum be an odd number, call it one less, to make it even, and in such cases the fraction must be $\frac{3}{4}$.

EXAMPLES FOR MENTAL OPERATIONS.

128. Multiply $3\frac{1}{2}$ by $7\frac{1}{2}$, thus, $7\times3+5=26$, to which annex $\frac{1}{4}$, and we have $26\frac{1}{4}$, the sum of 7 and 3 is 10, and half this sum is 5, so we simply say 7 times 3 are 21, and 5 are 26, to which we annex $\frac{1}{4}$.

129. Multiply $5\frac{1}{2}$ by $11\frac{1}{2}$. Ans. $63\frac{1}{4}$

130. Multiply mentally $4\frac{1}{2}$ by $8\frac{1}{2}$.

131. Multiply mentally $6\frac{1}{2}$ by $10\frac{1}{2}$.

132. Multiply mentally $3\frac{1}{2}$ by $9\frac{1}{2}$.

133. Multiply mentally $8\frac{1}{2}$ by $14\frac{1}{2}$.

134. Multiply mentally $12\frac{1}{2}$ by $18\frac{1}{2}$.

135. Multiply mentally $9\frac{1}{2}$ by $17\frac{1}{2}$.

136. Multiply mentally $11\frac{1}{2}$ by $19\frac{1}{2}$.

137. Multiply mentally $3\frac{1}{2}$ by $8\frac{1}{2}$.

138. Multiply mentally $5\frac{1}{2}$ by $7\frac{1}{2}$.

139. Multiply mentally $6\frac{1}{2}$ by $13\frac{1}{2}$.

140. What will $4\frac{1}{2}$ pounds of cheese cost, at $10\frac{1}{2}$ cents per pound?

141. What cost $7\frac{1}{2}$ dozen eggs, at $12\frac{1}{2}$ cents a dozen?

142. What are $10\frac{1}{2}$ yards of muslin worth, at $13\frac{1}{2}$ cents a yard?

143. What must I pay for $11\frac{1}{2}$ bushels of oats, at $37\frac{1}{2}$ cents a bushel?

USE THE SLATE FOR THE FOLLOWING.

144. What are $27\frac{1}{2}$ yards of linen worth, at $37\frac{1}{2}$ cents a yard? Ans. \$$10.31\frac{1}{4}$.

145. What are $39\frac{1}{2}$ bushels of potatoes, at $62\frac{1}{2}$ cents a bushel? Ans. \$$24.68\frac{3}{4}$.

146. What are $47\frac{1}{2}$ bushels of corn worth, at $87\frac{1}{2}$ cents a bushel? Ans. \$$41.56\frac{1}{4}$.

147. What are $137\frac{1}{2}$ bushels of wheat worth, at \$$1.12\frac{1}{2}$ a bushel? Ans. $154.68\frac{3}{4}$.

148. What will $47\frac{1}{2}$ days' work amount to, at \$$1.62\frac{1}{2}$ a day? Ans. \$$77.18\frac{3}{4}$.

149. If one yard of cloth cost \$3.87½, what will 62½ yards of the same quality of cloth amount to? Ans. \$242.18¾.

150. Bought 28½ bushels of potatoes, at 48½ cents a bushel, what is the cost of the whole? Ans. \$13.82¼.

To square any number involving the fraction ¼, as 7¼, 4¼, &c., square the whole number only, (disregarding the ¼,) and to the square add one-half of the given number; to this sum annex $\frac{9}{16}$ if the number be odd, or $\frac{1}{16}$ if even.

NOTE.—In taking the half if the given number be odd, call it one less, and then take its half.

151. For example the square

$$\text{of } 8\tfrac{1}{4} = 8\times8+4+\tfrac{1}{16} = 68\tfrac{1}{16}.$$
$$\text{of } 5\tfrac{1}{4} = 5\times5+2+\tfrac{9}{16} = 27\tfrac{9}{16}.$$

152. What is the square of 12¼ mentally?
153. What is the square of 6¼ mentally?
154. What is the square of 13¼ mentally?
155. What is the square of 7¼ mentally?
156. What is the square of 14¼ mentally?
157. What is the square of 4¼ mentally?
158. What is the square of 3¼ mentally?
159. What is the square of 15¼ mentally?
160. What is the square of 19¼ mentally?
161. What is the square of 27¼ mentally?
162. What is the square of 18¼ mentally?
163. What is the square of 16¼ mentally?
164. What is the square of 29¼ mentally?
165. What is the square of 38¼ mentally?
166. What will 6¼ yards of muslin cost, at 6¼ cts. a yard?
167. What will 7¼ yards of muslin cost, at 7¼ cts. a yard?

168. Bought 17¼ yards of lace, at 17¼ cents a yard, how much did it come to? Ans. \$2.97$\frac{9}{16}$.

169. What will 4¼ tons of coal cost, at \$4.25 per ton?

170. Sold 87¼ tons of iron, at \$87.25 per ton, what did it amount to? Ans. \$7612$\frac{9}{16}$.

171. If a ton of hay cost \$19.25, what will 19¼ tons cost, at the same rate? Ans. \$370$\frac{9}{16}$.

172. If one acre of land produce 47¼ bushels of corn, how many bushels would 47¼ acres produce at the same rate?
Ans. 2232$\frac{9}{16}$bus.

173. If one bushel of apples be worth $31\frac{1}{4}$ cents, what are $31\frac{1}{4}$ bushels worth at the same rate? Ans. $\$9.76\frac{9}{16}$.

To multiply any two numbers together, when the unit's figure of each is 5.—Omit the 5 in each factor, and to the product of the numbers, as they will then stand, add half their sum; to this result annex 25, if the sum of the numbers be an even number, or 75 if it be odd, and you have the true product.

NOTE.—If the sum of the numbers be odd, call it 1 less, and then take the half.

174. Multiply 145 by 85; thus $14\times8+11=123$ to which annex 25, the sum of 14 and 8 being an even number, and you have $12325=145\times85$.

175. Multiply 175 by 65; thus, $17\times6+11=113$ to which annex 75, the sum of 17 and 6 being an odd number, (viz. 23,) we call 23 1 less than it is, viz. 22, and add the half of 22 to the product of 17 by 6.

176. Multiply mentally 85 by 65.

177. Multiply mentally 95 by 45.

178. Multiply mentally 65 by 45.

179. Multiply mentally 75 by 35.

180. Multiply mentally 245 by 95.

181. Multiply mentally 375 by 135.

182. Multiply mentally 465 by 145.

183. Multiply mentally 785 by 165.

184. Multiply mentally 635 by 175.

185. What is the value of 385 bushels of wheat at \$1.35 per bushel?

186. Bought 265 cords of wood, at \$2.45 a cord; what did it amount to?

187. Calculate the value of 95 bushels of potatoes at 45 cents a bushel.

188. What is the value of 465 yards of broad cloth, at \$3.85 a yard?

189. What is the value of 175 tons of hay, at \$9.65 a ton?

190. What is the value of 245 barrels of flour, at \$6.45 a barrel?

191. What cost 235 barrels of wine, at \$45 a barrel?

To square any number ending in $2\frac{1}{2}$ *or* $7\frac{1}{2}$, *as* $12\frac{1}{2}$, $17\frac{1}{2}$, $22\frac{1}{2}$, $27\frac{1}{2}$, *&c.*—Call the given number $2\frac{1}{2}$ *more*, or $2\frac{1}{2}$ *less*, whichever will make it an odd number of 5's, and multiply the number *when thus* expressed, by its *left hand figure*, *or* by 1 *more* than its *left hand figure or figures*, according as addition or subtraction of $2\frac{1}{2}$ makes the given number an odd number of fives, and annex $6\frac{1}{4}$ to the right of the product.

192. Thus the sq. of $22\frac{1}{2}=25\times2$ with $6\frac{1}{4}$ annexed$=506\frac{1}{4}$.
193. And the sq. of $27\frac{1}{2}=25\times3$ with $6\frac{1}{4}$ annexed$=756\frac{1}{4}$.
194. And the sq. of $17\frac{1}{2}=15\times2$ with $6\frac{1}{4}$ annexed$=306\frac{1}{4}$.
195. And the sq. of $32\frac{1}{2}=35\times3$ with $6\frac{1}{4}$ annexed$=1056\frac{1}{4}$.
196. And the sq. of $37\frac{1}{2}=35\times4$ with $6\frac{1}{4}$ annexed$=1406\frac{1}{4}$.
197. What is the square of $42\frac{1}{2}$?
198. What is the square of $47\frac{1}{2}$?
199. What is the square of $52\frac{1}{2}$?
200. What is the square of $57\frac{1}{2}$?
201. What is the square of $62\frac{1}{2}$?
202. What is the square of $67\frac{1}{2}$?
203. What is the square of $72\frac{1}{2}$?
204. What is the square of $77\frac{1}{2}$?
205. What is the square of $82\frac{1}{2}$?
206. What is the square of $87\frac{1}{2}$?
207. What is the square of $92\frac{1}{2}$?
208. What is the square of $97\frac{1}{2}$?
209. What is the square of $102\frac{1}{2}$?
210. What is the square of $107\frac{1}{2}$?
211. What is the square of $112\frac{1}{2}$?
212. What is the square of $117\frac{1}{2}$?
213. What is the square of $122\frac{1}{2}$?
214. What is the square of $127\frac{1}{2}$?
215. What is the square of $132\frac{1}{2}$?
216. What is the square of $137\frac{1}{2}$?
217. What is the square of $142\frac{1}{2}$?
218. What is the square of $147\frac{1}{2}$?
219. What is the square of $152\frac{1}{2}$?
220. What is the square of $157\frac{1}{2}$?

MENTAL OPERATIONS CONTINUED.

221. Calculate the value of $17\frac{1}{2}$ bushels of lime, at $17\frac{1}{2}$ cents a bushel.

222. $22\frac{1}{2}$ yards of linen, at $22\frac{1}{2}$ cents a yard.
223. $27\frac{1}{2}$ bushels of apples, at $27\frac{1}{2}$ cents a bushel.
224. $32\frac{1}{2}$ bushels of oats, at $32\frac{1}{2}$ cents a bushel.
225. $37\frac{1}{2}$ acres of land at \$$37\frac{1}{2}$ an acre.
226. $42\frac{1}{2}$ pounds of tea, at $42\frac{1}{2}$ cents a pound.
227. $47\frac{1}{2}$ gallons of molasses, at $47\frac{1}{2}$ cents a gallon.
228. $52\frac{1}{2}$ days' work, at $52\frac{1}{2}$ cents a day.
229. $57\frac{1}{2}$ bushels of potatoes, at $57\frac{1}{2}$ cents bushel.
230. $62\frac{1}{2}$ yards of cloth, at $62\frac{1}{2}$ cents a yard.
231. $67\frac{1}{2}$ bushels of corn, at $67\frac{1}{2}$ cents a bushel.
232. $72\frac{1}{2}$ gallons of oil, at $72\frac{1}{2}$ cents a gallon.
233. $77\frac{1}{2}$ bushels of peaches, at $77\frac{1}{2}$ cents a bushel.
234. $82\frac{1}{2}$ pounds of indigo, at $82\frac{1}{2}$ cents a pound.
235. $87\frac{1}{2}$ bushels of wheat, at $87\frac{1}{2}$ cents a bushel.
236. $92\frac{1}{2}$ yards of satinet, at $92\frac{1}{2}$ cents a yard.
237. $97\frac{1}{2}$ acres of land at \$$97\frac{1}{2}$ an acre.

To square any number of nines, as 999, or 99999, *&c.*—Write from left to right as many nines, less one, as the given number contains, one 8, then as many ciphers, less one, as the given number contains nines, and lastly 1.

$$\text{Thus, the square of } \begin{cases} 999 = 998001 \\ 9999 = 99980001 \\ 99999 = 9999800001 \end{cases}$$

CONTRACTIONS IN MULTIPLICATION.

To multiply by any number of nines.—Annex as many ciphers to the right of the multiplicand as there are nines in the multiplier, and from this number subtract the multiplicand, the remainder will be the product required.

EXAMPLES.

1. Multiply 37645 by 9999.

$$\begin{array}{r} 376450000 \\ 37645 \\ \hline 376412355 \end{array}$$

Here, by annexing four ciphers, the multiplicand is made ten thousand times larger, which is once the multiplicand too many, hence, by subtracting the multiplicand, we get the true product.

2. Multiply 47036 by 999.
3. Multiply 26475 by 9999. Ans. 264723525.
4. Multiply 364364 by 99999.

To multiply by 25.—4 times 25 make 100; therefore annex two ciphers to the right of the multiplicand, and divide by 4, and you have the product.

EXAMPLES.

5. Multiply 3757 by 25.

Thus, 4)375700
93925

6. Multiply 437 by 25.
7. Multiply 6359 by 25.
8. Multiply 9742 by 25.
9. Multiply 7563 by 25.

To multiply by 125.—8 times 125 make 1000, therefore annex three ciphers to the right of the multiplicand and divide by 8.

EXAMPLES.

10. Multiply 3762 by 125.

8)3762000
470250

11. Multiply 6329 by 125.
12. Multiply 37637 by 125. Ans. 4704625.
13. Multiply 57539 by 125.
14. Multiply 976351 by 125.
15. Multiply 713659 by 125.

To multiply by 11.—Add the figures of the multiplicand, after the first, from right to left.

EXAMPLES.

16. Multiply 37643 by 11. 414073.

Here we write down the 3, and then say, 3 and 4 are 7, then 4 and 6 are 10, put down the 0 and carry 1, then 6

and 7 and 1 to carry are 14, set down 4 and carry 1, then 7 and 3 are 10 and 1 to carry make 11, set down 1, and carry 1 to the 3 makes 4.

17. Multiply 79632 by 11.
18. Multiply 47896 by 11.
19. Multiply 316943 by 11.
20. Multiply 137823 by 11.
21. Multiply 789645 by 11. Ans. 8686095.

To multiply by any number, from 12 *to* 20.—Multiply in regular succession, the figures of the multiplicand, by the unit figure of the multiplier, and add to the product of each multiplication that figure which stands next on the right-hand of the one which you multiply, and to the last figure add what you carry.

EXAMPLES.

22. Multiply 36435 by 13.

$$\frac{36435\times 13}{473655}$$

Here we say, 3 times 5 are 15, set down 5 and carry 1, then 3 times 3 are 9, and 1 to carry are 10, and 5 which stands on the right of the 3 make 15, set down 5 and carry 1, then 3 times 4 are 12, and 1 to carry, together with the 3 on the right of the 4 make 16, then 3 times 6 are 18, and 1 to carry, together with the 4 make 23, set down 3, and carry 2, then 3 times 3 are 9, and 2 to carry, together with the 6 on the right of the 3, make 17, set down 7, and carry 1 to 3, making 4.

23. Multiply 3792 by 14.
24. Multiply 37523 by 15.
25. Multiply 6579×16.
26. Multiply 7845×17.
27. Multiply 8732×18.
28. Multiply 2341×19.
29. Multiply 76374×13.
30. Multiply 56732×16.

To multiply by 21, 31, 41, 51, 61, 71, 81, *and* 91.—Write down the unit figure of the multiplicand, as the first figure of the product, then multiply in regular succession every figure of the multiplicand, by the left-hand figure of the multiplier, and to each product add the figure which stands

next on the left, and of that which you multiply, and you have the required product.

EXAMPLES.

31. Multiply 3725 by 21.

$$\begin{array}{r} 3725\times21 \\ \hline 78225 \end{array}$$

Here we set down the 5 and then multiply by the 2, thus, twice 5 are 10, and the 2 on the left of the 5 make 12, then twice 2 are 4 and 1 to carry, together with the 7 next on the left of the 2 make 12, then twice 7 are 14 and 1 to carry, together with the 3 on the left of the 7 make 18, then twice 3 are 6 and 1 to carry make 7.

32. Multiply 2793 by 31. Ans. 86583.
33. Multiply 3647×41.
34. Multiply 4793×51.
35. Multiply 7329×61.
36. Multiply 6482×71. Ans. 460222.
37. Multiply 3742×81. Ans. 303102.
38. Multiply 8732×91. Ans. 794612.
39. Multiply 3271×21. Ans. 68691.
40. Multiply 47695×51. Ans. 2432445.
41. Multiply 369217×71.
42. Multiply 3749×701.

$$\begin{array}{r} 3749\times701 \\ 26243 \\ \hline 2628049 \end{array}$$

Here we simply multiply by 7 and place the first product in the hundreds place, and then add the product to the multiplicand.

43. Multiply 3792 by 1007.

$$\begin{array}{r} 3792\times1007 \\ 26544 \\ \hline 3818544 \end{array}$$

Here observe that the multiplicand is made to occupy the

place of 1000, and the 7 being units its product must be placed 3 figures to the right.

44. Multiply 73496 by 9001.

$$\begin{array}{r} 73496 \times 9001 \\ 661464 \\ \hline 661537496 \end{array}$$

Here the only figure which it is necessary to multiply by is the 9, which being thousands, the first product must be made to occupy the place of thousands.

45. Multiply 5737 by 108.
46. Multiply 3796 by 801.
47. Multiply 49765 by 9001.
48. Multiply 63754 by 1009.
49. Multiply 38462 by 60001.
50. Multiply 39876 by 10006.
51. Multiply 74863 by 12001.
52. Multiply 97842 by 10012.
53. Multiply 54687 by 30001.
54. Multiply 45679 by 10003.
55. Multiply 79847 by 4001.
56. Multiply 49723 by 1004.
57. Multiply 64732 by 102.
58. Multiply 37475 by 201.
59. Multiply 4687 by 301.
60. Multiply 8643 by 103.
61. Multiply 3746 by 7, the product by 11, and the product of 11 again by 13.

$$\begin{array}{r} 3746 \\ 7 \\ \hline 26222 \times 11 \\ \hline 288442 \times 13 \\ \hline 3749746 \end{array}$$

A better method, $3746 \times 1001 = (7 \times 11 \times 13)$

$$\begin{array}{r} 3746 \\ \hline 3749746 \end{array}$$

2. Multiply 26375 by 7, 11, and 13, as in the last example.

63. Multiply 4637 by 17, and the product by 6.

$$\begin{array}{r} 4637\times17 \\ \hline 78829 \\ 6 \\ \hline 472974 \end{array} \qquad \text{Or thus, } \begin{array}{r} 4637\times102=6\times17 \\ 9274 \\ \hline 472974 \end{array}$$

64. Multiply 6239 by 143, and the product by 7.

$$\begin{array}{l} 6239\times1001=143\times7 \\ \quad 6239 \\ \hline 6245239 \end{array}$$

65. Multiply 36947 by 1429, and the product by 7.

$$\begin{array}{l} 3694\times10003=1429\times7. \\ \quad 11082 \\ \hline 36951082 \end{array}$$

From the above examples the learner will of course perceive the advantage which may be taken of those properties of numbers.

66. Multiply 27684 by 126 and the product by 8.

$$\begin{array}{l} 27684\times1008=126\times8 \\ \quad 221472 \\ \hline 27905472 \end{array}$$

67. Multiply 4673 by 89, and the product by 9.

$$\begin{array}{l} 4673\times801=89\times9 \\ 37384 \\ \hline 3743073 \end{array}$$

We will now give a table of easy multipliers, which may be committed to memory with but little labour.

Multiplied by 3.	Multiplied by 7.	Multiplied by 9.
34= 102	43= 301	89= 801
67= 201	143= 1001	889= 8001
167= 501	243= 1701	8889= 80001
267= 801	443= 3101	88889=800001
367=1101	1429=10003	&c.
467=1401	144= 1008	
667=2001		

When some of the figures of the multiplier are multiples of others, the operation may be contracted thus,

68. Multiply 34672 by 9612.

Common method.

```
        34672
         9612
        69344
       34672
      208032
     312048
Ans. 333267264
```

Operation.

```
              34672×12 units
Product by  12=416064×8 hundreds=9600÷12
              3328512= the foregoing product by 8 for 96
         Ans. 333267264
```

69. Multiply 37493 by 1284.

Common method.

```
        37493
         1284
       149972
      299944
      74986
     37493
Ans. 48141012
```

Easy method.

```
   37493×12 hundred
  449916×7 units=84÷12
 3149412
48141013     Ans.
```

70. Multiply 73649 by 80072.

```
   73649×8= tens of thousands
  589192×9 units=72÷8
     5302728
  5897222728
```

Note to the Learner.—You should be very particular at all times to observe the numerative value of the figure or number you multiply by; and to place the product accordingly: an example or two will show the importance of this suggestion.

71. Multiply 634275 by 24872.

Common method.	Easy method.
634275	634275×8 hundred
24872	5074200×3 thousand=24000÷8
1268550	15222600×3 units=72÷24
4439925	45667800
5074200	15775687800
2537100	
1268550	
15775687800	

72. Multiply 732492 by 729576.

Common method.	Easy method.
732492	732492×9 thousand
729576	6592428×8 ten thousands=720000÷9
4394952	52739424×8 units=576÷72
5127444	421915392
3662460	534408583392
6592428	
1464984	
5127444	
534408583392	

Numbers which afford these facilities in multiplication are of very frequent occurrence; we will give a few of them.

214	455	784	1224
216	459	824	1236
218	486	832	1248
222	497	819	1272
224	567	847	1284
315	568	856	1296
318	618	864	16848
321	624	872	2412192
324	636	896	287168
327	642	918	328192
426	648	981	427378
427	654	1122	4812432
432	672	1133	549648
436	728	1144	568168
448	726	1155	639378
452	763	1166	7212648

73. Multiply 4796 by 1272. Ans. 6000512.
74. Multiply 36737 by 16848. Ans. 618944976.
75. Multiply 27695 by 9612. Ans. 266204340.
76. Multiply 39463 by 972. Ans. 38358036.
77. Multiply 49872 by 847672. Ans. 42275097984.
78. Multiply 96254 by 549. Ans. 52843446.
79. Multiply 46397 by 568168. Ans. 26361290696.
80. Multiply 74621 by 729. Ans. 54398709.
81. Multiply 21374 by 1199. Ans. 25627426.
82. Multiply 463635 by 13212924. Ans. 6125974187740.

If the learner has studied the preceding contractions sufficiently to become familiar with their application, he will frequently derive from them great facilities in his subsequent calculations. And the following method of obtaining the product, in a single line, when the multiplier consists of several figures, will be found of important service in like manner.

Suppose we wish to multiply 5768 by 324, in a single line,—we will write the multiplier in an inverted order, on a narrow slip of paper, thus, 423,—then place the slip under the multiplicand, so that the 4 shall be directly under the 8, or the unit figure of the multiplicand, and multiply 8 by the 4—put down 2 and carry 3. Now slide the paper to the left, so that the 4 will come under the 6, and the 2 under the 8, and multiply 6 by the 4, and 8 by the 2, and you have 24 and 16, and 3 you had to carry make 43, put down 3 and carry 4. Again slide the paper to the left, so that the 4 stands under 7, 2 under 6, and 3 under the 8, and multiply 7 by the 4, 6 by 2, and 8 by 3, and you have 28, and 12 and 24, and 4 you had to carry make 68, put down 8 and carry 6. Slide the paper again, so that the 4 stands under the 5, 2 under the 7, and 3 under 6—multiply as before, and you have 20, and 14, and 18, and 6 you carried, make 58, put down 8 and carry 5. Now slide your paper again to the left, so that the 2 stands under the 5, and 3 under the 7—multiply, and you have 10, and 21, and 5 you carried, make 36, put down 6 and carry 3. Slide the paper once more to the left, so that the 3 stands under the 5, and multiply—and you have 15, and 3 you carried make 18. Thus you have obtained the complete product 1868832.

A little practice will enable any one to perform operations, astonishing to those who do not understand the process;—but none should stop with a little practice, as the utility of the process is very great. After becoming familiar with the slide, the operation can be performed without it.

NOTE.—The above method of multiplying is the same as that used by Deshong.

Multiply each of the following sums in a single line.

83. Multiply 3784 by 243. Ans. 919512.
84. Multiply 486 by 352. Ans. 171072.
85. Multiply 2634 by 76. Ans. 200184.
86. Multiply 364 by 265. Ans. 96460.
87. Multiply 1246 by 563. Ans. 701498.
88. Multiply 5248 by 162. Ans. 850176.
89. Multiply 14276 by 4562. Ans. 65127112.
90. Multiply 41365 by 2435. Ans. 100723775.
91. Multiply 2758 by 635. Ans. 1751330.
92. Multiply 7946 by 2842. Ans. 22582532.
93. Multiply 64325 by 12435. Ans. 799881375.
94. Multiply 19206 by 1643. Ans. 31555458.
95. Multiply 96485 by 4262. Ans. 411219070.
96. Multiply 236954 by 376. Ans. 89094704.
97. Multiply 384064 by 24635. Ans. 9461416640.
98. Multiply 287956 by 8725. Ans. 2512416100.
99. Multiply 79068 by 2642. Ans. 208897656.
100. Multiply 463 by 84. Ans. 38892.
101. Multiply 8764 by 269. Ans. 2357516.
102. Multiply 3721 by 138. Ans. 513498.

One more suggestion in reference to multiplication. (We take for granted that the multiplication table has been thoroughly committed to memory at least to 9 times 30.)

To multiply for example 3675 by 23—we proceed,

5 times 23 are 115—put down 5 and carry 11—then 7 times 23 are 161, and 11 to carry make 172—put down 2 and carry 17—then 6 times 23 are 138, and 17 to carry make 155—put down 5 and carry 15—lastly 3 times 23 are 69, and 15 to carry make 84.

Thus,
```
 3675
   23
-----
84525
```

CONTRACTIONS IN DIVISION.

To divide by 25.—Multiply the dividend by 4, and point off two figures to the right-hand of the product as so many hundredths, or take $\frac{1}{4}$ of the two right-hand figures of the product as so many twenty-fives.

EXAMPLES.

1. Divide 3757 by 25.

$$\begin{array}{r} \text{Thus, } 3757 \\ 4 \\ \hline 150\tfrac{28}{100}=150\tfrac{7}{25} \end{array}$$

2. Divide 4796 by 25. Ans. $191\frac{21}{25}$.
3. Divide 7639 by 25. Ans. $305\frac{14}{25}$.
4. Divide 97653 by 25. Ans. $3906\frac{3}{25}$.
5. Divide 86542 by 25. Ans. $3461\frac{17}{25}$.
6. Divide 92345 by 25. Ans. $3693\frac{4}{5}$.
7. Divide 249375 by 25. Ans. 9975.
8. Divide 797865 by 25.

To divide by 125.—Multiply the dividend by 8, and point off 3 figures to the right of the product as so many thousands, or take $\frac{1}{8}$ of those 3 figures as so many 125ths.

EXAMPLES.

9. Divide 437924 by 125.

Operation.

$$\begin{array}{r} 437924 \\ 8 \\ \hline 3503\tfrac{392}{1000}=3503\tfrac{49}{125}. \end{array}$$

10. Divide 39765 by 125.

Operation.

$$\begin{array}{r} 39765 \\ 8 \\ \hline 318\tfrac{120}{1000}=318\tfrac{3}{25} \end{array}$$

11. Divide 7643275 by 125. Ans. $61146\frac{1}{5}$.
12. Divide 4976372 by 125. Ans. $39810\frac{122}{125}$.
13. Divide 9874625 by 125. Ans. 78997.
14. Divide 9846375 by 125. Ans. 78771.
15. Divide 173964875 by 125. Ans. 1391719.

Note to the Learner.—Your attention will now be directed to an important consideration. We propose to teach you to perform long division with less than half the number of figures required by the ordinary process.

EXAMPLES.

16. Divide 3966424 by 7238.

Common method.	Short method.
7238)3966424(548	7238)3966424(548
36190	3474
34742	5790
28952	
57904	
57904	

17. Divide 76410905 by 23767.

Common method.	Short method.
23767)76410905(3215	23767)76410905(3215
71301	5109
51099	3565
47534	11883
35650	
23767	
118835	
118835	

Observe, that the first example requires 25 figures in the work, while the short method requires but 8. The second example requires 37 figures in the work by the common method, whilst the short method requires but 13.

The only difference between these two methods, is, that in the one case, the product of each quotient figure is set down, and then subtracted, whereas in the other, we subtract each figure of the product as it is formed. A little practice will enable any one to perform division by the short method, as

readily as in the ordinary way, and consequently in less than half the time, as he will need to make less than half the number of figures in the work.

TO DIVIDE BY THE SHORT METHOD.

Subtract each product figure, as it is formed, (that is the right hand figure of the product,) and when it is greater than the figure from which you subtract, carry *one more* to the next product figure than you would otherwise carry.

18. Divide 15341 by 29.

Here we say, 5 times 9 are 45—5 from 13 leaves 8, then 5 times 2 are 10, and 5 to carry make 15, being 1 more than the multiplication requires, because the 5 in the product was greater than the 3, from which you subtract, &c.

Operation.

```
29)15341(529
     8
    26
```

19. Divide 6283459 by 29.

Operation.

```
29)6283459(216671   Ans.
    4
    19
     19
      20
        2
```

20. Divide 3978 by 17.

Operation.

```
17)3978(234   Ans.
    5
     6
```

21. Divide 37872 by 526.

Operation.

```
526)37872(72   Ans.
      105
```

22. Divide 79316 by 316.

```
316)79316(251   Ans.
     161
      31
```

23. Divide 2738 by 37. Ans. 74—2 figs. in the work.
24. Divide 15552 by 36. Ans. 432—3 figs. in do.
25. Divide 74892 by 79. Ans. 948—4 figs. in do.
26. Divide 112896 by 84. Ans. 1344—6 figs. in do.
27. Divide 245551 by 89. Ans. 2759—6 figs. in do.
28. Divide 668039 by 97. Ans. 6887—6 figs. in do.
29. Divide 2248704 by 192. Ans. 11712—9 figs. do.
30. Divide 2755116 by 174. Ans. 15834—10 figs.
31. Divide 2686211248 by 296. Ans. 9075038—10 figs.
32. Divide 175152 by 246. Ans. 712—4 figs.
33. Divide 43859556 by 234. Ans. 187434—13 figs.
34. Divide 11133936 by 396. Ans. 28116—10 figs.
35. Divide 15351975 by 465. Ans. 33015—7 figs.
36. Divide 121932631112635269 by 123456789.
Ans. 987654321—57 figs. in the work.
37. Divide 3973580210754 by 32186.
Ans. 123456789—39 figs. in the work.

PROPERTIES OF NUMBERS.

The following properties of figures and numbers will render important assistance, by way of abridging the work in calculations, and the learner will be abundantly repaid for all the labour which it may be necessary to bestow upon them, in order to profit by their friendly services.

When one number will divide another without a remainder, it is said to measure that other number.

An odd number is one whose right-hand or unit figure is either 1, 3, 5, 7, or 9.

An even number is one whose right-hand or unit figure is either 2, 4, 6, 8, or 0.

1—Has this peculiar property, that as a multiplier or divisor it has no power.

2—Will measure all even numbers.

3—Will measure any number, if the sum of the figures added horizontally be a multiple of 3, as 42612, or 741, the

figures of the first of these numbers added horizontally make 15, and of the others, 12 each a multiple of 3.

4—Will measure any number, if it will measure the two right-hand figures, as 371432, here the two right-hand figures being 32, a multiple of 4, the number 371432 can be measured by 4.

5—Will measure any number of which the right-hand figure is 5 or 0.

6—Will measure any even number which can be measured by 3.

7—Will measure any number of 2 or 3 figures, if the right-hand figure be $\frac{1}{2}$ or $\frac{1}{9}$ of the left-hand figure or figures, as 84, 126, 168, 91, 364, 455, &c.

It will measure a number of 3 or 4 figures, if the two right-hand figures be either 5 times or $\frac{1}{3}$ of the left; when they are $\frac{1}{3}$, 43 will also measure the number; again if two ciphers be inclosed by equal figures, 7, 11, and 13 will also measure a number of 5 figures, if a cipher be inclosed by equal numbers.

Thus, 7, 11, and 13 will measure 8008, 3003, 43043, 79079, &c.

Again, 7 will measure a number of 5 figures, if the two right-hand figures be $\frac{1}{3}$ of the three left-hand ones, 43 will also measure in this case, as 16254, 28595, 29498, &c.

In a number of 5 or 6 figures, 7 will measure, if the difference between the three right-hand figures and the two or three left-hand ones be 0, 7, or a multiple of 7, as 65, 121, 79,863, 38,633, 458,458, 384,447, 594,685, 829,955, &c.

8—Will measure any number, if it will measure the *two right-hand* figures when the hundreds figure is *even*, or the *three* right-hand figures when the hundreds figure is odd; thus, 8 will measure 3197824, 273672, 5397656, because it will measure the two right-hand figures, the hundreds figure being even. It will measure 4573536, 3592784, &c., because it will measure the three right-hand figures, the hundreds figure being odd.

9—Will measure any number, if the sum of the figures added horizontally can be measured by 9, as 4575, the sum of whose figures is 18, 46782 the sum of whose figures is 27, 14684787, the sum of the figures being 45, 4375683, &c.

11—Will measure a number of 3 figures, if the middle

figure be equal to the sum of the other two, as 462, 792, 682, 594, 385, &c., and the division is performed by striking out the middle figure.

11—Will measure any number, if the sum of its periods of two figures can be measured by 11, thus, 29,46,24, is divisible by 11, because the sum of its periods of two figures $29+46+24=99$, a multiple of 11.

Or if the sum of the figures in the odd places be equal to the sum of those in the even places, or their difference be 11, 11 will measure, as 485672, here $(4+5+7)$ are the figures in the odd places, and $(8+6+2)$ those of the even, the sums being equal, 11 will therefore measure 485682.

Remark.—If a number can be measured by two other numbers prime to each other, (that is, numbers which have no common factor,) it can be measured by the product of those numbers, thus, 192 can be measured by 4, and also by 3, it can therefore be measured by 12, their product.

13—Will measure a number of 3 or 4 figures, if the two right-hand figures be 4 times the left-hand one or two, 936, 624, 1768, 1976, 2392, &c.

Or 13 will measure a number of 6 figures, if the alternate periods of 3 figures when subtracted leave a remainder of 0 or 13, or a multiple of 13, as 246, 207, 764, 673, 476,476, &c.

14—Will measure any even number which 7 measures.

15—Will measure any number which 3 and 5 both measure.

17—Will measure any number of 2 or 3 figures, if the right-hand figure be $\frac{1}{5}$ of the left-hand figure or figures, or a number of 3 or 4 figures, if the two right-hand figures be double the left, thus, 51, 255, 459, 357, &c., and 612, 918, 4386, 2754, &c.

Also in numbers of 4 or 5 figures, if the 3 right-hand figures be 3 times the left-hand figure or figures, as 39117, 24072, 74222, 87261, &c.

Again, we may cut off 2 or 3 figures to the right of a number, and divide the number thus cut off by the number of its figures, and if the difference between this quotient and the other part of the given number be either 0, 17, or a multiple of 17, 17 will measure, thus, 297,636, here cut off $636\div3=212$, which subtracted from the other part of the given num-

ber, 297—212 leaves 85, a multiple of 17, therefore 297,636 can be measured by 17; again 63478 can be measured by 17, because if we cut off 78 and take $\frac{1}{2}$ of it from 634 the remainder will be 595, which is a multiple of 17. If the figures cut off be not divisible by the required figure, add 17 to make it divisible, thus, 749,853 cut off three figures, 853, this is not divisible by 3, but by adding 17 it becomes so, thus, $853+17=870\div3=290$, this taken from 749 leaves 459, which can be measured by 17, therefore 749,853 can be measured by 17.

18—Will measure any even number which 9 measures.

19—Will measure any number, if twice the right-hand figure added to the left make 19, or a multiple of 19, as 76, 95, 133, 171, 266, 456, 779, &c., we will examine if 19 will measure 874, twice 4, the right-hand figure added to 87 make 95, a multiple of 19, therefore 19 will measure 874; again, suppose we try 56943, here twice 3 or 6 added to 4 make 10, twice 10 or 20 added to 9 make 29, take 19 from 29 leaves 10, then twice 10 added to 6 make 26, take 19 from this leaves 7, then twice 7 added to 5 make 19, hence 56943 can be measured by 19.

21—Will measure any number which 3 and 7 will measure.

22—Will measure any even number which 11 measures.

37—Has a peculiar property, it will measure any number of three figures when all the figures are the same, and the quotient will be the sum of the figures, thus, 37 will measure 444, 777, or 888, and the quotient in the first will be 12, in the second 21, and the third 24, that is, the first is three times 4, the second 3 times 7, the third 3 times 8. To divide several periods, as 333,888,777, &c., write a cipher between the sums, thus, $333,888,777\div37=9024021$.

43—Will measure numbers of three, four, or five figures, if the one or two right-hand figures be $\frac{1}{3}$ of the left, as 11137, 17759, 19264, 26187, &c., and the quotient will be 7 times the right-hand period; thus, to divide 14749 by 43 we multiply 49 (the right-hand period) by 7, and we have the quotient 343,—7 will measure all such numbers.

59—Will measure numbers of 4 or 5 figures, if the three right-hand figures be 3 times the left, as 74222, 38114, 69207, 87261, here we perceive 17 will also measure.

67—Will measure numbers of 3, 4, or 5 figures, if the two right-hand figures be $\frac{1}{2}$ of the left, as 16683, 7839, 13869 19497, 17889, &c. The quotient will always be 3 times the right-hand period, thus, if we wish to divide 17889 by 67 we have only to multiply 89 by 3=267.

MENTAL EXERCISES.

1. Multiply 14 by 12, subtract 18 from the product, and divide the remainder by 5, and tell the quotient.
2. If I divide 192 by 12, and multiply the quotient by 9, and then divide the product by 8, what will be the last quotient?
3. If $\frac{3}{4}$ of 48 be multiplied by 4, and the product be divided by 18, what will be the quotient?
4. If $\frac{2}{3}$ of 72 be divided by 16, and the quotient be multiplied by 14, and this product divided by 6, what will be the last quotient.
5. If 7 times 24 be divided by 8, what will be the square of the quotient?
6. If the square of 18 be divided by 12, and the quotient be multiplied by 8, and this product divided by 18, what will be the last quotient?
7. If the square of 24 be divided by 16, and the quotient be multiplied by 13, what will be the product?
8. If 27 be multiplied by 23, and the product by 25, what will be the result?
9. If 91 be divided by 7, and the square of the quotient be multiplied by 125, what will be the product?
10. If the square of 76 be divided by 19 what will the quotient be?
11. If 888 be divided by 37, what will be the square of the quotient?
12. What is the continued product of 24, 28, 7, 11, 13?
13. If 17889 be divided by 67, and the quotient be multiplied by $7\times11\times13$, what will be the result?
14. What is the result of $(34\times38)\div4$?
15. What is the result of $26\times27\times13$?
16. What is the result of $53\times57\times7$?
17. What is the result of $357\div17\times19$?

PROPORTION.

1. If 17 cords of wood be worth \$85, what are 13 cords worth?

Here we may say, if 17 cords cost \$85, 1 cord will cost $\frac{1}{17}$ of \$85, or \$5, and 13 cords will cost 13 times 5 or \$65; or we may say, if 17 cords cost \$85, 13 cords will cost a sum bearing the same relation to \$85, which 13 cords bear to 17 cords.

$$\begin{array}{r|l} & \$ \\ \text{Thus, } \cancel{17} & \cancel{85} \\ & 5 \\ & 13 \\ \hline \text{Ans.} & 65 \end{array}$$

Ratio is the relation which one quantity bears to another of the same kind with respect to magnitude; or the ratio of two numbers is the quotient resulting from the division of the first by the second. Thus the ratio of 13 to 17 is $\frac{13}{17}$, and of 65 to 85 it is $\frac{65}{85}=\frac{13}{17}$.

Proportion is equality of ratio.

Four numbers are said to be proportionals when the ratio of the first to the second is the same as the ratio of the third to the fourth; hence, 17, 13, 85, 65, are proportionals. The first and fourth are called the *extreme* terms, and the second and third are called the *means*. If four numbers be in proportion, the product of the *extremes* is equal to that of the *means*. Thus, 4, 7, 12, 21, are proportionals, and

Extremes. Means.

$$21\times4=12\times7$$

If we have three terms of a proportion given, the fourth is readily found. Thus, if we have given the three terms, 3, 12, and 8, we observe that the second term is 4 times the first, and in order that the same relation may exist between the third and fourth as that of the first and second, the fourth must be 4 times the third; now if we multiply the second and third together and divide the product by the first, we get $(12\times8)=96\div3=32$, which is 4 times the third term. This principle is one of the most important agents in the science of calculation. In simple proportion, three terms are given and a fourth required to be found.

2. If 8 bushels of corn cost \$5.60, what will 13 bushels cost?

13 bushels will cost a sum of money, bearing the same relation to \$5.60, which 13 bears to 8, the relation or ratio of 13 to 8 is $\frac{13}{8}$, and that of \$9.10 to \$5.60 is $\frac{910}{560}=\frac{13}{8}$.

$$\begin{array}{r|l} & \$ \\ \text{Thus } \cancel{8} & \cancel{5}.\cancel{60} \\ & 70 \\ & 13 \\ \hline \text{Ans.} & \$9.10 \end{array}$$

RULE—FOR THE STATEMENT AND SOLUTION OF QUESTIONS IN SIMPLE PROPORTION.

Place on the right of a vertical line that number which is of the same name or kind as the required answer, then ascertain from the reading of the question, whether the answer must be greater or less than this number (of its own name.) If it must be *greater*, place the *greater* of the two remaining terms or numbers on the *right* of the line, and the other on the left; but if the answer must be *less* than this first mentioned number, then place the *less* of the two remaining terms or numbers on the *right* of the line, and the other on the left. Now divide any number on one side of the line by any number on the opposite side, (if they measure each other,) then multiply all the numbers remaining on the right together for a dividend, or for the answer to the question, if no number remains on the left for a divisor; but if a divisor remains on the left the quotient will be the answer.

NOTE.—The two numbers on opposite sides of the line (differing in name from the answer) must be expressed in the same denomination.

3. If 18 tons of hay cost \$324 what will 23 tons cost at the same rate?

The required answer is evidently money; therefore the \$324 is the *number* of the same *name* as the required *answer*, and as this \$324 is the price of 18 tons, and we wish to know the price of 23 tons, the answer must be greater than \$324; therefore we place the 23, the *greater* of the two remaining numbers on the *right* of the line, and the other (18) on the left; now since we know 324 to be

Operation.

$$\begin{array}{r|l} \text{Thus, } \cancel{18} & \$\cancel{324} \\ & 23\times18 \\ \hline \text{Ans.} & \$414 \end{array}$$

the square of 18, we perceive that 18 will measure 324 and the quotient will be 18, by which we multiply 23 for the answer.

4. If 19 barrels of flour cost $123.50, what will 4 barrels cost?

$$\begin{array}{r|l} & \$ \\ \text{Thus,}\quad \cancel{2} & \cancel{247} \\ \cancel{19} & 13 \\ & \cancel{4}\ 2 \\ \hline \text{Ans.} & \$26 \end{array}$$

Here also the required answer is money,—therefore $123.50 is the number which is of the *same name* as the required answer; $\$123.50 = \$123\frac{1}{2} = \frac{247}{2}$. (When any of the numbers are fractions, you have only to arrange the numerators according to the rule, and as the denominators must always stand on the opposite side of the line, there will be no more difficulty in stating fractions than whole numbers. Now, since $123.50, or $\$\frac{247}{2}$ is the price of 19 bushels, and the answer required is to be the price of 4 bushels, it must be *less* than $123.50, or $\$\frac{247}{2}$; therefore, 4, the *less* of the two remaining numbers must be placed on the *right* of the line, and the other (19) on the left; 19 measures 247, and the quotient is 13; 2 measures the 4—the quotient being 2, then the product which is 26, is the answer.

Remark.—As the learner should accustom himself to use as few figures in the solutions as possible, the 13 and 2 in the above solution, need not be written; but after the statement is made, the answer should be obtained simply by a *mental* operation.

By pursuing the course here suggested, much time will be gained, a great amount of useless labour dispensed with; and the intellectual capacities of the learner invigorated at every stage of his progress.

5. If 17 yards of muslin cost $\$2.12\frac{1}{2}$, what will 72 yards cost?

$$\begin{array}{r|l} & \$ \\ \cancel{8} & \cancel{17} \\ \cancel{17} & \cancel{72} \\ \hline \text{Ans.} & \$9 \end{array}$$

Here $\$2.12\frac{1}{2} = 2\frac{1}{8} = \$\frac{17}{8}$.

6. If 24 men can do a piece of work in 65 days, in how many days can 13 men do the same.

Here the required answer must be days, and 65 being the number of days required by 24 men, 13 men will require a longer time; hence, 24, the greater of the remaining number, must be placed on the right of the line.

$$\begin{array}{r|l} & \text{Days.} \\ \cancel{13} & \cancel{65} \\ & 24 \\ \hline \text{Ans.} & 120 \text{ days} \end{array}$$

7. How many cows will consume as much pasture in 29 days as 63 will consume in 87 days?

Here we say, 63 is the number of cows which will consume a certain quantity of pasture in 29 days, and we wish to ascertain the number that will consume the same quantity in 87 days. It is evident that a less number of cows will consume it, because the time for its consumption is greater. 29 is contained 3 times in 87, and the 3, 21 times in 63—21 cows is, therefore, the answer to the question—and 21 bears the same relation to 63 that 29 does to 87.

$$\begin{array}{r|l} & \text{Cows.} \\ \cancel{3} & \cancel{63} \\ \cancel{87} & \cancel{29} \\ \hline \text{Ans.} & 21 \text{ cows.} \end{array}$$

NOTE.—3, the quotient of 87 by 29 need not be written, but disposed of mentally.

8. If 24 men can do a piece of work in $19\frac{1}{2}$ days, how many men must be employed to do the same in 13 days?

$$\begin{array}{r|l} & \text{Men.} \\ & \cancel{24} \\ \cancel{13} & \cancel{39} \\ \cancel{2} & \\ \hline \text{Ans.} & 36 \text{ men.} \end{array}$$

9. If 3 pounds of butter cost 69 cents, what will 8 pounds cost? 184

10. If a man can perform a journey in 60 days by travelling 9 hours a day, in how many days can he perform such a journey by travelling 12 hours a day? Ans. 45 days.

11. If 36 men can build a wall in 28 days, how many men can build a similar wall in 84 days? Ans. 12 men.

12. If 8 yards of cloth cost \$5.20, what will 72 yards cost? Ans. \$46.80.

13. If 72 yards of cambric cost \$113.36, what will 9 yards cost? Ans. 14.17.

14. If 36 pounds of sugar cost \$3.24, what will 47 pounds cost? Ans. \$4.23.

15. If 12 gallons of wine cost \$30, what will 39 gallons cost?

Here in order to multiply by 250 we omit the 0, and suppose two ciphers annexed to the right of 39, and divide by 4, and then annex the cipher we omitted to the right of the product. It will be useless to write down the 250 if the pupils are well taught in mental solutions.

	\$ cts.
~~12~~	~~30.00~~
	2.50
	39
Ans.	\$97.50

The same is true in reference to the preceding 15 questions, there being no necessity for writing any figure in the solutions, except the answers.

16. If 48 men build a boat in 24 days, how many men can do the same in 192 days? Ans. 6 men.

17. If 67 yards of cloth cost \$174.87, what will 17 yards cost, at the same rate?

It may be well to give a *solution* of this question in the *ordinary* way in order to afford the learner an opportunity of contrasting the two methods.

Ordinary process.

```
   yds.  yds.
As  67 : 17 :: 174.87
                   17
               ------
               122409
               17487
           67)297279($44.37  Ans.
               268
               ---
                292
                268
                ---
                 247
                 201
                 ---
                  469
                  469
                0 *
```

Improved method.

$\cancel{6}\cancel{7}$	$\cancel{1}\cancel{7}\cancel{4}.\cancel{8}\cancel{7}$
	261
	17
Ans.	\$44.37

Here observe that the sum of the periods gives the quotient. The most convenient way to multiply 261 by 17, is to multiply 17 by each figure of the multiplicand. In this solution there is no necessity for making more than three figures, not counting those in the statement or answer. The ordinary process requires 42.

18. If a pole 28 feet high cast a shadow 42 feet in length, what is the height of a steeple whose shadow at the same time is 294 feet? Ans. 196 feet.

19. If 19 yards of silk cost \$14.25, how many yards can I buy for \$75, at the same rate? Ans. 100 yds.

20. If 48 yards of cloth cost \$67.25, what will 144 yards of the same cloth cost? Ans. \$201.75.

21. If $14\frac{1}{2}$ yards of cloth cost \$$19.33\frac{1}{3}$, how much will $19\frac{7}{8}$ yards cost? Ans. \$26.50.

22. If 5 yards of cloth cost 28s. 4d., what is the value of 18 pieces each containing 21 yards 1 quarter?

Here we say, 28s. 4d.$=28\frac{1}{3}$s. $=\frac{85}{3}$s., now as £1=20s., and this $=\frac{60}{3}$s., therefore 28s. 4d.$=\frac{85}{60}$, or £$\frac{17}{12}$. and 21 yards 1 quarter$=\frac{85}{4}$ yds.

	£	
2 $\cancel{1}\cancel{2}$	17	
4	$\cancel{8}\cancel{5}$	17
$\cancel{5}$	$\cancel{1}\cancel{8}$	3
8		
	)867	
Ans.	£108	7s. 6d.

If this question be solved in the ordinary way, the solution will require 57 figures in the work, whereas this solution requires but 8, and even these are unnecessary, as it may be solved very conveniently with 3 figures, or by a purely mental process, (requiring no figures but those in the *statement* and *answer*,) as it will afford an opportunity for both teachers and pupils to judge of the relative merits of our system, and the ordinary method, the number of figures which the solutions by each process requires will be hereafter generally placed in contrast, thus, (57—8,) the number on the left denoting the number required by the common process.

23. If 225 cwt. be carried 512 miles for a certain sum, how many cwt. may be carried 64 miles for the same sum?

Ans. 1800 cwt. (31—1.)

24. At \$1.76 per yard, what is the value of a piece of cloth containing 52 English ells, 3 quarters?

Ans. \$115.72. (26—0.)

25. If 1 cwt of iron be worth £1 8s., what is the value of 33 cwt. 1 quarter 22 pounds?

Ans. £46 16s. 6d. (78—0.)

Explanation.—Observe £1 8s.$=£\frac{7}{5}$, 33 cwt. 1 qr. 22 lbs. $=133\frac{11}{14}$ qrs., then the statement will stand

Here if you divide the 7 into 14, the product of the numbers remaining on the left of the line multiplied together will produce 40, this however need not be written, the 40 will go 46 times into 1873, leaving a remainder of 33, now instead of multiplying this remainder by 20, to change it to shillings, cancel the 20 into the 40, on the opposite side of the line, and the quotient into 33, &c.

Thus,

	£
5	$\cancel{7}$
$\cancel{14}$	1873
4	

26. If when wheat is 83 cents per bushel, the cent loaf weighs 9 ounces, what ought it to weigh when wheat is \$1.24½ a bushel? Ans. 6 oz. (20—0.)

27. If 5 yards of cloth cost 14s. 2d., what must be paid for 9 pieces, each containing 21.25 yards?

Ans. £27 1s. 10½d. (43—15.)

28. What is the value of a silver tankard weighing 1 pound, 7 ounces, 14 pennyweights, at 6s. 4d. per ounce?

Ans. £6 4s. $9\frac{1}{5}$d. (49—4.)

Statement. £

60	19
10	197

Here 6s. 4d.$=\frac{19}{3}$s.
£1$=$20s.$=\frac{60}{3}$s.
1 lb. 7 oz. 14 dwt.$=19\frac{7}{10}$ oz.

29. If 3 quarters of a yard of velvet cost 99 cents, how many yards can I buy for \$37.62?

Ans. 28 yds. 2 qrs. (25—6.)

30. If \$4292.32½ be paid for 476 acres, 3 roods, 28 rods of land, how much will 47 acres come to, at that rate?

Ans. \$423. (108—0.)

Observe 32½ cts.$=\frac{65}{2}$ cts.$=\$\frac{13}{40}$, and 3 roods, 28 rods$=\frac{37}{40}$ of an acre.

In the above question you observe 108 figures will be required in the work, if solved in the ordinary way, whereas

our solution requires not a single figure to be put down, with the exception of those in the statement and answer.

We will give both solutions.

Usual method.

```
      acre. rood. rod.   acre.
   As 476   3    28  :  47  ::  4292.32½
         4               4              2
      ----             ---      ---------
      1907             188         858465
        40              40           7520
      ----             ----     ---------
     76308             7520      17169300
                                4292325
                               6009255
                   76308)6455656800(2)846.00
                         610464      -------
                         ------      $423.00  Ans.
                          351016
                          306232
                          ------
                           457848
                           457848
                           ------
                               00
```

Improved method.

$$\begin{array}{r|l} \cancel{40} & \cancel{171693} \\ \cancel{19077} & \cancel{40} \\ & 47 \\ \hline \text{Ans.} & \$423 \end{array}$$

31. If 19 yards of linen cost $14.25, what will 435.6 yards amount to? Ans. $326.70 (49—4.)

32. If 1cwt. 2 quarters of sugar cost $11.76, what is the value of 16 cwt., 3 quarters, 14 pounds?
Ans. $132.30. (72—2.)

33. What quantity of sugar may be bought for $70.75, when the price of 43 cwt., 2 quarters is $424.50.
Ans. 7 cwt. 1 qr. (50—0.)

34. If 125 bushels of wheat grow on 4 acres, 2 roods, 4 perches, how much land will be necessary to produce 650 bushels, supposing the crop to be equally good?
Ans. 23 acres, 2 roods, 4.8 prs. (39—4.)

35. If a man pays \$28.12½ for 18 bushels, 3 pecks of wheat, what can he get 136 bushels for at the same rate?
Ans. \$204. (52—0.)

36. If a pole 21 feet high cast a shadow 24 feet, 6 inches, what is the height of a steeple whose shadow at the same time is 189 feet? Ans. 162 ft. (46—0.)

37. If 76 yards of muslin cost \$8.55, what must I pay for 28 yards at the same rate? Ans. \$3.15. (31—0.)

38. If 24 barrels of flour cost \$153.60, what will 125 barrels come to? Ans. \$800. (31—0.)

39. At \$110 a ton, what will 5 cwt. 3 quarters, 12 pounds of iron come to? Ans. \$32.21$\frac{3}{7}$. (64—3.)

40. If 125 bushels, 3 pecks of wheat cost \$130.78, what will 34 bushels cost at the same rate?
Ans. \$35.36. (64—4.)

41. If a staff 5 feet, 6 inches cast a shadow 6 feet, what is the height of a steeple whose shadow at the same time measures 153? Ans. 140¼ ft. (25—3.)

The rule generally given directs the learner to reduce the *terms* of the statement to the *lowest denomination mentioned*, which in effect is teaching to *express* those terms by the *greatest possible* number of figures. Now the opposite of this is certainly the *only rational* course to pursue; that is to express the terms of the statement, by the least number of figures which the proportion will admit of. To do this, *Consider each lower denomination as a fraction of the next higher*—thus in the 39th question, the term (5 cwt. 3 quarters, 12 pounds)$=\frac{41}{7}$ cwt., we arrive at this result by a very simple mental process; thus 12 pounds$=\frac{3}{7}$ of a quarter, then 3$\frac{3}{7}$ quarters are$=\frac{24}{7}$ quarters, and 1 cwt.$=\frac{28}{7}$ quarters; hence, 24 parts are $\frac{6}{7}$ of 28 parts, (disregarding the denominators,) and finally 5$\frac{6}{7}$ cwt.$=\frac{41}{7}$ cwt.

42. If a person whose rent is \$147 pays \$9.38 parish tax, how much should a person pay whose rent is \$189?
Ans. \$12.06. (38—3.)

43. What must I pay for 1 cwt. 2 quarters, 14 pounds of sugar, at the rate of \$1.12½ for 9 pounds?
Ans. \$22.75. (34—3.)

44. If 418 bushels of wheat cost \$789.80, what will be the value of 874 bushels at the same rate?
Ans. \$1651.40. (63—8.)

45. If 2.75 yards of cloth cost £4 13.5s. what are 12.25 yards worth at the same rate?
Ans. £20 16s. 6d. (63—3.)

46. If 37 acres of land can be purchased for $451.40, how much must I pay for a farm containing 125 acres at that rate? Ans. $1525. (45—2.)

47. If a pint of rum a day will kill a man in a year and a half, how many men would a cargo of 2190 barrels of rum kill in the same time? Ans. 1008 men. (66—0.)

48. The clothing of a regiment of soldiers, consisting of 750 men, amounts to £2831 5s. what will it cost at the same rate to clothe a body of 3500 men?
Ans. £13212 10s. (70—6.)

49. If 39 horses eat 91 bushels of oats in three weeks, how many bushels will 36 horses eat in the same time?
Ans. 84 bus. (23—0.)

50. If 3 cwt. of hay cost $5.88, what will 2 tons, 5 cwt. 20 pounds cost at that rate? Ans. $88.55. (79—0.)

51. If 5 cwt. 3 quarters, 14 pounds of sugar cost $25.33, what will 35 cwt. 1 quarter cost? Ans. 151.98. (98—0.)

52. A goldsmith sold a tankard for £31 16s. at the rate of 5s. 4d. per ounce, how many ounces did it weigh?
Ans. 119 oz. 5 dwts. (38—3.)

53. If $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a gallon of wine cost $\$\frac{3}{16}$, what will $\frac{5}{6}$ of $\frac{6}{7}$ of $3\frac{1}{2}$ times 560 gallons cost? Ans. $1312.50.* (2.)

54. The length of a wall, ascertained by a measuring line was found to be 643 feet, 8 inches; but on examination the line was found to be 25 feet, 5.25 inches in length instead of 25 feet, its supposed length; required the true length of the wall. Ans. 654 ft. 11.17 ins. (65—9.)

NOTE.—In this question the numbers are prime to each other, and yet we save 56 figures.

55. At $69.875 for 5 cwt. 1 quarter, 14 pounds of raisins, what is that per cwt.? Ans. $13. (30—0.)

56. If 360 pounds of sugar cost $27.45, what will 1760 pounds cost at the same rate? Ans. $134.20. (46—0.)

*We cannot say definitely what number of figures an ordinary solution of this question would require; ours requires but (2.)

57. A man gave \$5.30 for 6.25 cwt. of hay, how much would 1 ton cost at the same rate? Ans. \$16.96. (37—0.)

58. If 22 cwt. 3 quarters of sugar cost \$158.20, what will 146 cwt. 1 quarter cost? Ans. \$1017. (47—3.)

59. If $17\frac{1}{2}$ bushels of corn be worth as much as $22\frac{1}{2}$ bushels of potatoes, how many bushels of corn are 315 bushels of potatoes worth? Ans. 245 bus. (36—0.)

60. If a pole 12 feet, 8 inches high, cast a shadow on a level ground 19 feet long, what is the height of a steeple whose shadow at the same time measures 235 feet 6 inches? Ans. 157 ft. (72—0.)

61. If £1 10s. and 40 groats* will buy a load of hay, how many pounds, and 19 crowns sterling for 20 loads will pay? Ans. £38 11s. 8d. (44—4.)

62. A man bought a cask of wine, containing 126 gallons, for \$283.50, and sold it out at the rate of \$2.75 per gallon, how much per cent. did he gain by this transaction? Ans. $22\frac{2}{9}$ per cent. (45—0.)

63. A merchant bought a quantity of goods for \$649, and paid \$39 for their transportation; he sold them so as to gain $37\frac{1}{2}$ per cent. on the whole cost; for how much did he sell them? Ans. \$946. (28—0.)

64. A merchant bought 3 tierces of rice, each tierce containing 6 cwt. 3 quarters, 27 pounds, for \$135, how many pounds could he have bought for $\$87.33\frac{1}{3}$ at that rate? Ans. $1519\frac{3}{5}$ lbs. (70—4.)

65. A farmer gave $\$838.56\frac{1}{4}$ for 83 acres, 3 roods, 17 rods of land, for what sum can he sell 5 acres, 2 roods, 38 rods, so as neither to gain nor loose? Ans. $\$57.37\frac{1}{2}$. (101—0.)

66. A ship's crew of 16 men is on an allowance of 6 ounces of bread per day, when meeting with a vessel they are supplied with 2 cwt. of bread; what addition will this make to their daily allowance, if the voyage will last 28 days? Ans. 8 oz. (0.)

NOTE.—Where but one figure or number is given, it is that required by our solution, the other being omitted, as it is somewhat uncertain.

67. What will be the value of 5 hogsheads of sugar, each

* A groat is equal to 4 pence, and a crown to 5 shillings.

weighing 7 cwt. 3 quarters, 27 pounds, at $8.96 per hundred weight. Ans. $358. (67—0.)

68. If 1 ounce of fine gold is sold for $20, what will 15 ingots come to, each weighing 14 pounds, 7 ounces, 3 pennyweights. Ans. $52545. (26—0.)

69. If $\frac{7}{8}$ of a gallon of wine cost $\$\frac{5}{8}$, what will $\frac{5}{9}$ of $1\frac{3}{4}$ tuns cost? Ans. $175. (0.)

70. A merchant bought 5 tuns of wine for $1767, but by misfortune of one pipe staving, he lost 120 gallons; how must he sell the remainder per gallon to get the first cost?

Ans. $1.55. (35—4.)

71. How many cords of wood worth $2.50 a cord must be given in exchange for 360 cords worth $2.75 a cord?

Ans. 396 cords. (33—0.)

72. What is the value of 7 cheeses, each weighing 26 pounds, 12 ounces, at 8 cents per pound?

Ans. $14.98. (43—0.)

73. If the carriage of 6 cwt. 2 quarters, 14 pounds cost $2.50, what should be paid for 1 ton, 19 cwt. 3 quarters, at the same rate? Ans. $15.00. (36—0.)

This question is solved in Vogdes' Arithmetic, page 141, and the difference between the number of figures required by his solution and ours is 36 in our favour.

74. Bought 5.8 tuns of oil for $338.64, whereof 50.6 gallons leaked out; how much must the remainder be sold for per gallon, that I may sustain no loss?

Ans. 24 cents. (38—7.)

75. The hind wheels of a coach which are 225 inches in circumference, will turn round 4634 times in running a certain distance, how many times will the forward wheels, which are 175 inches in circumference turn round in running the same distance? Ans. 5958 times. (52—3.)

76. If 144 gross of buttons cost £22 19s., how many gross can be bought for £12 5s. $5\frac{1}{4}$d.?

Ans. 77 gross. (82—5)=77 dif.

77. A wall was to be built by 48 men in 18 days, but an extraordinary occasion calls off $\frac{1}{3}$ of the men; how long will the remainder require to complete the work?

Ans. 27 days. (0.)

78. Bought 96 ounces of silver, for which I gave £24

sterling, what must I pay in Federal money for 480 ounces, at the same rate? Ans. $533.33\frac{1}{3}$. (0.)

NOTE.—A dollar is equal to $\frac{9}{40}$ of a pound sterling.

79. If 36 pounds of hyson tea cost £21 12s. sterling, what will be the price of a chest containing 6 cwt. 1 quarter, at the same rate in Federal money?

Ans. $1866.66\frac{2}{3}$. (51—5)=46 dif.

80. If 22 cwt. of sugar cost £12 6s. 8d. sterling, what will 1188 cwt. cost in Federal money, at the same rate?

Ans. $2960. (83—0)=83 dif.

81. Sold 16 yards of sheeting for £3 6s. 8d. sterling, and having a further order for 96 pieces, each containing 36 yards, at the same rate, I wish to know the amount in Federal money? Ans. $3200. (0.)

82. If $2\frac{3}{4}$ times $5\frac{1}{2}$ pounds of coffee cost $\frac{3}{4}$ of $2.75, what will $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of 495 pounds cost? Ans. $13.50. (0.)

83. How many yards of paper, 3 quarters wide, will cover the walls of a room that is 80 feet in compass, and 10 feet $1\frac{1}{2}$ inches high? Ans. 120 yds.

84. If for a certain sum I can have 16 cwt. 2 quarters carried 156 miles, what distance should 198 cwt. be carried for the same money? Ans. 13 miles. (0.)

85. If a man can perform a journey in 36 days by riding 42 miles a day, how many days will he require if he rides but 28 miles a day? Ans. 54 days. (0.)

86. A merchant bought $3\frac{1}{2}$ pieces of silk, each piece containing $29\frac{1}{3}$ yards, at 45 cents a yard; what did the whole amount to? Ans. $46.20. (0.)

87. If 144 pounds Avoirdupois be equal to 175 pounds Troy, how many pounds Troy are equal to 1296 pounds Avoirdupois? Ans. 1575 lbs. Troy. (0.)

88. At $7\frac{1}{2}$ cents per pounds, what will 10 cwt. 2 quarters, 16 pounds of rice come to? Ans. $8940. (0.)

89. If $14\frac{1}{4}$ cwt. be carried 171 miles for $25, how many pounds can I have carried 133 miles for the same money?

Ans. 2052 lbs. (54—0.)

90. If 49 cwt. 2 quarters, be carried 256 miles for a certain sum, what distance may 792 cwt. be carried for the same sum? Ans. 16 miles. (46—0.)

91. The earth is 360 degrees in circumference, and makes a complete revolution on its axis in 24 hours; how far are the inhabitants of the equator carried in 8 minutes, a degree being $69\frac{1}{2}$ miles? Ans. 139 miles. (0.)

92. If 3 cwt. of sugar cost £9 2s. 0d., what will 4 cwt. 3 quarters, 26 pounds cost at the same rate?

Ans. £15 2s. 3d.

This question is taken from Davies' Universal Arithmetic, page 198, where the solution is given, and 107 figures used in the work, our solution requires but 6 figures in the work.

93. A merchant bought a quantity of broad cloth and baize for £124, there were $117\frac{1}{2}$ yards of broad cloth, at 17s. 9d. per yard, for every 5 yards of broad cloth he had $1\frac{1}{2}$ yards of baize; what was the price of the baize per yard?

Ans. 11s. 2d. $1\frac{1}{47}$ far. (138—16)=122 dif.

94. A grocer bought a puncheon of rum for £41 14s. 6d., to which he added as much water as reduced its cost to 5s. 6d. per gallon; how much water did he add?

Ans. $67\frac{8}{11}$ gals. (39—3)=36 dif.

95. If 1 pound of tea be equal in value to 36 oranges, and 60 oranges be worth 84 lemons, what is the value of $3\frac{1}{3}$ pounds of tea when a lemon is worth 2 cents?

Ans. \$3.36. (0.)

96. A merchant bought 21 pieces of cloth, each containing 41 yards, for which he paid \$1260, he sold the cloth at \$1.75 per yard; did he gain or lose, and how much on the whole?

Ans. He gained \$246.75. (33—8.)

97. If \$29 75 will pay for $59\frac{1}{2}$ yards of cloth, how many yards of the same kind, will \$87.50 pay for?

Ans. 175 yds. (0.)

98. If $15\frac{1}{2}$ yards of cloth cost \$75, what will 403 yards cost? Ans. \$1950. (44—0.)

99. What is the value per yard of silk velvet, 240 yards of which I purchased for £1350 10s. 11d.

Ans. £5 12s. 6d. 2 qrs. (62—19)=43 dif.

100. If $93\frac{3}{4}$ yards of muslin cost \$8.75, what will $12\frac{6}{7}$ yards cost? Ans. \$1.20. (0.)

101. If $7\frac{5}{7}$ barrels of flour be consumed by a company in 18 days, how long will 84 barrels last?

Ans. 196 days. (0.)

102. If a mill grind $18\frac{1}{16}$ bushels of corn in 1 hour and 25 minutes, in what time will it grind $38\frac{1}{4}$ bushels?

Ans. 3 hours. (0.)

103. If $\frac{3}{4}$ of an acre of land be worth \$54, what will $5\frac{1}{8}$ acres cost? Ans. \$368. (0.)

104. Bought 24 yards, 3 quarters, 3 nails of cloth, at the rate of \$19.20 for 14 yards and 1 quarter, what did it amount to? Ans. \$33.60. (55—0.)

105. If 12.8 yards of cloth cost \$10.125, what will be the cost of 20.48 English ells? Ans. \$20.25. (60—0.)

106. If 5.75 barrels of flour cost \$32.20, what will 443.875 barrels cost, at the same rate?

Ans. \$2485.70. (70—0.)

107. If the carriage of 37 cwt. for 56 miles come to \$26, how far can I have 259 cwt. carried for the same money?

Ans. 8 miles. (0.)

108. In exchanging $20\frac{1}{2}$ yards of cloth $1\frac{1}{4}$ yards wide, for some of the same kind $\frac{3}{4}$ of a yard wide, what quantity of the latter will make an equal barter?

Ans. $34\frac{1}{6}$ yards. (0.)

109. Bought 4 yards, 3 quarters of cloth for \$6.84, what will 5 yards, 3 quarters, 3 nails cost, at the same rate?

Ans. \$8.55. (44—0.)

110. How many yards of cloth worth 17s. 4d. per yard are equal 728 yards worth 24s. a yard?

Ans. 1008 yds. (45—0.)

111. What cost 30 pieces of lead, each weighing 1 cwt. 12 pounds, at the rate of 16s. 4d. per cwt.?

Ans. £27 2s. 6d. (74—3.)

112. What quantity of sugar will £23 10s. buy at the rate of 26s. 8d. per cwt?

Ans. 17 cwt. 2 qrs. 14 lbs. (54—3.)

113. If \$850.50 be paid for 18 pieces of cloth, at the rate of \$11.25 for 5 yards, how many yards were there in each piece, allowing each to contain an equal number?

Ans. 21 yds. (43—3.)

114. If by travelling 12.3 hours per day, a journey is performed in 24.55 days, in how many days will it be accomplished by travelling 10.25 hours per day?

Ans. 29.46 days. (53—0.)

115. It has been found that 100 cubic inches of alcohol, and 82.5 cubic inches of water, each at the temperature of 60 degrees, when mingled and reduced to the same temperature, measure only 177.41 cubic inches; if, therefore, 125 gallons of the former, and 103.125 gallons of the latter be mingled under these circumstances, how many gallons will the mixture contain? Ans. 221.7625 gals.

This solution requires 107 figures in the work by the ordinary process; our solution does not require any. (107—0.)

116. If 9 yards, 2 quarters, 2 nails of broad cloth cost $68.09, what must I pay for 42 yards at that rate?
Ans. $297.12. (74—7)=67 dif.

117. Bought $5\frac{1}{3}$ pieces of muslin, each containing 27 yards, 3 quarters, at $37\frac{1}{2}$ cents per yard, what was the amount?
Ans. $55.50. (43—0.)

118. A merchant had $5\frac{9}{8}$ cwt. of sugar, at $6\frac{3}{4}$d. per pound, which he bartered for tea, at $8\frac{5}{8}$s. per pound, how many pounds of tea did he receive for the sugar? Ans. $43\frac{1}{69}$lbs.

The solution of this question will require 118 figures in the work by the common process; our solution requires but 10. (118—10)=108 dif.

119. How many yards of cloth, at 17s. 6d. per yard, can I have for 13 cwt. 2 quarters of wool, at 14d. per pound?
Ans. 100 yds. $3\frac{1}{5}$ qrs. (50—0.)

120. Bought 17 acres of land for $791.01, what would 98 acres, 3 roods, 14 perches cost at that rate?
Ans. $4598.90.$8\frac{7}{8}$.

This question is taken from Greenleaf's Arithmetic, (late edition,) page 163. The solution is given, or rather indicated, in his Key, page 77, and when completed will require 113 figures in the work; our solution requires but 12.
(113—12)=101 dif.

The following also from Greenleaf, page 164, solution in the Key, page 78, requires 74 figures; our solution requires none.

121. Sound, uninterrupted, moves 1142 feet in a second, how long after a cannon's being discharged at Boston is the time before it is heard at Bradford, the nearest distance being $25\frac{21}{22}$ miles? Ans. 2 min. (74—0.)

122. What must I pay for 50 acres, 2 roods, 20 perches of land, at $18.25 per acre? Ans. 923.90\frac{5}{8}$. (60—6.)

123. A silversmith bought 73 pounds, 5 ounces, 15 pennyweights of silver, for which he paid 5s. 9d. per ounce, what was the amount? Ans. £253 10s. $0\frac{3}{4}$d. (61—13.)

124. A grocer bartered 9 cwt. of coffee at 9d. per pound, for tea, at 5s. $7\frac{1}{2}$d. per pound, how many pounds of tea did he receive? Ans. $134\frac{2}{5}$ lbs. (65—3.)

125. If for 18 acres, 2 roods of land, I pay \$666.74, how much will a lot containing 7 acres, 3 roods, 24 perches cost, at the same rate? Ans. \$$284.71\frac{3}{5}$. (106—4.)

126. How many pieces of flannel, worth \$$18.66\frac{2}{3}$ per piece, are equal in value to $224\frac{2}{5}$ pieces worth \$12.50 per piece? Ans. $150\frac{15}{56}$. (65—6.)

127. In how many minutes will a locomotive run 49.9 miles, allowing it to run at the rate of 2.5 miles in 5.75 minutes? Ans. 114.77 minutes. (47—7.)

128. What is the value of 172 pigs of lead, each weighing 3 cwt. 2 quarters, $17\frac{1}{2}$ pounds, at the rate of \$$29.58\frac{1}{3}$ per fother of $19\frac{1}{2}$ cwt? Ans. \$$954.06\frac{1}{4}$.

If this question be solved in the ordinary way, it will require 211 figures in the work, our solution requires but 8. (211—8)=203 dif.

129. If a man buy 257 chaldrons, 15 bushels, 3 pecks of Sidney coal for \$2000, and sells it so as to make a profit of 25 per cent., for what sum should he sell $13\frac{1}{2}$ chaldrons? Ans. \$$131.09\frac{1343}{1373}$. (134—16.)

130. Bought 10 gallons, 3 quarts, 1 pint of wine for \$$16.16\frac{2}{3}$, what sum would 27 gallons, 2 quarts cost, at the same rate? Ans. \$$40.88\frac{32}{261}$. (70—12.)

131. A man who owes \$3000, for which he pays interest at 6 per cent. per annum, receives \$4.50 a day, and spends \$1.45 a day, how long before he will be worth \$1000 reckoning the year 365 days?

Ans. 4 years, $104\frac{1588}{3733}$ days. (99—35.)

132. A merchant bought a hogshead of sugar, containing 5 cwt. 3 quarters, $7\frac{1}{4}$ pounds for \$50, what should he give for 2 other hogsheads, one of which contains 6 cwt. 18 pounds, and the other 4 cwt. 3 quarters?

Ans. \$$93.81\frac{499}{521}$. (84—20.)

133. A trader bought 10 puncheons of rum for \$800; 2 puncheons were stove through accident, and he lost 28 gal-

lons besides from leakage; if he sell the remainder so as to lose 9 per cent. of the whole cost, for what sum can you buy 3 puncheons? Ans. \$284.86$\frac{22}{23}$. (92—12.)

134. If when the price of wheat is 6s. 3d. a bushel, the penny loaf weighs 9 ounces, what ought it to weigh when wheat is 8s. 2$\frac{1}{2}$d. a bushel? Ans. 6$\frac{168}{197}$ oz. (27—4.)

135. If 159 yards of muslin can be purchased for \$20.67, how many yards may be purchased for \$34.71.

Ans. 267 yds. (51—0.)

136. If 29 cwt. 14 pounds of beef cost \$87.50, how many pounds may be bought at the same rate for \$31.75?

Ans. 1183$\frac{16}{25}$ lbs. (88—7.)

137. If \$25.75 will buy 20$\frac{3}{5}$ bushels of wheat, how many bushels can be bought for \$17.62$\frac{1}{2}$?

Ans. 14.1 bushels. (47—0.)

138. If 2 yards, 1 quarter, 2 nails of cloth sell for £3 16s. 7$\frac{1}{2}$d. what will 8 yards, 2 quarters, 3 nails cost?

Ans. £14 0s. 3d. 1$\frac{14}{19}$ qrs. (99—19.)

139. When a bushel of corn is worth 3 pecks, 4 quarts of wheat; if the price of a bushel of corn be 87$\frac{1}{2}$ cents, how many bushels of wheat can be had for \$19.75?

Ans. 19$\frac{3}{4}$ bushels. (51—0.)

140. If \$14 will pay for the carriage of a ton of hay 75.6 miles, what distance may a ton be carried for \$16.75?

Ans. 90.45 miles. (37—0.)

141. If 2 hogsheads, 19 gallons, 2 quarts of wine cost £93 1s. 2$\frac{1}{4}$d. what will 25 hogsheads, 36 gallons cost?

Ans. £1030 7s. 4d. 2qrs. (152—12.)=140 dif.

This question, solved in the ordinary way, will require 152 figures in the fork; our solution requires but 12, being a gain or saving of 140 figures.

142. If the earth 360 degrees in circumference, and each degree 69$\frac{1}{2}$ miles, how long would a man be in travelling round it, who advances 20 miles a day, reckoning the year at 365$\frac{1}{4}$ days? Ans. 3 years, 155$\frac{1}{4}$ days. (104—7)=97 dif.

143. A merchant failing in trade owes in all \$17,280, his effects are sold for \$15,120; what does A. receive to whom he owes \$5670? Ans. \$4961.25. (50—10.)

144. How many yards of cloth, at 17s. 6d. per yard, can I have for 17 cwt. 2 quarters, 24 pounds of wool, at 15d. per pound? Ans. 141$\frac{5}{7}$ yds. (47—3.)

145. If $1.75 will buy 7 pounds of loaf sugar, how much will $213.50 buy? Ans. 7 cwt. 2 qrs. 14 lbs. (40—0.)

146. A blacksmith gave $5.60 for 1 cwt. of iron, how much must he pay, at that rate, for 15 cwt. 2 quarters, 8 pounds? Ans. $87.20. (58—0.)

147. If I pay $18 for 80 gallons of molasses, what should I pay for 2 hogsheads, 1 tierce, 23 quarts?

Ans. $42.97½. (29—4.)

148. If 18 bushels, 3 pecks of wheat cost $28.12½, what will 28 bushels cost, at the same rate?

Ans. $42. (45—0.)

149. A man gave $5.30 for 6.25 cwt. of hay; how much would 1 ton, 7 cwt. 3 quarters, 14 pounds cost, at the same rate? Ans. 23.63\frac{4}{5}$. (65—5.)

Here we suppose the common process will be to change 3 quarters, 14 pounds to a decimal.

150. If a loaf of bread weighing 28 ounces, be sold for 8 cents, when flour is worth $7.50 per barrel, what ought the 8 cent loaf to weigh when flour is worth $4.37.5 per barrel? Ans. 48 oz. (34—0.)

151. How many yards of paper 2 feet, 8 inches wide will cover the wall of a room 27 yards in circuit and 9 feet 8 inches high? Ans. 97⅞ yds. (32—0.)

152. If 8 yards, 3 quarters of cloth cost $16.80, what will 37 yards, 1 quarter, cost at the same rate?

Ans. $71.52. (46—0.)

153. A country trader agreed to exchange 670 pounds of butter which he valued at 22 cents a pound, for a hogshead of white sugar, containing 7 cwt. 3 quarters, 16.4 pounds, at how much a pound was the sugar valued?

Ans. 16¾ cts. (64—0.)

154. What must be paid for 53 English ells, 1 quarter of holland, at the rate of 7s. 9½d. for 2¾ yards?

Ans. £9 8s. 5d. (53—6.)

155. A young hare starts 40 yards before a grayhound, and is not perceived by him till she has been up 40 seconds, she scuds away at the rate of 10 miles an hour, and the hound in view makes after her, at the rate of 18 miles an hour, what distance must the dog run to overtake the hare?

Ans. 530 yds.

Since the hare runs at the rate of 10 miles an hour, in 40 seconds she will run $\frac{1}{90}$ of 10 miles, or $\frac{1}{9}$ of *one* mile, or of 1760 yards, and the 40 yards, which is equal to $\frac{360}{9}$, added to $\frac{1760}{9}=\frac{2120}{9}$ yards, this distance the dog has to gain; now since the dog runs 18 miles in order to gain 8, the distance he will have to run to overtake the hare, will be $\frac{9}{4}$ of the distance to be gained. There is evidently no necessity for finding the time occupied in the race in order to get the distance the hound has to run.

Solution.

yards.

$$\begin{array}{r|l} & \cancel{2120} \\ \cancel{9} & 1 \\ \cancel{4} & \cancel{9} \\ \hline \text{Ans.} & 530 \text{ yds.} \end{array}$$

Emerson's solution of this question (see Emerson's Key, page 62) will require 141 figures in the work, independent of those contained in two statements and in the answer; our solution requires *none* but those in the statement and answer. (141—0.)

156. If the corn contained in 8 bags, holding 2 bushels, 3 pecks each, be worth \$14.25, what is the value of the corn contained in 7 bags, each holding 2 bushels, 3 pecks, 7 qts.? Ans. \$13.46$\frac{41}{704}$. (73—26.)

157. If 7 bushels, 3 pecks of corn cost \$6, what will 25 bushels, 2 pks. 6 qts. cost? Ans. \$19.88$\frac{22}{31}$. (60—12.)

158. Suppose a fox to have 499$\frac{1}{2}$ yards the start of a hound, and to run at the rate of 19$\frac{3}{4}$ yards, whilst the hound runs 29 yards, how far must the hound run to catch the fox? Ans. 1566 yds. (46—0.)

NOTE—As the labour of solving the questions by the common method is considerable, we will give only the *number* of *figures* required by our own solution hereafter, except some particular cases, when both numbers will be given.

159. How must I retail molasses by the gallon, for which I paid \$28 a hogshead to gain 12$\frac{1}{2}$ per cent.?

Here we say, since \$28 is the price of 63 gallons, 1 gallon will cost a less sum than \$28, therefore we place 1 on the right of the line and the 63 on the left. This statement would give us the cost of one gallon, but we wish to sell it so as to gain 12$\frac{1}{2}$ per

Solution.

$$\begin{array}{r|l} & \$ \\ \cancel{63} & \cancel{28} \\ & \cancel{1} \\ \cancel{8} & \cancel{9} \\ \hline \text{Ans.} & 50 \text{ cents.} \end{array}$$

cent., now $12\frac{1}{2}$ being $\frac{1}{8}$ of 100, we are to increase the cost in the proportion of 9 to 8, this we will effect by placing the 9 on the right of the line and 8 on the left. In the solution we know 7 and 4 to be factors of 28; then the 7 and 9 on the right of the line being multiplied together (mentally of course) destroy the 63 on the left, this leaves 4 on the right and 8 on the left, and 4 being $\frac{1}{2}$ of 8 the answer is half a dollar, or 50 cents.

160. Bought 1 ton of steel for \$182, for how much must I sell 3 cwt. 2 quarters, 16 pounds, to gain 20 per cent.?

Solution.

	\$
	$\cancel{182}$
$\cancel{14}$	51
$\cancel{20}$	39
5	$\cancel{6}$
Ans.	\$39.78

Here since money is required for the answer, we write \$182 at the head, on the right of the line, this being the price of 1 ton or 20 cwt. the answer, which is to be the price of 3 cwt. 2 quarters, 16 pounds, must be less than \$180, therefore the less number must be written on the right of the line, 16 lbs.$=\frac{4}{7}$ of a quarter, then $2\frac{4}{7}$ qrs.$=\frac{18}{7}$ of a quarter, or $\frac{18}{28}=\frac{9}{14}$ of a cwt., hence $3\frac{9}{14}$ cwt.$=(\frac{51}{14}$ cwt.) (This process of changing the lower denomination to fractions of the higher should always be done mentally.)

Having made the statement which expresses the cost price of 3 cwt. 2 quarters, 16 pounds, or $\frac{51}{14}$ cwt. and wishing to gain the 20 per cent. we must increase this cost price, in the ratio of 120 to 100, but as 20 is $\frac{1}{5}$ of 100 we will increase it in the ratio of 6 to 5, that is, write the 6 on the right of the line and 5 on the left.

161. Suppose a merchant to pay \$85 per ton for iron, at what price per cwt. must he sell it to gain 12 per cent.?

Ans. \$4.76. (0.)

162. If I pay \$525 for 90 barrels of flour, at what price must I sell it per barrel to gain 7 per cent.?

Ans. $\$6.24\frac{1}{8}$. (4.)

163. What will an iron master receive for 94 tons, 17 cwt. 3 quarters of iron, at \$96 per ton, if he allow a deduction of $2\frac{1}{2}$ per cent. for cash payment? Ans. \$8881.47. (3.)

$2\frac{1}{2}=\frac{1}{40}$ of 100, 17 cwt. 3 qrs$=\frac{71}{80}$ of a ton.

164. If I buy 297 acres of western land for \$3740, for how much must I sell 135 acres to clear 40 per cent.?
Ans. \$2380. (0.)

165. A grocer sold 29 pounds of coffee for \$5.07$\frac{1}{2}$ making on the sale a profit of 25 per cent., what did the grocer pay for 60 pounds of this coffee? Ans. \$8.40. (0.)

166. A merchant bought 10 tons of iron for \$950, the freight and duties amounted to \$145, and his own charges to \$25, how must he sell it per pound to gain 20 per cent.?
Ans. 6 cents. (0.)

167. If 4$\frac{1}{4}$ yards of broad cloth cost \$17, what will 13$\frac{7}{8}$ yards cost at the same rate? Ans. \$55.50. (0.)

168. A trader having $\frac{7}{8}$ of a cwt. of coffee sold $\frac{4}{7}$ of it for \$19.37$\frac{1}{2}$, at what price per pound was it sold?
Ans. 34$\frac{67}{112}$ cents. (10.)

169. 16,000 cubic feet of water were found to flow over a mill-dam in 1 hour, 23 minutes, 10 seconds, how much at that rate will flow over the dam in 3 days?
Ans. 831,102$\frac{1}{4}\frac{0}{9}\frac{2}{9}$ cubic ft.

In the above question the numbers of the dividend and divisor are all prime to each other, and our solution requires 18 figures in the work, the common method will require 90.

170. Tubes may be made of gold, weighing not more than $\frac{1}{1625}$ of a grain per foot; what would be the weight of such a tube which would extend across the Atlantic ocean, from Boston to London, estimating the distance at 3000 miles?
Ans. 1 lb. 8 oz. 6 dwt. 3$\frac{9}{13}$ grs. (4.)

171. A man sold 345 pounds of beef, at 7$\frac{1}{2}$ cents a pound, and received his pay in molasses, at 37$\frac{1}{2}$ cents a gallon; how many gallons did he receive? Ans. 69 gals. (0.)

172. A trader bought 10 puncheons of rum for \$800, 2 puncheons were stove through accident, and he lost 28 gallons besides by leakage; if he should sell the remainder so as to lose 8 per cent. on the whole first cost, for what sum can you buy 3 puncheons? Ans. \$288. (0.)

173. If 5 tons of hay will last 160 sheep 120 days, how long will it last 150 sheep? Ans. 128 days. (0.)

174. If 3.25 pounds of tabacco cost \$0.975, what quantity can I purchase for \$5.46? Ans. 18$\frac{1}{5}$ lbs. (3.)

175. What is the value of 7 tons, 11 cwt. 3 quarters of

figs at $4.16 per cwt. allowing a deduction of 14 pounds per cwt. for boxes? Ans. $552.37. (0.)

176. In what time will $858 gain as much interest as $572 will gain in 13 months? Ans. $8\frac{2}{3}$ months. (0.)

177. If $6\frac{1}{2}$ pounds of rice cost 63 cents, and 18 pounds of rice are worth as much as 7 pounds of coffee, what are 78 pounds of coffee worth? Ans. $19.44. (0.)

178. If $8\frac{2}{9}$ yards of cloth cost $33.30, what will 62 yards, 2 quarters cost at the same rate? Ans. 253.12\frac{1}{2}$. (6.)

179. What will 43 pieces of cloth, each containing 40 yards, come to at $62\frac{1}{2}$ cents per yard? Ans. $1075. (0.)

180. How many yards of silk may be bought for $37.62, if $\frac{3}{4}$ of a yard cost 66 cents? Ans. 42 yds. 3 qrs. (3.)

181. If a ship sail $92\frac{4}{7}$ miles in $9\frac{1}{3}$ hours, in how many hours would she sail $65\frac{1}{10}$ miles, at that rate?

Ans. $6\frac{913}{1620}$ hours. (13.)

182. If 3 pounds of sugar cost 25 cents, and 15 pounds of sugar are worth 8 pounds of coffee, how much will 96 pounds of coffee cost? Ans. $15. (0.)

183. If 7 cwt. 1 quarter of sugar cost $64.96, what will be the price of 4 cwt. 2 quarters, at the same rate?

Ans. $40.32. (3.)

184. How many yards of carpeting, 3 quarters of a yard wide, are sufficient to cover the floor of a room that is 18 feet wide and 60 feet long? Ans. 160 yds. (0.)

185. If $6\frac{1}{2}$ yards of cloth cost $23, what will 9 yards, 3 quarters cost, at that rate? Ans. $34.50. (0.)

186. If a man perform a journey in $22\frac{1}{2}$ days, when the days are 12 hours long, how many days will it take him to perform the same journey when the days are 15 hours long, if his rate of travelling be the same? Ans. 18 days. (0.)

187. If $14\frac{1}{4}$ cwt. be carried 200 miles for $32, how many pounds can I have carried 72 miles for the same money?

Ans. $4433\frac{1}{3}$ pounds. (5.)

188. A goldsmith sold a tankard for £10 12s. at the rate of 5s. 4d. per ounce; I demand the weight of it.

Ans. 39 oz. 15 dwts. (3.)

189. A gave B 5 cwt. of sugar, at 6 cents a pound, for a quantity of cinnamon, at $1.28 a pound; how many pounds of cinnamon did A receive? Ans. 26 lbs. 4 oz. (3.)

190. If \$783.42 gain \$27.45 in 12.75 months, what principal will gain the same amount of interest in 15.3 months?
Ans. \$652.85. (0.)

The common method will require 70 figures.

191. If a man drink daily a dram which costs $6\frac{1}{4}$ cents, how much will he expend in this manner in 20 years of $365\frac{1}{4}$ days each? Ans. \$$456.56\frac{1}{4}$. (6.)

192. If I invest \$1250 in wood, at \$4.25 a cord, and sell it at an average of \$5.10 a cord, how much do I make by the transaction? Ans. \$250. (0.)

193. Laid out £250 in cloth, at 30s. per yard, and afterwards finding it was damaged, sold it at 26s. 3d. per yard, how much did I lose? Ans. £31 5s. (0.)

194. What is the cost of 37 tons, 14 cwt. 2 quarters, 14 pounds of hemp, at £89 6s. 8d. per ton?
Ans. £3370 13s. 2d. (10.)

195. E has flannel worth 50 cents a yard for ready money, but in trade he will charge 56 cents. H has muslin worth $31\frac{1}{4}$ cents a yard in ready money, at what price ought the muslin to be rated in trade? Ans. 35 cts. (0.)

196. A bankrupt owes in all \$26,789, and delivers up his whole property amounting to \$11,739, how much does that creditor receive to whom he owes \$1869?
Ans. \$819. (6.)

197. If $3\frac{1}{2}$ times $8\frac{1}{2}$ pounds of sugar cost $1\frac{1}{2}$ times $1\frac{1}{2}$ dollars, what will $\frac{1}{2}$ of $\frac{1}{3}$ of $12\frac{1}{4}$ pounds cost?
Ans. $37\frac{1}{2}$ cents. (0.)

198. If a mill grind $18\frac{11}{16}$ bushels of corn in one hour and 18 minutes, in what time will it grind 690 bushels?
Ans. 48 hours. (0.)

199. A merchant bought $\frac{7}{18}$ of a company's stock, for \$89,900, what would be the price of $\frac{14}{29}$ of the stock, at the same rate? Ans. \$111,600. (0.)

200. If $\frac{3}{5}$ of a yard of cloth cost £$\frac{7}{15}$, what will $\frac{3}{14}$ of a yard come to at the same rate? Ans. 3s. 4d. (0.)

201. A person having $\frac{4}{5}$ of a sloop, sells $\frac{2}{3}$ of his share for £319, what was the value of the whole sloop, at that rate?
£598 2s. 6d. (4.)

202. A merchant sold $5\frac{2}{5}$ pieces of cloth, each containing $24\frac{2}{3}$ yards, at \$$6\frac{2}{3}$ per yard, what did he get for the whole?
Ans. \$888. (0.)

203. Bought a cask of wine, containing 126 gallons, at 40 cents a gallon; but by accident 18 gallons leaked out; at what price per gallon must the remainder be sold to gain $12\frac{1}{2}$ per cent. on the whole first cost? Ans. $52\frac{1}{2}$ cts. (0.)

204. How much wine at $1.26 per gallon must I give for 26 cwt. 2 quarters, 14 pounds of raisins, valued at $9.44.4 per cwt.? Ans. 199 gals. $2\frac{17}{70}$ qts. (16.)

To solve the above question in the ordinary way, and reduce the fraction to its lowest terms, will require 151 figures, being 135 figures more than our solution requires.

205. If a Spanish dollar weighs 17 pennyweights, 6 grains, what will be the weight of $480? Ans. 34 lbs. 6 oz. (0.)

206. The circumference of the earth, measured on the equator, being 24,896.16 miles, what is the length of an arc 57° 17′ 45″? Ans. 3962.35065 miles. (20.)

Here our solution requires 20 figures, the common method will require 155, making a difference of 135 figures.

207. How much sugar at 8d. per pound must I give for 20 cwt. of tobacco, at £3 per cwt.?

Ans. 16 cwt. 8 lbs. (3.)

208. My tailor informs me it will take $10\frac{1}{4}$ square yards of cloth to make me a full suit of clothes. The cloth I am about to purchase is $1\frac{7}{8}$ yards wide, and on sponging it will shrink 5 per cent. in width and length; how many yards of the above cloth must I purchase for my new suit?

Ans. $6\frac{62}{1083}$ yds.

Solution.

	Yds.
$\cancel{4}$	41
$\cancel{15}$	8
19	$\cancel{20}$
19	20

1083)6560($6\frac{62}{1083}$ Ans.

Here $10\frac{1}{4}$ yards$=\frac{41}{4}$ which we place at the head of the line. Now if the cloth were a yard wide, then every yard in length would contain one square yard; but being $1\frac{7}{8}$ yards wide, it will require *less* in *length* in the same *ratio* as the *width* is *greater* than a yard, (that is in the ratio of 8 to 15.) This statement would give the proper quantity of cloth were there no other conditions; but in consequence of its shrinking 5 per cent. which is equal to $\frac{1}{20}$, we perceive that 20 parts in length and width, before the cloth is sponged, will only be equal to 19 parts after; hence we must *increase*

the quantity which the first part of our statement would give, in the ratio of 20 to 19, both in length and width.

The solution of the above question, as indicated in Greenleaf's Key, page 130, will require *three* statements, which when worked out and the fractional part reduced to its lowest terms, 101 figures will be the smallest number which will complete the work; and unless the pupil is pretty skilful in reducing the fraction, he may not get through much short of 200 figures. In our solution it is unnecessary to make more than 8 figures.

COMPOUND PROPORTION.

1. If the wages of 6 men for 14 days be $84, what will be the wages of 9 men for 11 days?

Here we say, $84 being the wages of 6 men, the wages of 9 men must be more than $84; hence we place the 9 which is greater than 6 on the right of the line, and (of course) 6 on the left. The statement, thus far, would give us the wages of 9 men, but it would be for the *same time* which 6 men require, to earn $84, that is, 14 days, which would be too much, as we are required to find the wages for only 11 days, which will be less than the first part of the statement would give, in the ratio of 11 to 14; hence we place 11 on the right of the line, and 14 on the left, then 6 times 14 cancels the 84, and we have only to multiply 9 and 11 together for the answer.

Solution.

$$\begin{array}{r|l} & \$ \\ \cancel{6} & \cancel{84} \\ \cancel{14} & 9 \\ & 11 \\ \hline \text{Ans.} & \$99 \end{array}$$

When the answer to a question depends upon several conditions, as in the last two examples, the process by which we effect the solution is called Compound Proportion.

RULE FOR COMPOUND PROPORTION.

As in simple proportion, place on the right of a vertical line, that number which is of the same name or kind as the required answer; then of the remaining numbers take any two that are of the same kind, and ascertain from the reading

of the question, whether an answer depending upon *these alone* would be greater or less than the number first written, and place the greater or less on the right of the line accordingly, (as in simple proportion;) then take any other two of the same kind, and again ascertain from the reading of the question, whether an answer depending upon *these alone* would be greater or less than the number first written, and place the greater or less of these numbers on the right of the line accordingly; and so on until all the numbers are stated, then proceed as in simple proportion.

2. Suppose 4 men in 12 days can mow 48 acres of grass, how many acres ought 13 men to mow in 9 days?

Here you will at once perceive that 48 acres is of the same name as the required answer; then of the remaining numbers, first take the men, and pay no regard to the time; and you will readily perceive that since 48 is the number of acres which 4 men can mow, 13 men should of course mow a greater number of acres; therefore, you place the 13 on the right of the line, and the 4 on the left; in the next place take the days, and pay no attention to the number of men, and say if the mowing of 48 acres will afford labour for 12 days, the mowing of a less number of acres will afford labour for 9 days; therefore, place 9, the less of these two numbers, on the right of the line; then 4 times 12 cancels the 48, and the product of 13 and 9 gives the answer.

$$\begin{array}{r|l} & \text{Acres.} \\ & \cancel{48} \\ \cancel{4} & 13 \\ \cancel{12} & 9 \\ \hline \text{Ans.} & 117 \text{ acres.} \end{array}$$

3. If 16 men receive $256 for 18 days' work, how much ought 27 men receive for 24 days?

In this solution, observe 9 and 2 are factors of 18, the 9 we divide (mentally) into 27, and the 2 into 24; the product of their quotients is 36; now to multiply 36 and 16 together, multiply 16 by each figure of 36; thus, 6 times 16 are 96, put down 6 and carry 9, then 3 times 16 are 48, and 9 are 57 This could have been done very conveniently without writing either 16 or 36.

$$\begin{array}{r|l} & \$ \\ & \cancel{256} \\ \cancel{16} & 16 \\ & \cancel{27} \\ \cancel{18} & 36 \\ & \cancel{24} \\ \hline \text{Ans.} & \$576 \end{array}$$

The pupil who adopts the course of making his solutions to as great an extent as possible, a purely mental operation, will make much greater progress than by solving his question by the mere mechanical process of making figures, whilst at the same time he will thus strengthen his memory and develop the reasoning powers of his mind more in one day than would result from the common method of pursuing the study of this science in a week.

The solution of the following question will be found in Professor Vogdes' Arithmetic, page 148, it will be observed that he makes use of the *cancelling* process. We will place his statement and solution on the "left hand, and our own on the right."

4. If £50 in $\frac{5}{12}$ of a year gain £2 5s. 1d. $2\frac{2}{3}$qrs. in what time will £$13\frac{1}{3}$ gain £$1\frac{1}{12}$?

$$\left.\begin{array}{rcr}
 & \pounds & & \pounds \\
\text{As} & 13\frac{1}{3} & : & 50 \\
 & 3 & & \\
 & \overline{40} & & \\
 & \overline{3} & & \\
\\
 & \pounds\ \text{s.}\ \text{d.}\ \text{qr.} & & \pounds \\
 & 2\ \ 5\ \ 1\ \ 2\frac{2}{3} & : & 1\frac{1}{12} \\
 & 20 & & 12 \\
 & \overline{45} & & \overline{13} \\
 & 12 & & 20 \\
 & \overline{541} & & \overline{260} \\
 & 4 & & 12 \\
 & \overline{2166} & & \overline{3120} \\
 & 3 & & 4 \\
 & \overline{6500} & & \overline{12480} \\
 & \overline{3} & & \overline{12}
\end{array}\right\} :: \frac{5}{12}\text{ yrs.}
\qquad
\begin{array}{r|l}
\multicolumn{2}{c}{\text{years.}} \\
\cancel{12} & \cancel{5} \\
\cancel{12} & \cancel{13} \\
\cancel{1625} & \cancel{30} \\
 & \cancel{20} \\
4\ \cancel{0} & \cancel{50} \\
 & 3 \\
\hline
 & \frac{3}{4}\text{ yrs. Ans.}
\end{array}$$

Cancelled.

$$\frac{\cancel{3}}{\cancel{40}} \times \frac{\cancel{3}}{\underset{1300}{\cancel{6500}}} \times \frac{50}{1} \times \frac{\overset{\overset{78}{\cancel{312}}}{\cancel{12480}}}{\underset{4}{\cancel{12}}} \times \frac{\cancel{5}}{\underset{\cancel{4}}{\cancel{12}}} = \frac{50\times78}{1300\times4} = \frac{3900}{5200} = \frac{3}{4}\text{ yrs. Ans.}$$

Here you will perceive that 95 figures are used in the work, (of Vogdes' solution.)

Notwithstanding the application
Of the process of cancellation,
Whilst in our solution there are none;
The statement's made, and the work is done.

5. If 18 men can build 24 rods of wall in 16 days, how many men should be employed to build 72 rods of the same kind of wall in 12 days?

$$\begin{array}{ll|ll} & & 18 & \text{men} \\ \text{rods} & \cancel{24} & \cancel{72} & \text{rods} \\ \text{days} & \cancel{12} & \cancel{16} & \text{days} \\ \hline & \text{Ans.} & 72 & \text{rods.} \end{array}$$

6. If the use of $100 for 90 days be worth $1.50, what will the use of $78 for 85 days be worth?

$$\begin{array}{ll|ll} & \cancel{2} & \$\ \cancel{3} & \\ \text{days} & 100 & \cancel{78} & \\ & \cancel{90} & 85 & \text{days} \\ \hline & \text{Ans.} & \$1.105 & \end{array}$$

Here 3 measures the 9, then you say, 3 times 2 or 6, measures 78, and you have 13 to multiply 85 by, and as there are 3 ciphers on the left of the line, you point off 3 in the product, and the true result is obtained without the least necessity of writing a single figure, except those in the statement and answer.

7. If a class of 25 girls perform 1750 examples in Arithmetic in 15 hours, how many similar examples may a class of 30 girls perform in 18 hours?

Ans. 2520 examples. (0.)

8. If a man travel 217 miles in 7 days, travelling 6 hours a day, how many miles will he travel at that rate, in 9 days, if he travel 11 hours a day? Ans. $511\frac{1}{2}$ miles. (4.)

9. If 19 students spend $551 in 29 weeks, how much at that rate, would 48 students spend in 16 weeks?

Ans. $768. (0.)

10. If 17 persons drink $23\frac{4}{5}$ gallons of beer in $3\frac{1}{2}$ weeks, how much, at the same rate, will 35 persons drink in 8 weeks?

Ans. 112 gallons. (0.)

11. If 129 loads of hay be carried 39 miles for \$559, how much should be paid at the same rate, for the carriage of 47 loads 27 miles? Ans. \$141. (0.)

12. If the interest of \$347 for $3\frac{1}{2}$ years be \$72.87, what will be the interest at that rate, on \$537 for $2\frac{1}{2}$ years?
Ans. \$80.55. (0.)

13. If the carriage of 60 cwt. 20 miles cost \$14.50, what weight can I have carried 30 miles for $\$5\frac{7}{16}$?
Ans. 15 cwt. (0.)

14. If $4\frac{1}{2}$ pounds of thread make $13\frac{1}{2}$ yards of linen, 3 quarters wide, how many pounds will be required to make 81 yards, 5 quarters wide? Ans. 45 lbs. (0.)

15. If it requires 750 tiles each 16 inches long and $12\frac{1}{2}$ inches wide to pave a gentleman's yard, how many tiles, each 15 inches long and 8 inches wide would pave the same yard? Ans. 1250 tiles. (0.)

16. If 35 men can mow 208 acres, 1 rood, 24 perches of grass in 24 days, how many men can mow 260 acres, 2 roods, in 25 days? Ans. 42 men. (0.)

17. How many days of $8\frac{1}{2}$ hours each, will 42 men require to build a wall $98\frac{3}{4}$ feet long, 8 feet high, and $2\frac{3}{4}$ feet thick, if 63 men can build a wall $45\frac{1}{3}$ feet long $6\frac{7}{12}$ feet high and $3\frac{1}{3}$ feet thick, in 68 days of $11\frac{1}{3}$ hours each?
Ans. 297 days. (0.)

Solution.

	$\cancel{6}\cancel{8}$ days	
$\cancel{3}$	$\cancel{3}\cancel{4}$	
$\cancel{1}\cancel{7}$	$\cancel{2}$	
$\cancel{4}\cancel{2}$	$\cancel{6}\cancel{3}$	
$\cancel{4}$	$\cancel{1}\cancel{9}\cancel{5}$	
$\cancel{1}\cancel{3}\cancel{6}$		3
	$\cancel{8}$	
$\cancel{7}\cancel{9}$	$\cancel{1}\cancel{2}$	3
$\cancel{4}$		11
$\cancel{1}\cancel{0}$		3
Ans.	297 days	

Here 68 days is the term of the same name as the required answer; then to state the other terms we say, in the first place, 68 is the number of days of $11\frac{1}{3}$ hours long, required to build a wall, and it will require a greater number of days of $8\frac{1}{2}$ hours long, (to build a wall,) therefore we place the greater of *these* two numbers on the right of the line. Secondly, 68 is the number of days required for 63 men to build a wall, and a longer time will be required for 42 men to build a wall, therefore we place the greater of *these* two numbers on the right of the line.

Thirdly, 68 is the number of days required to build a wall $45\frac{1}{3}$ feet long, and a greater number of days will be required to build a wall $98\frac{2}{3}$ feet long, therefore we place the greater of *these* two numbers on the *right* of the line. Fourthly, 68 is the number of days required to build a wall $6\frac{7}{12}$ feet high, and a greater number of days will be required to build a wall 8 feet high, therefore we place the greater of *these* two numbers on the right of the line. Lastly, 68 is the number of days required to build a wall $3\frac{1}{3}$ feet thick, and a *less* number of days will be required to build a wall $2\frac{3}{4}$ feet thick, therefore we place the *less* of *these* two numbers on the right of the line.

Now, in the solution we say, 17 into 34, twice, (though we do not write down the 2,) but multiply the 68 by it, and the product will concel 136 on the other side of the line, then 79 into 395, 5 times, this 5 we do not write down, but multiply the 8 by it, and the product *cancels* the product of 4 and 10 on the other side, next cancel a 3 on both sides; then 4 into 12, 3 times, by which 3 multiply the 2, and say, 6 into 42, 7 times, 7 into 63, 9 times, (mentally,) then the product of 9, 11, and the remaining 3 (performed mentally) will give 297, the answer.

18. If a footman perform a journey of 177 miles, in 12 days, when the days are 12 hours long, in how many days travelling at the same rate, 16 hours a day, would he accomplish a journey of 1003 miles? Ans. 51 days. (5.)

19. If a pile of stones $16\frac{1}{2}$ feet long and $4\frac{1}{2}$ feet wide cost \$2.75, what will another pile of the same height, 132 feet long and $22\frac{1}{2}$ feet wide of the same quality cost?

Ans. \$110. (0.)

20. If 4 pounds of cotton make 11 yards of cloth 7 quarters wide, how many pounds will it take to make 462 yards 6 quarters wide? Ans. 144 lbs. (0.)

21. If 3 men receive £$8\frac{9}{10}$ for $19\frac{1}{2}$ days' work, how much should 20 men receive for $100\frac{1}{4}$ days?

Ans. £305 0s. $8\frac{8}{39}$d. (12.)

22. If 16 compositors set 150 pages of types, each page consisting of 48 lines, and each line 50 letters, in 3 days of 10 hours, how many compositors will be required to set 500 pages of 72 lines each, and 45 letters in each line, in 6 days of 8 hours each? Ans. 45 compositors. (0.)

23. A cistern $17\frac{1}{2}$ feet in length, $10\frac{1}{2}$ feet in breadth, and 13 feet deep, holds 546 barrels of water, how many barrels will fill a cistern that is 16 feet long, 7 feet broad, and 15 feet deep? Ans. 384 barrels. (0.)

24. If a stream of water running into a pond of 190 acres will raise the pond 10 inches in 12 hours, how much would a pond of 57 acres be raised by the same stream in 8 hours? Ans. $22\frac{2}{9}$ inches. (0.)

25. If 15 weavers by working 10 hours a day for 12 days can make 250 yards of cloth, how many must work 9 hours a day at the same rate for 15 days to make $607\frac{1}{2}$ yards? Ans. 27 men. (3.)

26. If 248 men in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardness, $232\frac{1}{2}$ yards long, $3\frac{3}{4}$ yards wide, and $2\frac{1}{3}$ deep, in how many days of 9 hours each will 24 men dig a trench of 4 degrees of hardness, $337\frac{1}{2}$ yards long, $5\frac{3}{5}$ wide, and $3\frac{1}{2}$ deep? Ans. 132 days. (0.)

27. If 36 men in $127\frac{1}{2}$ days of $13\frac{1}{2}$ hours dig a trench $33\frac{3}{4}$ yards long, $10\frac{1}{2}$ feet deep, and $15\frac{3}{5}$ feet wide, how many men in $7\frac{2}{7}$ days of $12\frac{8}{11}$ hours will dig a similar trench $82\frac{8}{11}$ yards long, $7\frac{2}{5}$ feet deep, and 10 feet wide? Ans. 740 men. (0.)

28. If 10 cows eat $7\frac{1}{2}$ tons of hay in 14 weeks, how many cows at the same rate, will eat $22\frac{1}{2}$ tons in 28 weeks? Ans. 15 cows. (0.)

29. If 3 pounds of yarn make 9 yards of cloth 5 quarters wide, how many pounds will be required to make a piece of cloth 45 yards long and 4 quarters wide? Ans. 12 lbs. (0.)

30. If 29 men can cut 87 cords of wood in 3 days, when the days are 15 hours long, how many men, working at the same rate, will cut 462 cords in 14 days of 11 hours each? Ans. 45 men. (0.)

31. If \$625 in 16 months gain \$37.50 interest, how much will \$400 gain in 5 years and 4 months? Ans. \$96. (0.)

32. How many stones $1\frac{3}{4}$ feet long, $\frac{2}{3}$ of a foot wide, and $\frac{3}{4}$ of a foot thick are equal in weight to 50 stones of the same quality $3\frac{1}{3}$ feet long, $2\frac{1}{2}$ feet wide, and $1\frac{1}{5}$ feet thick? Ans. $571\frac{3}{7}$ stones. (0.)

33. When 12 oxen graze $16\frac{1}{4}$ acres in 20 days, how much of the like pasture will serve 24 such cattle 100 days? Ans. $162\frac{1}{2}$ acres. (0.)

34, How many pieces of patch-work 4 inches long and 3 inches wide, will make a quilt 8 feet 6 inches long and 7 feet 10 inches wide? Ans. 799 pieces. (0.)

35. If $971.25 worth of provisions will serve 25 men 35 weeks, how much will serve 59 men 17 weeks at the same rate? Ans. $1113.33. (4.)

36. If 6915 men, by working 14 hours in the day, can make a railroad from Philadelphia to Pittsburg in 108 days, how many men ought to do the same in 945 days, by working 12 hours in the day? Ans. 922 men. (3.)

37. If 7 bars of iron, each 4 feet long, 3 inches wide, and 2 inches thick, weigh 546 pounds, what will 13 bars, each 6 feet long, 4 inches wide, and 3 inches thick weigh? Ans. 3042 lbs. (0.)

38. A garrison of 400 men had a supply of provisions for 34 weeks, each receiving 35 ounces a day, at the end of 20 weeks, 50 of the men were killed in an assault, what quantity of food can each of the remaining men receive a day, so that the provisions may last 28 weeks beyond the time first specified? Ans. $13\frac{1}{3}$ oz. (0.)

39. If 96 plank, 15 feet 6 inches long, and 14 inches wide, will floor a certain hall, how many will it take if the plank are only 11 feet, 4 inches long, and 9 inches wide? Ans. $204\frac{4}{17}$ plank. (4.)

40. If the transportation of 2000 pounds 47 miles cost $10, what must be paid at the same rate for the transportation of $7261\frac{1}{2}$ pounds 60 miles? Ans. $46.35. (3.)

41. If 260 stones, 15 inches long and 10 wide, will pave 30 square yards, how many stones, 25 inches long and 18 wide, will pave 72 square yards? Ans. 208 stones. (0.)

42. If the freight of 9 hogsheads of sugar, each weighing $12\frac{1}{2}$ cwt. for 20 miles, cost $18, what must be paid for the freight of 50 hogsheads, each weighing $2\frac{1}{2}$ cwt. 125 miles? Ans. $125. (0.)

43. If a canal, 1500 yards long, 8 yards wide, and 3 yards deep, be dug in 30 days of $12\frac{1}{2}$ hours each, by 280 men, how many men, working at the same rate, would be required

to dig another canal 7½ miles long, 5 yards wide, and 2½ yards deep, in 125 days of 10 hours each?

Ans. 385 men. (2.)

44. If the expenses of 9 horses for oats be $45 for 4½ months, when the price of oats is 36⅔ cents per bushel, how many horses will, at that rate, consume the value of $216 in 6 months, when oats is 33 cents per bushel?

Ans. 36 horses. (0.)

45. If 19 men eat $2.21 worth of bread in 18 days, when the price of wheat is 85 cents per bushel, how many days will 63 men require to eat $2.73 worth of bread, when wheat is sold at 95 cents per bushel? Ans. 6 days. (0.)

46. If a regiment of soldiers, consisting of 864 men, can eat 351 barrels of flour in 13 months, how many soldiers will eat 1464 barrels in 8 months, at the same rate?

Ans. 5856 men. (0.)

47. If the freight of 3 packages, each 9½ feet long, 6 feet wide, and 4 feet deep, be $23.75 for the distance of 45 miles, what must be paid at that rate for 4 packages, each 7½ feet long, 5 feet wide, and 3 feet deep, the distance being 72 miles? Ans. $25. (0.)

48. If $157.50 in 16 months gain $12.60 interest, in what time will $293.75 gain $11.75 interest, at the same rate?

Ans. 8 months. (0.)

49. If 7 compositors in 16 days of 12 hours each, can compose a work of 14 sheets of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line, in how many days of 10 hours each, may 9 compositors compose a volume to be printed on the same letter, consisting of 30 sheets, 18 pages in each sheet, 48 lines in a page, and 55 letters in a line? Ans. 36 ~~compositors.~~ (0.) days

50. If 200 pounds be carried 40 miles for 40 cents, how far may 20,200 pounds be carried for $60.60?

Ans. 60 miles. (0.)

51. How many bricks 9 inches long, 4½ inches wide, and 2 inches thick, will build a wall 6 feet high and 13½ inches thick, around a yard, each side of which will measure 280 feet, on the outside of the wall.

Ans. 160,632 bricks. (0.)

52. If 29 men can reap 116 acres of wheat in 8 days,

how many men should be employed to reap 96 acres in 12 days? Ans. 16 men. (0.)

53. A man paid \$3.90 for the use of \$80 for 9 months, how much at that rate should he pay for \$375 for 16 months?
Ans. \$32.50. (0.)

54. A yard, 24 feet 6 inches, by 17 feet 4 inches, is to be paved with stones 28 inches square, how many stones are necessary for the purpose? Ans. 78 stones. (0.)

Solution.

$$\begin{array}{r|l} & \text{stones.} \\ & 1 \\ \cancel{2} & \cancel{49} \\ \cancel{7} & \cancel{3} \\ \cancel{3} & \cancel{52} \\ \cancel{7} & 3 \\ \hline \text{Ans.} & 78 \text{ stones.} \end{array}$$

Here we say, one stone is the term of the same name as the required answer; then since it requires one stone to pave a space of 28 inches in length, to pave a space 24 feet 6 inches ($=\frac{49}{2}$ feet) in length will require more than one stone, therefore we place the greater of these two numbers on the right of the line. Again, since one stone will pave a space but 28 inches ($=\frac{7}{3}$ feet) in width, a greater number will be required to pave a space 17 feet 4 inches ($=\frac{52}{3}$ feet) in width; therefore, we place the greater of these two numbers on the right of the line.

55. Suppose the average width of a river to be 1760 feet, its depth 6 feet, and that the rate of its current is uniformly 3 miles an hour. in what time would a cistern, 2 miles in length, width, and depth, be filled by the river flowing into it?
Ans. 293 days, 8 hours. (0.)

$$\begin{array}{r|l} & 1 \text{ hour} \\ \text{Miles } \cancel{3} & \cancel{2} \text{ miles} \\ \cancel{1760} & \cancel{1760} \\ & \cancel{3} \\ & \cancel{2} \\ \cancel{6} & 1760 \\ & \cancel{3} \\ \cancel{24} & \cancel{2} \\ \hline \text{Ans.} & 293 \text{ days, 8 hours.} \end{array}$$

There being 1760 yards in a mile, the 3 is to express it in feet, and the 2 to give the number of feet in 2 miles.

56. If $6\frac{3}{4}$ yards of cloth $1\frac{3}{4}$ yards wide cost $25.50, what should be paid for $17\frac{1}{2}$ yards of the same quality of cloth $1\frac{1}{4}$ yards wide? Ans. 47.22\frac{2}{9}$. (0.)

57. A wall to be built to the height of 27 feet, has been raised 9 feet by 12 men in 6 days, how many men must be employed to finish it in 4 days? Ans. 36 men. (0.)

58. If the freight of 9 hogsheads of sugar, weighing each 12 cwt. for 20 leagues, cost £27, what must be paid at the same rate, for the freight of 50 boxes, each weighing $2\frac{1}{2}$ cwt. for 120 leagues? Ans. £187 10s. (0.)

59. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains a cord, what is a pile 37 feet 6 inches long, 8 feet 4 inches high, and 3 feet 9 inches wide, worth at the rate of $2.56 a cord? Ans. 23.43\frac{3}{4}$. (4.)

Solution.

	$\cancel{2}\cancel{5}\cancel{6}$
$\cancel{2}$	$\cancel{7}\cancel{5}$
$\cancel{8}$	
$\cancel{4}$	25
$\cancel{3}$	
$\cancel{4}$	15
$\cancel{4}$	93.75

Ans. 23.43\frac{3}{4}$

60. A company of 75 men are to be clothed, each suit is to contain $3\frac{1}{4}$ yards of cloth 6 quarters wide, and to be lined with flannel $\frac{5}{8}$ of a yard wide, how many yards of flannel will be required? Ans. 585 yds. (0.)

Solution.

$\cancel{4}$	13
1	$\cancel{7}\cancel{5}$
$\cancel{2}$	3
$\cancel{5}$	$\cancel{8}$

Ans. 585

61. A merchant bought a number of pieces of cloth, each piece containing $129\frac{17}{27}$ yards, at the rate of $7.00 for 5 yards,

and sold the same at the rate of $11.00 for 7 yards, and gained $200, how many pieces did he buy?

Ans. 9 pieces. (0.)

62. If a cellar 22.5 feet long, 17.3 feet wide, and 10.25 feet deep, be dug in 2.5 days, by 6 men, working 12.3 hours per day, how many days of 8.2 hours, would 9 men require at that rate, to dig another which will measure 56.25 feet long, 25.95 feet wide, and 16.4 feet deep?

Ans. 15 days. (0.)

63. If the interest of $76.48 for 1.25 years, be $5.736, what will be the interest of $2576.8 for 2.75 years, at the same rate? Ans. $425.172. (0.)

The solution of the above question will be found in Enoch Lewis' Arithmetic, page 120, and though much abridged by cancelling, yet the completion of the solution will require 112 figures in the work, independent of those in the statement and answer. Our solution requires none in the work.

64. If a piece of cloth 28.75 yards long, and 1.5 yards wide, cost $129.375, what will be the price of another piece of the same quality of 43.125 yards in length, and 1.2 yards wide? Ans. $155.25. (0.)

65. If a man travel 273 miles in 13 days, travelling only 7 hours a day, how many miles will he travel at the same rate in 23 days, if he travel 9 hours a day?

Ans. 621 miles. (0.)

66. 354 men are to be clothed, each suit to contain $3\frac{3}{4}$ yards of cloth that is $1\frac{1}{4}$ yards wide, and to be lined with holland, which is 3 quarters of a yard wide, how many yards of holland will be required? Ans. $2212\frac{1}{2}$ yds. (0.)

67. If a pile of wood 74 feet long, 11 feet wide, and 9 feet high, be worth $343.75, what is a pile of the same quality of wood 45 feet long, $18\frac{1}{2}$ feet wide, and 8 feet high worth? Ans. $312.50. (3.)

68. If a load of hay $10\frac{1}{2}$ feet long, 6 feet wide, and 8 feet high be sold for $15.75, what must you pay for a load 10 feet long, 4 feet wide, and 6 feet high, at the same rate?

Ans. $7.50. (0.)

69. If 27 compositors set 300 pages of 40 lines, with 35 letters in a line, in $3\frac{1}{2}$ days by working 8 hours a day, how many pages of 60 lines and 50 letters in a line will 33 com-

positors set in 11 days, at the same rate, if they work 9 hours a day? Ans. 605 pages. (0.)

70. If 13 barrels of flour, at 3 cents a pound cost $76.44, what will 27 barrels cost, at $2\frac{3}{4}$ cents a pound?

Ans. $145.53. (3.)

71. If a pile of wood $31\frac{1}{2}$ feet long, $9\frac{2}{3}$ feet high, and $7\frac{4}{5}$ feet wide cost $101.50, what must be the length of a pile 13 feet high and $6\frac{3}{4}$ feet wide to be worth $$97.87\frac{1}{2}$ at the same rate? Ans. $26\frac{1}{10}$ feet long. (0.)

72. If a man perform a journey of 286 miles, in 13 days, by travelling 14 hours in the day, in what time at the same rate of travelling, can he perform a journey of 495 miles, by travelling 15 hours in the day? Ans. 21 days. (0.)

73. 1200 men besieged in a town, have provisions for 5 weeks, allowing each man 16 ounces a day, if they are reinforced by 400 men and no relief can be offered till the end of 8 weeks, how many ounces must be given daily to each man? Ans. $7\frac{1}{2}$ ounces. (0.)

74. If 145 men can build a wall 360 feet long, 32 feet high, and 2 feet 9 inches thick in 27 days, by working 12 hours in the day, how many men, working at the same rate, can build a wall 495 feet long, 24 feet high, and 3 feet 4 inches thick, in 25 days, working 9 hours a day? Ans. 261 men. (0.)

75. If 248 men in 5 days of 11 hours each, can dig a trench 230 yards long, 3 yards wide, and 2 yards deep, in how many days of 9 hours each, will 24 men dig a trench 420 yards long, 5 yards wide, and 3 yards deep?

Ans. $288\frac{59}{207}$ days. (20.)

The solution of the above question, as given by Adams, Emerson, and others, requires to complete the work, not less than 126 figures; our solution requires but 20, making a difference of 106 figures.

76. If $7\frac{1}{4}$ ounces of bread be bought for $4\frac{3}{4}$d. when corn is 4s. 2d. per bushel, what weight of bread may be bought for 1s. 2d. when the price per bushel is 5s. 6d.?

Ans. $16\frac{118}{627}$ oz. (10.)

Greenleaf's solution of this question (by cancelling) requires 45 figures in the work; ours 10.

77. What will 12 hogsheads of sugar cost, at $$5\frac{4}{9}$ per cwt. each hogshead, weighing 6 cwt. 2 quarters, 12 pounds, if a

deduction of 16 pounds on the hundred weight be made for casks and waste? Ans. $370. (0.)

Explanation.—16 pounds being $\frac{1}{7}$ of a cwt., the whole price will be but $\frac{6}{7}$ of what it would be, if no deduction were made.

Solution.

$$\begin{array}{r|l} & \$ \\ \cancel{9} & 4\cancel{0} \\ \cancel{28} & 185 \\ & \cancel{12} \\ \cancel{7} & \cancel{6} \\ \hline \text{Ans.} & \$370 \end{array}$$

78. What will 25 barrels of potash, each weighing 1 cwt. 3 quarters, 14 pounds, cost at $3.36 per cwt. if a deduction of 12 lbs. on the cwt. be allowed?

Ans. $140.62½. (4.)

Here observe 12 pounds is $\frac{3}{28}$ of a hundred weight, therefore the whole cost will be but $\frac{25}{28}$ of what it would have been if no reduction had been made.

Solution.

$$\begin{array}{r|r} & \$ \\ & \cancel{3.36} \\ \cancel{8} & 15 \\ & 25 \\ \cancel{28} & 25 \\ \hline & 1125 \\ \hline \text{Ans.} & \$140.62\tfrac{1}{2} \end{array}$$

79. Required the value of 16 hogsheads of tobacco, each weighing 15 cwt. 3 quarters, 16 pounds, at $28 per cwt. allowing a deduction of 7 pounds per cwt.? Ans. $66.75. (0.)

80. What will 237 cwt. 3 quarters, 16 pounds of sugar amount to, at $8.50 per cwt. if a deduction of 8 pounds on the cwt. be allowed? Ans. 1877.65\frac{85}{198}$. (28.)

81. What is the value of 4 hogsheads of sugar, each weighing 12 cwt. 1 quarter, 14 pounds, at $12.20 per cwt., allowing a deduction of 12 pounds on the cwt.?

Ans. 539.19\frac{9}{14}$. (17.)

82. Sold 12 butts of currants, each weighing 7 cwt. 1 quarter, 10 pounds gross; if a deduction of 16 pounds per cwt. be allowed, what will the whole amount to, at the rate of $9.20 per cwt.? Ans. 694.50\frac{30}{49}$. (17.)

Gross weight, is the whole weight of merchandise or goods with the dust and dross, hogshead, box, chest, bag, &c., in which they are contained.

Tare is an allowance made to the buyer for the weight of the hogshead, box, &c., which is either so much per box, &c., or so much per cwt., or so much on the whole.

Tret is an allowance of 4 pounds on every 104 pounds for dust, waste, &c.

If the tare be subtracted from the gross, the remainder is sometimes called suttle, and when the tret is subtracted from the suttle, the remainder is called neat or net weight, sometimes tare only is allowed, the remainder is then called neat or net weight, or net weight is what remains after all allowances are made.

83. What is the value of 56 cwt. 0 quarters, 26 pounds gross, tare 110 pounds in all, tret 4 pounds per 104, at $7.16 per cwt.?

We will solve this question in the ordinary way and contrast our own solution therewith.

Common method.

```
   cwt.  qr.  lbs.
    56   0    26
     4
   ---
   224 qrs.
    28
  ----
  1818
  448
  ----
  6298 lbs. gross        6188
   110 tare               238 tret
26)6188 suttle        28)5950 lbs. net
   52                    56                4)212 qrs.—14 lbs.
   --                    --                ------------------
    98                    35                 53    0   14
    78                    28
   ---                    --
   208                     70
   208                     56
                           --
                           14
```

```
         $
|14|1/8|  716
         53
       ----
       2148
      3580
      -----
      37948
        89 1/2
      ------
      380.37 1/2   Ans.
```

Our solution.

```
      $
   |7̸1̸6̸
  4̸|2̸2̸1̸
 2̸6̸|     25
        179
       ----
       3043
   ------------
   $380.37 1/2   Ans.
```

The learner need have no difficulty in understanding our solution; observe that the tare 110 pounds, is 2 pounds less than 1 cwt., or is equal to 3 quarters, 26 pounds, hence we have only to take 3 quarters from 56 cwt. (mentally) and we have $\frac{221}{4}$ cwt., then the tret being $\frac{1}{26}$ part of the weight after the tare is deducted, we diminish the price, which the first part of our statement would give, in the ratio of 25 to 26.

In the solution observe that 13 will measure 26 and 221, the quotient being 2 and 17, which we do not write, but multiply 179 (the quotient of 716 by 4) by 17, and our product is 3043, now we have 25 yet on the right of the line to multiply by, and 2 on the left for a divisor, but we always use the short method of multiplying by 25, (that is, annex two ciphers and divide by 4,) but in this case we divide by 8, which is equivalent to multiplying by 25 and dividing the product by 2.

Remark.—There is no necessity for the special rules for the solution of questions in Tare and Tret, Loss and Gain, Barter, Interest, Discount, &c., which we find in the Arithmetics generally used in our schools.

If the pupil has learned the proper exercises of his reasoning faculties, he will ascertain generally without much assistance from the teacher what is required, as the answer to a question, and will make his statement and solution accordingly.

84. What is the value of 8 hogsheads of sugar, at 72s. 6d. per cwt., each hogshead weighing 8 cwt. 3 quarters, 7 pounds, if a deduction of 12 pounds per cwt. be allowed?

Ans. £228 3s. 7d. $1\frac{4}{7}$ qrs. (32.)

85. What is the value of 14 tierces of rice, each containing 5 cwt. 1 quarter, 12 pounds gross, tare 14 pounds per cwt., and tret 4 pounds per 104 pounds, at $4.94 per cwt.?

Ans. \$311.71$\frac{7}{8}$. (12.)

86 What is the value of 3 tierces of rice, each weighing 14 cwt. 2 quarters, 14 pounds gross, tare 10 pounds per cwt. at $5.36 per cwt.? Ans. \$214.17$\frac{15}{56}$. (18.)

87. What is the value of 68 barrels of figs, each weighing 126 pounds gross, tare 12 pounds per cwt. at $7.25 per cwt.

Ans. \$495.20$\frac{5}{56}$. (25.)

88. Purchased in London, 16 cwt. of tea, at £28 sterling

per cwt. net weight, tare 12 pounds per cwt., how much must I receive in federal money for the whole quantity to realize a profit of 12 per cent. ?

Ans. \$1991.11$\frac{1}{9}$. (5.) (£9=\$40.)

89. If I purchase 8 hogsheads of sugar, each weighing 8 cwt. 3 quarters, at \$9 per cwt. net, being allowed 12 pounds per cwt. for tare, for how much must I sell the whole to gain 30 per cent. ? Ans. \$731.25. (3.)

90. A ship's crew of 300 men were so supplied with provisions for 12 months, that each man was allowed 30 ounces a day, but after having been 6 months on their voyage, they find it will take 9 months more to finish it, and 50 of their number have been lost, it is required to find the daily allowance of each man during the last 9 months?

Ans. 24 oz. (0.)

91. If I buy 8 pieces of cloth, each containing 15 yards, at 62$\frac{1}{2}$ cents per yard, for how much must I sell the whole in order to gain 33 per cent. ? Ans. \$99.75. (3.)

92. A sold to B 20 hogsheads of molasses, at a loss of 7$\frac{1}{2}$ per cent. B sold the same to C and gained 20 per cent. C then sold the whole to D for \$666, and thereby gained 12$\frac{1}{2}$ per cent.; how much did A give per hogshead for it?

Ans. \$26.66$\frac{2}{3}$. (3.)

Solution.

	\$
	$\cancel{6}\cancel{6}\cancel{6}$
$\cancel{9}$	8
6	5
$\cancel{3}\cancel{7}$	4$\cancel{0}$
$\cancel{2}\cancel{0}$	160

Ans. \$26.66$\frac{2}{3}$

In making this statement, we reason thus; since C by selling the whole for \$666, gained 12$\frac{1}{2}$ per cent. (which was $\frac{1}{8}$ of what it cost him,) he sold it for 9 parts, whereas he bought it for 8, we therefore put the 8 on the right of the line, and we have the statement of the cost price to C; next we perceive that B sold to C, so as to gain 20 per cent. (which was $\frac{1}{5}$) of what it cost; *he* sold it, *therefore*, for 6 parts, whereas it cost him but 5, hence we place the 5 on the right of the line, and the statement thus far will give the cost price to B. Now we perceive that A sold to B. at a *loss* of 7$\frac{1}{2}$ per cent., which was $\frac{3}{40}$ of what it cost him, he therefore sold it for 37 parts, whereas it cost him 40 parts, hence we place the 40 on the right of the line.

93. What will the plastering of a room 98 feet 8 inches in compass, and 8 feet 3 inches high, come to at 25 cents per square yard, allowing a deduction of $\frac{1}{10}$ for doors and windows? Ans. \$20.35. (3.)

Here you may say, 25 cents or $\frac{1}{4}$ of a dollar will pay for plastering a space 3 feet in length, and more will be required to pay for plastering a space 98 feet 8 inches $=\frac{296}{3}$ feet in length, and again $\frac{1}{4}$ of a dollar will pay for plastering a space 3 feet in width, and more will be required to pay for plastering a space 8 feet 3 inches $=\frac{33}{4}$ feet in width.

Solution.

$$\begin{array}{r|l} & \$ \\ \cancel{4} & 1 \\ \cancel{3} & \cancel{296} \\ \cancel{3} & \\ \cancel{4} & \cancel{33} \\ \cancel{3} & \\ \cancel{10} & \cancel{9} \\ \hline & 40.7 \\ \hline & \$20.35 \end{array}$$

94. A wall was to be built 700 yards long, in 29 days; after 12 men had been employed on it during 11 days, it was found they had built only 220 yards; how many more men must be put on to finish it in the given time?

Men.

$$\begin{array}{r|l} & \quad \cancel{12} \\ \cancel{220} & 48 \quad \cancel{0} \\ \cancel{18} & \quad \cancel{11} \\ \hline \end{array}$$

16—12=4 more men.

95. If $2\frac{1}{4}$ yards of cloth $1\frac{3}{5}$ yards wide cost £$3\frac{3}{5}$, what is the value of $38\frac{1}{4}$ yards, 2 yards wide?

Ans. £76 10s. (0.)

96. If $\frac{3}{4}$ of a yard of cloth, $\frac{7}{8}$ yards wide, cost £$\frac{2}{5}$, what is the value, at the same rate, of $\frac{5}{8}$ of a yard $1\frac{3}{4}$ yards wide?

Ans. 13s. 4d. (0.)

97. If \$100 in 12 months gain \$8 interest, what sum, at that rate, will gain \$8.60 interest in 5 months?

Ans. \$258. (0.)

98. How many plank, 15 feet long and 15 inches wide, will floor a barn that is 62 feet 6 inches long, and 37 feet 6 inches wide? Ans. 125 plank. (0.)

99. A cubic foot of air weighs $1\frac{1}{4}$ oz., how many pounds of air does a room contain which is 16 feet long, 14 feet wide, and 10 feet high?

$$
\begin{array}{r|l}
 & \text{oz.} \\
\cancel{4} & 5 \\
1 & \cancel{16} \\
1 & \cancel{14} \\
1 & \cancel{10} \\
\cancel{16} & \\
\hline
 & 175
\end{array}
$$

Ans. 175 lbs.

100. A has 608 yards of cloth, at 14s. per yard, for which B is to give him £125 12s. in money, and 85 cwt. 2 quarters, 24 pounds of beeswax; at how much is the beeswax valued per cwt.? Ans. £3 10s. (0.)

101. If 12 ounces of wool make 2½ yards of cloth, that is 6 quarters wide, how many pounds would make 150 yards, 4 quarters wide? Ans. 30 lbs. (0.)

102. If 21 cows eat 9⅓ tons of hay in 15 weeks, how many cows, at that rate, will eat 32 tons in 45 weeks?

Ans. 24 cows. (0.)

103. A man sold a house for $255.85, which was only $\frac{7}{11}$ of what it cost him; when he bought it, he paid for it with wood at $2.75 per cord, how many cords did he give?

Ans. 146⅕ cords. (3.)

104. A man sold 19 cows for £197.60, by which he lost $\frac{4}{17}$ of what they cost him; for how much a head should he have sold them to gain on the first cost 20 per cent.?

Ans. $16.32. (3.)

105. A merchant received from Lisbon, 180 casks of raisins, each containing 80$\frac{8}{9}$ pounds, which cost him $218 a cask; at what price per cwt. must he sell them to gain 25 per cent.? Ans. 3.77\frac{4}{13}$. (6.)

106. If I purchase 13 cwt. of coffee, at 12½ cents a pound, at what price per pound must I sell it to gain $80.08 on the whole? Ans. 18 cents. (6.)

$$
\begin{array}{r|l}
 & \$ \\
\cancel{13} & \cancel{80.08} \\
\cancel{4} & \cancel{616} \\
\cancel{28} & \cancel{154} \\
 & 5\tfrac{1}{2}
\end{array}
$$

5½ cents gained per pound, which being added to the cost, 12½ cts.=18 cts.

107. If $4\frac{1}{2}$ cwt. be carried 36 miles for \$3.50, how many pounds can I have carried 28 miles for \$2.80?

Ans. $518\frac{2}{5}$ lbs. (0.)

MISCELLANEOUS QUESTIONS.

1. A merchant imported from England, 847 ells of cloth, at 7s. 6d. per ell. The commission and duty amounted to 40 per cent. exchange 10 per cent. after paying these expenses, in addition to the first cost of the cloth, for how much per yard must he sell his cloth (in federal money) to gain $12\frac{1}{2}$ per cent. Ans. \$2.31. (0.)

2. Bought 27 bags of ginger, each weighing gross $84\frac{3}{4}$ pounds, tare $1\frac{3}{8}$ pound per bag, tret 4 pounds per 104, what does the whole net weight amount to at $8\frac{1}{2}$d. per pound?

Ans. £76 13s. $2\frac{2}{4}\frac{07}{16}$. (56.)

3. If I buy a quantity of goods in London, for \$500, and pay \$165 for transportation and duty, for how much must I sell them in Philadelphia, to gain 24 per cent.?

Ans. \$824.60. (5.)

4. A younger brother received £1560, which was just $\frac{7}{12}$ of his elder brother's share, and $5\frac{3}{8}$ times the elder brother's fortune was $\frac{3}{4}$ of all the father was worth; what was the value of his whole estate? Ans. £19165 14s. $3\frac{3}{7}$d. (5.)

5. A merchant has 1260 yards of canvass, worth $8\frac{3}{4}$ cents a yard, which he wishes to trade for muslin, at $6\frac{1}{4}$ cents a yard; how many yards of muslin should he receive?

Ans. 1764 yds. (0.)

6. A fox starts 80 yards before a grayhound, and is not perceived by him till he has been up 45 seconds, he scuds away at the rate of 9 miles an hour, and the hound pursues him at the rate of 18 miles an hour, how far must the hound run to overtake the fox?

The following solution of the above question, we take from a little work called "The Plain Calculator," by Lewis Joerries, Professor of Mathematics, from Prussia. To Mr. Joerries' work we are indebted for the idea of using the vertical line, as well as for several useful properties of numbers.

In common with all others whose works we have consulted, (and they are not a few,) Mr. Joerries appears to think it

essential that several statements should be made, and results obtained to satisfy various intermediate conditions of the question, in order to arrive at the final result in the solution of problems of this kind.

His first statement and solution are as follows:

Statement.

	45 seconds
60	1 minute
60	1 hour
1	9 miles
1	1760 yards.

Solution.

	~~45~~	9
~~4~~ ~~20~~ ~~60~~	~~5~~	~~3~~
~~3~~ ~~60~~	~~1760~~	~~88~~ 22
	198	Ans.
	80	

Total advance before the hound starts 278 yards.

The next process is to ascertain in how many seconds the hound will gain 278 yards.

Statement.

	278 yards
yards 1760	1
9	1
1	60
1	60

Solution.

11 ~~22~~ ~~44~~ ~~1760~~	~~278~~	139
3 ~~6~~	~~60~~	~~2~~
	~~60~~ ~~20~~	5
11	695	
	$63\frac{2}{11}$ secs.	

Finally, to ascertain how many yards the hound will run in $63\frac{2}{11}$ seconds.

Statement.

	$63\frac{2}{11}$ seconds
yards 60	1 minute
60	1 hour
1	18 miles
1	1760 yards.

Solution.

11	695	139
4 60	18	3
12 60	1760	44
Ans.	556 yards.	

Now we give our solution.

We also ascertain what distance the fox runs in 45 seconds, but it is certainly unnecessary to make a statement for this purpose, 45 seconds being $\frac{1}{80}$ of an hour, and $\frac{1}{80}$ of 1760 yards being 22

Thus,

	yards.
	278
miles ~~9~~	~~18~~ miles
Ans.	556 yards.

yards, and 9 times 22 yards=198 yards=$\frac{1}{80}$ of 9 miles, to which we add the 80 yards, and we have the whole distance to be gained by the hound, and since he must run 18 miles to gain 9 miles, the distance he must run to gain 278 yards, must have the same ratio to 278 yards that 18 miles has to 9 miles; and hence we perceive there is no necessity for even the statement we have given, but having ascertained the distance the hound has to gain, we see at once that he must run twice this distance to gain it; we may observe therefore that Mr. Joerries has been at the trouble of making 140 figures in order to obtain a result which the pupil if rightly instructed may arrive at by a purely mental process, in less time than that occupied in making the first statement.

We will give another.

7. A hare starts up 12 rods before a hound, but is not perceived by him till she has been up $1\frac{1}{4}$ minutes, she scuds away at the rate of 36 rods in a minute, and the dog in view makes after her at the rate of 40 rods in a minute, what distance must the dog run to overtake the hare?

Here the learner will readily perceive that the dog has 57 rods to gain, and as he must run 40 rods to gain 4, he must run 10 times the distance to be gained, viz., 570 rods.

8. If I buy 765 yards of baize, at 3s. $4\frac{1}{2}$d. per yard, and sell it at 3s. 9d. per yard, how much do I gain on the whole?

Ans. £14 6s. $10\frac{1}{2}$d. (4.)

9. If a bar of iron 7 feet long, 4 inches wide, and 3 inches thick weighs 84 pounds, what will be the weight of a bar 9 feet long, 3 inches wide, and 2 inches thick?

Ans. 54 lbs. (0.)

10. If the transportation of 12 cwt. 2 quarters, 6 pounds, 275 miles cost $27.78, how far at that rate, may 3 tons, 0 cwt. 3 quarters be carried for $234.78?

Ans. $480\frac{60035}{225018}$ miles. (39.)

11. If 4 men or 5 women can do a certain piece of work in 63 hours, in how many hours, at that rate, can one man and one woman do the same work? Ans. 140 hours. (0.)

12. If 16 pounds of beef or 20 pounds of pork will pay for $1\frac{1}{2}$ bushels of potatoes, how many bushels of potatoes will 100 pounds of beef and 100 pounds of pork pay for, at the same rate? Ans. 16 bus. 3 pks. 4 qts. (0.)

13. If 25 pears can be bought for 10 lemons, and 28 lemons for 18 pomegranates, and 1 pomegranate for 48 almonds, and 50 almonds for 70 chestnuts, and 108 chestnuts for $2\frac{1}{2}$ cents, how many pears can I buy for \$1.35.

Ans. $337\frac{1}{2}$ pears. (0.)

14. If a man earn \$20.87$\frac{1}{2}$ in 15 days, how much would he earn, at that rate, in a year, there being 26 working days in a month? Ans. \$434.20. (4.)

15. If A can mow an acre of grass in $5\frac{1}{3}$ hours, and B can mow $1\frac{3}{4}$ acres is $9\frac{1}{3}$ hours, in what time can they jointly mow $8\frac{1}{4}$ acres? Ans. 22 h. 40 m. (2 statements, 0.)

16. A merchant bought 28 pieces of muslin, at \$6.43$\frac{1}{2}$ each, and by selling 112 yards for \$10.92 he gained 25 per cent., how many yards did each piece contain?

Ans. $82\frac{1}{2}$ yds. (3.)

17. If the hind wheels of a coach which are 180 inches in circumference will turn round 4825 times in running a certain distance, how many times will the forward wheels turn round in going the same distance, they being 145 inches in circumference? Ans. $5989\frac{19}{29}$ times. (17.)

18. If $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of $13\frac{1}{2}$ yards of muslin cost $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of \$6.75, what will $\frac{9}{10}$ of $\frac{11}{12}$ of $\frac{8}{9}$ of $19\frac{2}{3}$ yards cost? Ans. \$10.81$\frac{2}{3}$. (4.)

Statement.

$\cancel{3}$	$\cancel{2}$
$\cancel{7}$	$\cancel{5}$
$\cancel{5}$	$\cancel{4}$
$\cancel{9}$	$\cancel{7}$
$\cancel{4}$	\$$\cancel{27}$
$\cancel{10}$	$\cancel{9}$
$\cancel{12}$	$\cancel{11}$
$\cancel{9}$	$\cancel{8}$
$\cancel{3}$	59 yds.
1	$\cancel{2}$
$\cancel{2}$	$\cancel{3}$
$\cancel{3}$	$\cancel{4}$
$\cancel{8}$	$\cancel{9}$
yds. $\cancel{27}$	$\cancel{2}$
6	64.9

Ans. \$10.81$\frac{2}{3}$

19. How many barrels of flour, at $6.66 a barrel are equal in value to 16 hogsheads of molasses worth 37 cents a gallon? Ans. 56 bars. (0.)

20. What quantity of tea, at $37\frac{1}{2}$ cents per pound must be given for 450 pounds of chocolate, at 22 cents per pound? Ans. 264 lbs. (0.)

21. Bought 375 yards of cloth for $1275, how must I sell it per yard to gain 25 per cent.? Ans. $4.25. (0.)

22. If 7 pounds of beef and 5 pounds of bread be the daily allowance of each man engaged in erecting a fortification, the beef costing $5\frac{3}{4}$ cents per pound, and the bread 6 cents a pound, and the daily expense for beef be found to be $34.50, what is the daily expense for bread? Ans. 25.71\frac{3}{7}$. (3.)

23. If 40 rods in length, and 4 rods wide make an acre, what must be the length of a piece of land 24 rods wide to contain 14 acres? Ans. 93 rods, 1 yd. 2 ft. 6 in. (0.)

24. Bought a cask of wine, at 87.5 cents a gallon, and paid $130 for it; how many gallons did the cask contain? Ans. 148 gals. 2 qts. $2\frac{2}{7}$ gills. (4.)

25. A room 48 feet long and 27 feet wide is to be covered with painted cloth $\frac{3}{4}$ of a yard wide, how many yards of cloth will be required? Ans. 192 yds.

26. If $\frac{5}{6}$ of a gallon of wine cost $\frac{5}{8}$ of a pound, what will $\frac{5}{9}$ of a tun cost? Ans. £105. (0.)

27. If $1\frac{1}{3}$ bushels of apples cost $78\frac{1}{3}$ cents, what will $3\frac{1}{5}$ bushels cost at the same rate? Ans. $1.88. (0.)

28. If $\frac{7}{8}$ of a yard of cloth cost 2.62\frac{1}{2}$, what will $5\frac{5}{12}$ yards cost, at the same rate? Ans. $16.25. (0.)

29. If 16 men finish a piece of work in $28\frac{3}{5}$ days, how many days will 12 men require to do the same? Ans. $38\frac{2}{15}$ days. (3.)

30. How many pieces of cloth, at 20.12\frac{1}{2}$ per piece, are equal in value to $240\frac{1}{7}$ pieces, at $12.50 per piece? Ans. $149\frac{177}{1127}$ pieces. (11.)

31. A trader had 24,000 pounds of cotton which he exchanged for linen, giving $12\frac{1}{2}$ pounds of cotton for 3 yards of linen, the linen he exchanged for coffee, giving 4 yards of linen for 7 pounds of coffee, he then exchanged his coffee for cheese, at the rate of 8 pounds of coffee for 14 pounds of

cheese, and sold the cheese at 9 cents a pound; what did he get at this rate, for his 24,000 pounds of cotton?

Ans. $1587.60. (4.)

In stating this question we proceed thus, 9 cents being the price of 1 pound of cheese, 14 pounds will cost more, therefore we place 14 on the right of the line; as the statement now stands it will give us the price of 8 pounds of coffee, (8 pounds of coffee being equal in value to 14 pounds of cheese,) and 7 pounds of coffee will cost less, hence 7 goes on the right of the line; now the result of our statement will give the price of 4 yards of linen, and 3 yards will cost less, so we place 3 on the right of the line; the statement thus far will give the price of 12½ pounds of cotton, and 24,000 pounds will cost more, hence 24,000 pounds must go on the right of the line.

$$
\begin{array}{r|l}
 & \text{Cents.} \\
 & 9 \\
1 & 14 \\
\cancel{8} & 7 \\
\cancel{4} & 3 \\
\cancel{25} & 2 \\
 & \cancel{24000} \\
\hline
 & 1960 \\
\hline
\text{Ans.} & \$1587.60
\end{array}
$$

Such questions as the above are generally worked by a special rule called "Conjoined Proportion," which we think unnecessary.

32. If 17 pounds of raisins are worth 20 pounds of almonds, and 5 pounds of almonds are worth 8½ pounds of figs, and 37½ pounds of figs are worth 30 pounds of tamarinds; how many pounds of tamarinds are equal in value to 42½ pounds of raisins? Ans. 68 lbs. (0.)

33. If Isaac can earn as much in 7 days as William can in 9, and William as much in 12 days as John can in 14, and John as much in 18 days as Thomas can in 10½ days, and Thomas as much in 8 days as Moses can in 20, if Moses gets 40 cents a day, how much ought Isaac to get a day?

Ans. 87½ cents. (0.)

34. If 6 sheep are worth as much as 14 turkeys, and 35 turkeys as much as 54 geese, and 9 geese as much as 16 ducks, and 7 ducks as much as 4 bushels of oats, and 3 bushels of oats are worth $1.05, for how much can I buy 96 sheep? Ans. $122.88. (4.)

35. If 1 French crown is equal in value to 80 pence of

Holland, and 83 pence of Holland to 48 of England, and 40 pence of England to 70 pence of Hamburg, and 64 pence of Hamburg to 1 florin of Frankfort, how many florins of Frankfort are equal to 166 French crowns?

Ans. 210 florins of Frankfort. (0.)

36. If in California 18 barrels of salt are worth 4 barrels of flour, and 8 barrels of flour are worth 40 barrels of apples, and 15 barrels of apples are worth 24 cwt. of pork, and 10 cwt. of pork are worth 6 cwt. of beef, and 27 cwt. of beef are worth 144 bushels of potatoes, and 16 bushels of potatoes are worth 96 pounds of butter, and 36 pounds of butter are worth 60 dozen of eggs, and 8 dozen of eggs are worth 81 cents; what are 72 barrels of salt worth, and what is flour worth a barrel?

Ans. $\left\{\begin{array}{l}\$414.72=\text{worth of 72 bls. of salt.}\\ \$25.92=\text{worth of 1 bl. of flour.}\end{array}\right\}$ (3.)

Since 4 barrels of flour are equal to 18 barrels of salt, 16 barrels of flour will be equal to 72 barrels of salt, hence we have only to divide the $414.72 by 16 to get the price of a barrel of flour.

37. If 30 cwt. be equal to 120 quarters, and 7 quarters equal to 196 pounds, and 5 pounds equal to 80 ounces, and 9 ounces equal to 144 drams; how many drams are 13 cwt. equal to? Ans. 372,736 drams. (7.)

38. If a wheel 48 feet in diameter has on the other end of its shaft one of 56 feet in diameter propelling a wheel 18 feet in diameter being connected with one 36 feet in diameter, which propels a wheel 8 feet in diameter, being connected with one 24 feet in diameter, propelling a wheel 2 feet in diameter; how many revolutions will the 2 feet wheel make for every one of the 48 feet wheel?

Ans. 168 revolutions. (0.)

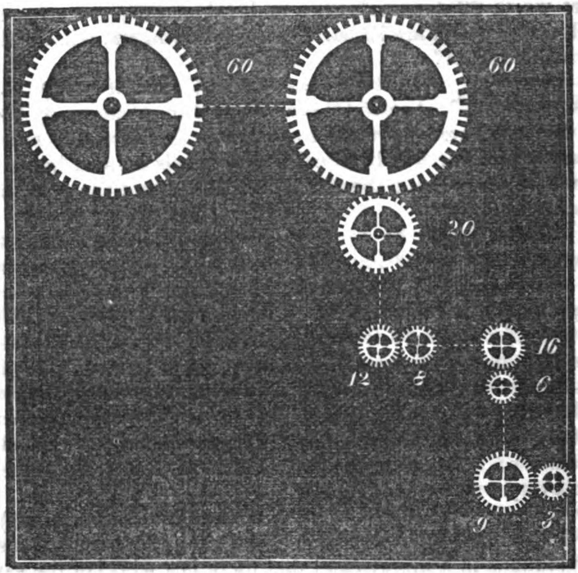
39. Suppose a water-wheel 60 feet in diameter makes 8 revolutions in a minute, and that there is a wheel of the same diameter on the other end of its shaft, which propels a wheel 20 feet in diameter having a wheel 12 feet in diameter on the other end of the same shaft which runs in one 8 feet in diameter, connected by a shaft with one 16 feet in diameter, and this again propels a wheel 6 feet in diameter, connected with one 9 feet in diameter at the other end of the same

shaft, and this last propels a wheel 3 feet in diameter; now how many revolutions does the 3 feet wheel make in a minute? Ans. 288 revolutions. (O.)

Solution.

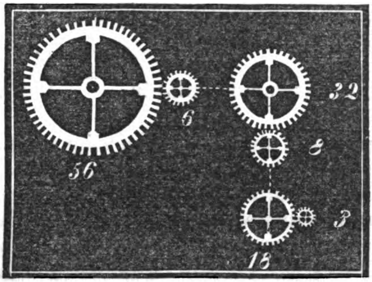
	$\cancel{8}$ revolutions
$\cancel{20}$	$\cancel{60}$
$\cancel{8}$	$\cancel{12}$
$\cancel{6}$	16
$\cancel{3}$	9

Ans. 288 revolutions.



40. Suppose a wheel 3 feet in diameter (for propelling machinery) makes 2016 revolutions in a minute, and that this wheel is propelled by one 18 feet in diameter, having on the other end of its shaft a wheel 8 feet in diameter, being propelled by one 32 feet in diameter, having on the other end of its shaft a wheel 6 feet in diameter, which is propelled by

one 56 feet in diameter; now if the 3 feet wheel makes 2016 revolutions in a minute, how many does the 56 feet wheel make in a minute? Ans. 9 revolutions. (O.)



41. Suppose Mary can make a vest in 8 hours, Jane in 12 hours, and that Sarah can make one in 16 hours, and a gentleman who is about to start on a journey wishes one made in the shortest possible time; now if they all three work at the same, in what time will his vest be finished?

To solve this question, we take the least common multiple of 8, 12, and 16, which is 48, and reason thus—since Mary can make a vest in 8 hours, in 48 hours she can make 6 vests; Jane in 48 hours can make 4, and Sarah 3 vests; therefore they will all make 13 vests in 48 hours, and consequently they can make one vest in $\frac{1}{13}$ of 48 hours, which is $3\frac{9}{13}$ hours. Ans.

42. If Mr. Jackson can make a coat in 36 hours, and his wife can make one in 48 hours, and his apprentice in 72 hours; in what time at this rate, should they make a coat if all work at the same one?

Here 144 being the smallest common multiple of 36, 48, and 72, we see at once, that Mr. Jackson by working 144 hours will make 4 coats, his wife will make 3, and his apprentice 2; altogether then they will make 9 coats in 144 hours, hence they will make one coat in 16 hours. Ans.

43. There are 4 large lamps used for lighting a hall, the first of which will consume a pint of oil in 48 minutes, the second in 64 minutes, the third in 96 minutes, the fourth in 128 minutes. In what time will a pint be consumed at this rate by the 4 lamps?

The smallest common multiple in this case is 384; then in 384 minutes the first lamp will consume 8 pints, the second 6 pints, the third 4, and the fourth 3 pints; so they will altogether consume 21 pints in 384 minutes.

$$\text{Hence tney will consume one pint in } \frac{\cancel{21}|\cancel{384}}{\quad 18\frac{2}{7}} \text{ min. Ans.}$$

44. Three rooms of the same dimensions are to be plastered; the first by John, the second by Thomas, and the third by William; John completed his in 32 hours, Thomas in 24 hours, and William in 40 hours; had they all been working at the same room at this rate, in what time would they have finished it? Ans. $10\frac{1}{4}\frac{9}{7}$ hours. (0.)

45. If A can mow an acre of grass in 6 hours, B in 8 hours, and C in 12 hours, in how many days of 12 hours each, can they altogether mow $31\frac{1}{2}$ acres?

Ans. 7 days. (0.)

46. If David can chop a cord of wood in 4 hours, Edwin in 6 hours, Charles in 8 hours, and Caleb in 9 hours, in what time can they altogether chop 141 cords, if they work at this rate 9 hours a day? Ans. In 24 days. (0.)

47. A lion of bronze, placed upon the basin of a fountain, can spout water into the basin through his throat, his eyes, and his right foot. If he spouts through his throat only, he will fill the basin in 6 hours; if through his right eye only, he will fill it in 48 hours; if through his left eye only, in 72 hours; if through his foot only, he will fill it in 4 hours. In what time will the basin be filled if the water flows through all these apertures at once?

Ans. 2 hrs. 12 min. $55\frac{5}{13}$ s. (7.

48. A reservoir for water has two pipes to supply it; by the first alone it may be filled in 40 minutes; by the second alone in 50 minutes; and it has a discharging pipe by which it may when full be emptied in 30 minutes; now if these three pipes were all left open, the influx and efflux of the water being always at the aforesaid rates, in what time would the reservoir be filled?

Here 600 being the smallest common multiple, we say the first pipe would fill 15 such reservoirs in 600 minutes, and

the second would fill 12 in the same time; hence, both would fill 27, and the third in the same time would empty 20 such reservoirs; therefore, in 600 minutes the gain of the first and second over the third would be sufficient to fill 7 such reservoirs, and the time required to gain sufficient to fill one such reservoir will be

$$\begin{array}{r|l} 7 & 6\cancel{0}\cancel{0} \\ \cancel{6}\cancel{0} & \\ \hline \end{array}$$

Ans. 1 h. 25 min. 42$\frac{6}{7}$ s.

49. A cistern for water has two cocks to supply it; by the first it may be filled in 45 minutes, and by the second in 55 minutes; it has likewise a discharging cock, by which it may, when full, be emptied in 30 minutes; now if these apertures be all left open when the water comes in, in what time will the cistern be filled? Ans. 2 hrs. 21 m. 25$\frac{5}{7}$ s. (3.)

INTEREST.

Interest is a premium paid for the use of money by the borrower to the lender.

There are four parts to be considered in computing interest, viz., the principal, rate, time, and amount.

Principal is the sum for which interest is to be paid.

The *Rate* or *per cent.* per annum, is the interest of one hundred dollars for one year. The words "*per cent.*" or centum "per annum," being Latin expressions, meaning *for a hundred, for a year.*

Time is the number of years, months, days, &c., for which interest is to be computed.

The *Amount* is the sum of the principal and interest.

1. What is the interest of \$743 for one year, at 5 per cent.; that is, what is $\frac{5}{100}$ or $\frac{1}{20}$ of 743 dollars? If we point off 2 figures to the right, we have $\frac{1}{100}$ or one part, and 5 parts will be 5 times as much.

$$\text{Thus,}\quad \begin{array}{r} \$7.43 = 1 \text{ part} \\ 5 \\ \hline \$37.15 = 5 \text{ parts} \end{array} \qquad \text{or } \frac{74.3}{2\cancel{0}} = \$37.15$$

2. What is the interest of $476 for 2 years, at 6 per cent. per annum. If we pay 6 parts for 1 year, for 2 years we must pay twice as much, that is, 12 parts, or $\frac{12}{100}$. By pointing off 2 figures for decimals of a dollar, that is, by considering 2 figures on the right, which are dollars in the given sum, as cents, we have the expression for one hundredth part, which being multiplied by the number of parts will give the interest required.

Thus, $476 by pointing off 2 figures=$4.76
which being multiplied by 12
gives the interest =$57.12

3. What will be the interest of $748 for $3\frac{1}{2}$ years at 6 per cent. per annum; if the interest for one year be 6 parts, for $3\frac{1}{2}$ years it will be $3\frac{1}{2}$ times 6 parts, that is, 21 parts.

Therefore $7.48×21
Ans. 157.08

How do you find the interest of any given sum for years, or for years and parts of a year?

Ans. Point off two figures, that is, the units and tens of the dollars of the given sum, as decimals of a dollar, or as cents; then multiply this sum by the product of the *rate* of interest for one year, and the *time* or number of years.

NOTE 1.—If there be cents in the given sum, observe that after 2 figures are pointed off from the dollars there will be four decimals.

NOTE 2.—When no rate of interest is mentioned, 6 per cent. is always to be understood.

4. What is the interest of $1236 for 7 years, 9 months, at 6 per cent. per annum?

$
| $\cancel{1}\cancel{2}.\cancel{3}\cancel{6}$
| 309 7 yrs. 9 ms.=$7\frac{3}{4}$=$\frac{31}{4}$ yr.
| 6
$\cancel{4}$| 31
| 18.54

Ans. $514.74

5. What is the interest of $374.28 for 3 years and 5 months, at 6 per cent. per annum?

$$\begin{array}{r|r}
 & \$ \\
 & \cancel{374.28} \quad 3\text{ yrs. } 5\text{ mos.}=3\tfrac{5}{12}=\tfrac{41}{12}. \\
\cancel{12} & 18714 \\
 & 41 \\
 & \cancel{6} \\
\hline
\text{Ans.} & \$76.7274
\end{array}$$

6. What is the interest of $3994.37 for 8 years and 11 months, at 6 per cent. per annum?

$$\begin{array}{r|r}
 & 3994.37 \\
\cancel{12} & 2796059 \\
\cancel{2} & 107 \\
 & \cancel{6} \\
\hline
 & 4273.9759 \\
\text{Ans.} & \$2136.9879+
\end{array}$$

7. What is the interest of $1346 for 1 year? Ans. $80.76. (0.)

8. What is the interest of $736 for 3 years? Ans. $132.48. (0.)

9. What is the interest of $376.25 for 4 years? Ans. $90.30. (0.)

10. What is the interest of $964.75 for 7 years? Ans. $405.19½. (5.)

11. What is the interest of $1728 for 6 years and 11 months? Ans. $717.12. (3.)

12. What is the interest of $3784 for 12 years and 6 months? Ans. $2838. (4.)

13. What is the interest of $78.34 for 16 years and 8 months? Ans. $78.34. (0.)

14. What is the interest of $462.48 for 17 years and 9 months? Ans. $492.5412. (8.)

15. What is the interest of $24.75 for 28 years and 8 months? Ans. $42.57. (2.)

16. What is the interest of $9462 for 13 years and 4 months? Ans. $7569.60. (0.)

17. What is the interest of $18,472 for 2 years and 7 months? Ans. $2863.16. (4.)

18. What is the interest of $147 for 3 years and 8 months? Ans. $32.34. (0.)

19. What is the interest of $94 for 5 years and 11 months? Ans. $33.37. (2.)

20. What is the interest of $639.56 for 4 years and 3 months? $163.0878. (5.)

21. What is the interest of $568.36 for 27 years and 3 months? Ans. $929.2686. (8.)

22. What is the interest of $6827 for 5 years? Ans. $2048.10. (0.)

23. What is the amount of a note for $168, which has been on interest for 7 years and 7 months, at 6 per cent. per annum? Ans. $244.44. (6.)

The learner must recollect that the interest must be *added to* the principal, to get the amount.

24. What is the amount of $1346 for 3 years and 5 months? Ans. $1621.93. (5.)

25. What is the amount of $4762.69 for 4 years and 2 months, at 6 per cent. per annum? Ans. 5953.3625. (8.)

26. What is the amount of $9647.18 for 9 years and 11 months, at 6 per cent. per annum? Ans. $15387.2521. (14.)

27. What will $148.92 amount to in 3 years and 7 months, at 6 per cent. per annum? Ans. $180.9378. (10.)

28. What is the interest of $4632 for 2 years and 9 months, at 5 per cent.? Ans. $636.90. (4.)

29. What is the interest of $79.75 for 3 years and 7 months, at 7 per cent. per annum? 20.0039\frac{7}{12}$. (10.)

30. What is the interest of $891.28 for 1 year and 3 months, at 8 per cent.? Ans. 89.12\frac{4}{5}$. (0.)

31. What is the interest of $4732 for 7 years and 9 months, at 4 per cent. per annum? Ans. $1466.92. (0.)

32. What is the amount of $6853.68 for 2 years and 5 months, at 7 per cent. per annum? Ans. $8013.0942. (16.)

33. To what sum will $247.48 amount in 2 years and 9 months, at 5 per cent. per annum? Ans. $281.5085. (10.)

INTEREST FOR YEARS, MONTHS, AND DAYS.

It is becoming a general practice in calculating interest for days to consider the year 360 days, instead of 365, the months will then average 30 days each.

This is the mode adopted in all banks and institutions of this kind, though it gives $\frac{1}{73}$ part more than the true interest.

34. What is the interest of \$80 for 1 year, 5 months, and 12 days, at 6 per cent. per annum?

$$\begin{array}{r|l} & \$ \\ & \cancel{8}\cancel{0} \\ \cancel{5} & 87 \\ \cancel{1}\cancel{2} & \\ & \cancel{6} \\ \hline \text{Ans.} & \$6.96 \end{array} \qquad 1\text{ yr. }5\text{ mos. }12\text{ d.}=17\tfrac{2}{5}\text{ mos.}=\tfrac{87}{5}\text{ mo.}$$

By placing the 12 as a divisor we express the time in years, this may be done in all cases.

35. What is the interest of \$248 for 2 years, 7 months, and 18 days, at 6 per cent. per annum?

The learner will observe that $\frac{158}{5}$ months is inverted (that is, the figures of the numerator) in order to multiply in a single line, also that 3 figures are pointed off as decimals, one of which is to balance a cipher on the left of the line, which will result from 2×5.

$$\begin{array}{r|l} & \$ \\ & 248\times851 \\ 5 & \\ \cancel{1}\cancel{2} & \\ & \cancel{6} \\ \hline \text{Ans.} & \$39.184 \end{array}$$

36. What is the interest of \$436.68 for 1 year, 4 months, and 25 days, at 6 per cent. per annum?

$$\begin{array}{r|l} & \$ \\ & \cancel{4}.\cancel{3}\cancel{6}\cancel{6}\cancel{8}\times101 \\ \cancel{1}\cancel{2} & \quad 6 \\ \cancel{6} & \ 3639 \\ \hline \text{Ans.} & \$36.7539 \end{array}$$

37. What is the interest of \$764 for 2 years, 7 months, and 14 days, at $7\frac{1}{2}$ per cent. per annum?

Ans. \$150.25$\frac{1}{3}$. (6.)

38. What is the interest of \$371.52 for 1 year, 9 months, and 19 days at $7\frac{1}{2}$ per cent. a year?

$$
\begin{array}{r|l}
 & \quad\$ \\
 & 3.71.52 \\
 & \cancel{3}\cancel{0}\cancel{9}\cancel{6} \\
\cancel{3}\cancel{0} & 649\times477 \\
\cancel{2} & \cancel{1}\cancel{5} \\
\cancel{1}\cancel{2} & \\
\hline
\text{Ans.} & \$50.2326
\end{array}
$$

See Emerson's Arithmetic, page 99, for a solution of this question, in which 63 figures are used in the work, whereas ours requires but 17.

A solution of the following question will also be found in Emerson's Arithmetic, page 95; if we count the figures used in obtaining the *sum* of the several parts which make up his multiplier, allowing what precedes this to be performed mentally, 60 figures are used in the work; our solution requires 12 figures in the work, making a difference of 48, (ours gives the exact decimal which Emerson's does not.)

39. What is the amount of \$546.72 for 4 years, 7 months, and 19 days, at 6 per cent. a year?

$$
\begin{array}{r|l}
 & \quad\$ \\
 & \cancel{5}\cancel{4}\cancel{6}.\cancel{7}\cancel{2} \\
\cancel{3}\cancel{0} & 9112\times9661 \\
\cancel{1}\cancel{2} & \cancel{6} \\
\hline
 & \quad 152.07928 \\
\hline
\text{Ans.} & \$698.79928
\end{array}
$$

We consider the years and months all months, and the days as so many 30ths of a month, and place 12 as a divisor to express the time in years.

40. What is the interest of \$174 for 2 years, 3 months, and 21 days, at 6 per cent. a year? Ans. \$24.099. (5.)

41. What is the interest of \$360 for 1 year 11 months and 24 days, at 6 per cent. per annum? Ans. \$42.84. (0.)

42. What is the interest of \$314.36 for 1 year, 1 month, and 6 days, at 6 per cent. per annum?

Ans. \$20.74776. (0.)

43. What is the interest of $512.38 for 7 months and 10 days, at 6 per cent. a year? Ans. $18.7872+ (7.)

44. What is the interest of $817.44 for 11 months and 12 days, at 6 per cent. per annum? Ans. $46.59408. (0.)

45. What is the interest of $396.15 for 17 months and 13 days, at 6 per cent. a year? Ans. $34.53107+ (8.)

46. What is the interest of $16.75 for 7 months and 17 days, at 7 per cent. a year? Ans. $0.739+ (18.)

47. What is the interest of $976.18 for 29 months and 23 days, at 9 per cent. a year? Ans. $217.93.2+ (15.)

48. What is the interest of $51.17 for 9 months and 29 days, at 4 per cent. per annum? Ans. $1.69.99+ (7.)

49. What is the interest of $348 for 17 months and 15 days, at 6 per cent. a year? Ans. $30.45. (2.)

50. To what sum will $245 amount in 3 years and 4 months, at 6 per cent. a year? Ans. $294. (0.)

51. To what sum will $216 amount in 5 years, 4 months, and 20 days, at 5 per cent. a year? Ans. $274.20. (0.)

52. What is the amount of $4119.20 for 1 year and 5 days, at 6 per cent. per annum? Ans. 4369.78.46. (14.)

53. What is the interest of $263.48 for 2 months and 21 days, at 6 per cent. a year? Ans. $3.55698. (5.)

54. What is the interest of $418 for 19 months and 17 days, at 6 per cent.? Ans. 40.894\frac{1}{3}$. (6.)

55. What is the interest of $1680.60 for 11 months and 15 days, at 6 per cent.? Ans. $96.6345. (6.)

56. What is the amount of $999.99 for 19 months and 29 days, at 6 per cent.? Ans. $1099.822+ (10.)

57. What is the interest of $3010.75 for 3 months and 1 day, at 6 per cent. a year? Ans. $45.663+ (8.)

58. What is the interest of $964 for 19 months and 24 days, at 5$\frac{1}{2}$ per cent. a year? Ans. $87.483. (9.)

59. What is the interest of $596 for 3 years, 7 months, and 9 days, at 12$\frac{1}{2}$ per cent. a year? Ans. 268.82\frac{1}{12}$. (9.)

60. What is the interest of $1760 for 2 years, 9 months, and 15 days, at 6 per cent. a year? Ans. $294.80. (0.)

61. What will $3267 amount to in 1 year, 8 months, and 25 days, at 8 per cent. a year? Ans. $3720.75. (8.)

62. To what sum will $7452 amount in 1 year, 11 months, and 6 days, at 8$\frac{1}{4}$ per cent. a year? Ans. $8640.594. (17.)

63. What is the interest of $272 for 41 days, at 6 per cent. a year?

$$\begin{array}{r|l} & \$ \\ & 272 \\ \cancel{30} & 41 \\ \cancel{12} & \cancel{6} \\ 6 & \overline{11.152} \\ \hline \text{Ans.} & \$1.85.8\frac{1}{2} \end{array}$$

The rule usually given for computing interest by days, is to multiply the principal by the number of days, and divide the product by 6 for the interest in mills, when the principal consists of dollars only, the rate being 6 per cent., but as our statement gives the same result, and is just as applicable to any other rate of interest as to 6 per cent. the rule is unnecessary.

64. What is the interest of $168.42 for 32 days, at 7½ per cent. a year?

$$\begin{array}{r|l} & \$ \\ & \cancel{16842} \\ & 2807 \\ \cancel{30} & \cancel{32} \\ \cancel{12} & 4 \\ \cancel{2} & \cancel{15} \\ \hline \text{Ans.} & \$1.1228 \end{array}$$

Here we point off 2 figures from the dollars, making in all 4 decimals, when the cipher on the left of the line is not destroyed by cancelling, an additional one must be pointed off.

65. What is the interest of $346.75 for 84 days, at 6 per cent. a year?

$$\begin{array}{r|l} & \$ \\ & 346.75 \\ \cancel{5} & 14 \\ \cancel{12} & \cancel{6} \\ \hline \text{Ans.} & \$4.8345 \end{array}$$

Here instead of writing the days $\frac{84}{30}$, we write ($\frac{14}{5}$) to express the fraction in its lowest terms; the first figure in the product being 0, we do not write it.

66. To what sum will \$75.69 amount in 4 years, 5 months, and 10 days, at $4\frac{1}{2}$ per cent. a year? Ans. \$90.828. (5.)

67. What is the interest of \$6432 for 3 years, 9 months, and 12 days, at $17\frac{1}{2}$ per cent. a year? Ans. \$4258.52. (7.)

68. What is the interest of \$48.62$\frac{1}{2}$ for 7 years, 7 months, at 8 per cent. a year? Ans. \$29.499+ (5.)

69. What is the interest of \$378 for 45 days, at 6 per cent. a year? Ans. \$2.83$\frac{1}{2}$. (4.)

70. What is the interest of \$747 for 57 days, at 6 per cent. a year? Ans. \$7.09.6$\frac{1}{2}$. (5.)

71. What is the interest of \$864 for 18 days, at 6 per cent. a year? Ans. \$2.59.2. (0.)

72. What is the interest of \$476.46 for 42 days, at 5 per cent. a year? Ans. 2.77.9+ (3.)

73. What is the interest of \$68.75 for 24 days, at 6 per cent. a year? Ans. \$0.27$\frac{1}{2}$. (0.)

74. What is the interest of \$97.20 for 41 days, at $8\frac{1}{4}$ per cent. a year? Ans. \$0.913+ (3.)

75. What is the interest of \$1296 for 29 days, at 6 per cent. a year? Ans. \$6.26.4. (3.)

76. What is the interest of \$873 for 37 days, at $7\frac{1}{3}$ per cent. a year? Ans. \$6.579$\frac{5}{6}$. (10.)

77. What is the interest of \$496 for 39 days, at $12\frac{1}{2}$ per cent. a year? Ans. \$6.716$\frac{2}{3}$. (5.)

78. What is the interest of \$1760.80 for 53 days, at $8\frac{3}{4}$ per cent. a year? Ans. \$22.682+ (15.)

79. What is the interest of \$1000 for 49 days, at $7\frac{1}{2}$ per cent. a year? Ans. \$10.20$\frac{5}{6}$. (4.)

80. What is the interest of \$276 for 31 days, at $4\frac{1}{2}$ per cent. a year? Ans. \$1.069$\frac{1}{2}$. (4.)

81. What is the interest of \$560 for 54 days, at $6\frac{1}{4}$ per cent. a year? Ans. \$5.25. (0.)

82. What is the amount of \$948 for 17 days, at 9 per cent. a year? Ans. \$952.029. (7.)

83. What is the interest of \$650 for 36 days, at 6 per cent. a year? Ans. \$3.90. (0.)

84. What is the amount of \$384 for 59 days, at $7\frac{3}{4}$ per cent. a year? Ans. \$388.877+ (12.)

85. What is the amount of \$947.70 for 51 days, at 6 per cent. a year? Ans. \$955.755+ (10.)

TO COMPUTE INTEREST BETWEEN DIFFERENT DATES.

Find the time by subtracting one date from the other, reckoning 30 days to the month, and 12 months to the year, and then calculate the interest.

86. What is the interest of $480 from May 12, 1841, to August 3, 1843?

year.	mo.	d.
1843	8	3
1841	5	12
2	2	21

$$\begin{array}{r|l} & \$ \\ & \cancel{4}.8\cancel{0} \quad 21\text{ ds.} = \tfrac{7}{10}\text{ m.} \\ \cancel{10} & 267 \\ \cancel{12} & 24 \\ & \cancel{6} \\ \hline \text{Ans.} & \$64.08 \end{array}$$

We multiply 24 by each figure of 267 separately.

NOTE.—The learner will bear in mind that 6 per cent. is always to be understood when no rate is mentioned.

87. What is the interest of $768.69 from April 1, 1842, to September 21, 1847?

year.	mo.	d.
1847	9	21
1842	4	1
5	5	20

$$\begin{array}{r|l} & \$ \\ & 768.69 \times 791 \\ 3 & \\ \cancel{12} & \cancel{6} \\ \hline & 1514.3193 \\ \hline \text{Ans.} & \$252.386+ \end{array}$$

88. What is the interest of $375 from March 6, 1843, to June 15, 1845? Ans. $51.187½. (6.)

89. What is the interest of $196 from September 21, 1839, to June 6, 1843? Ans. $43.61. (2.)

90. What is the interest of $632 from May 18, 1845, to March 24, 1846? Ans. $32.232. (0.)

91. What is the interest of $825 from July 17, 1846, to July 3, 1848? Ans. $97.07½. (3.)

92. What is the interest of $75.84 from August 9, 1837, to May 4, 1843? Ans. $26.1016. (3.)

93. What is the interest of $87.50 from November 14, 1840, to April 21, 1847? Ans. $33.789+ (7.)

94. What is the interest of \$97.86 from May 17, 1821, to Dec. 19, 1828? Ans. \$44.55892. (4.)

95. What is the interest of \$786.96 from October 19, 1827, to August 17, 1831, at 7½ per cent. a year?
Ans. \$225.92.31. (4.)

96. What is the interest of \$96.84 from November 27, 1829, to July 3, 1832, at 7½ per cent. a year?
Ans. \$18.88.38. (7.)

97. What is the amount of \$83.33 from March 11, 1831, to January 1, 1833, at 7½ per cent. a year?
Ans. \$94.614+ (18.)

98. What is the amount of \$144.89 from August 25, 1832, to March 5, 1845? Ans. \$253.7989+ (14.)

99. What is the interest of \$69.75 from January 11, 1833, to June 29, 1833, at 17½ per cent. a year?
Ans. \$5.69625. (8.)

100. What is the interest of \$1876 from March 13, 1841, to August 7, 1846, at 8⅓ per cent. a year?
Ans. \$844.20. (2.)

101. What is the interest of \$672 from September 19, 1843, to January 4, 1849? Ans. \$213.36. (3.)

102. What will \$1396 amount to, from July 17, 1845, to December 29, 1847? Ans. \$1601.212. (6.)

All the foregoing calculations in interest are made upon the assumption of reckoning but 360 days to the year. This principle being adopted by all mercantile men, and also by banks; yet it is necessary sometimes to compute interest for days, according to the actual *number* of days in the year, in that case we place 365 on the left of the line, instead of 12 and 30.

103. What is the interest of \$146 for 85 days, at 7½ per cent. a year of 365 days?

$$\begin{array}{r|l} & \$ \\ & \cancel{146} \\ \cancel{365} & \cancel{85} \\ & \quad 17 \\ \cancel{2} & \quad 15 \\ \cline{2-2} \text{Ans.} & \$2.55 \end{array}$$

Here we observe 5 will measure 85, and also 365, the last quotient being 73, which we multiply by the 2, and the product cancels 146.

104. What is the interest of \$114.80 for 136 days, at 6 per cent. a year of 365 days?

Here we express \$114.80 in dollars, 80 being $\frac{4}{5}$ of a dollar, 73 measures 365, and 574, as it goes 5 times into 365, we cross 36, leaving the 5; we then multiply 136 by 8, and 24 by each figure of the product, (1088,) which is equivalent to multiplying 1088 by 6, and dividing the product by 25; observing to allow 2 more figures for decimals on this account.

$$\begin{array}{r|r} & \$ \\ 5 & \cancel{5}\cancel{7}\cancel{4} \\ \cancel{3}\cancel{6}5 & 8 \\ & 136 \\ & 6 \\ \hline & 1088 \\ \end{array}$$

Ans. \$26.112

105. What is the interest of \$730 for 48 days, at 6 per cent. a year of 365 days? Ans. \$5.76. (0.)

106. What is the interest of \$487 for 146 days at 5 per cent. a year of 365 days? Ans. \$9.74. (0.)

107. What is the interest of \$642.50 for 95 days, at 6 per cent. a year of 365 days? Ans. \$10.03$\frac{26}{73}$. (9.)

108. What is the interest of \$275.25 for 71 days, at 5 per cent. a year of 365 days? Ans. \$2.6770$\frac{65}{73}$. (16.)

109. What is the interest of \$438 for 41 days, at 6 per cent. a year of 365 days? Ans. \$2.952. (0.)

110. What is the interest of \$30.86 for 180 days, at 7$\frac{1}{2}$ per cent. a year of 365 days? Ans. \$1.134. (4.)

111. What is the interest of \$469.32 for 114 days, at 12$\frac{1}{2}$ per cent. a year of 365 days? Ans. \$18.322$\frac{56}{73}$. (21.)

112. What is the interest of \$12.84 for 219 days, at 8$\frac{1}{3}$ per cent. a year of 365 days? Ans. \$0.64$\frac{1}{5}$. (0.)

113. What is the interest of \$97.64 for 35 days, at 8$\frac{3}{4}$ per cent. a year of 365 days? Ans. \$0.8192$\frac{29}{73}$. (21.)

114. What is the interest of \$1438 for 11 days, at 10 per cent. a year of 365 days? Ans. \$4.33$\frac{27}{73}$. (11.)

115. What is the interest of \$8.96 for 17 days, at 5 per cent. a year of 365 days? Ans. \$2.08$\frac{48}{73}$. (8.)

We find in all, or nearly all our arithmetics, special rules for finding the rate per cent. per annum, when the principal, interest, and time are given, also for finding the time when

the other conditions are given, and again for finding the principal, when the amount, time, and rate are given.

These rules are unnecessary, if the pupil understands proportion, and if he does not, he will probably forget the rules in less time than that required to learn them.

116. At what rate per cent. a year, will \$480 amount to \$660, in 7 years and 6 months?

$$
\begin{array}{r|l}
 & \$ \\
 & \cancel{180} \\
\cancel{480} & \cancel{100} \\
\cancel{15} & \cancel{2} \\
\hline
\text{Ans.} & 5 \text{ per cent. a year.}
\end{array}
$$

The learner will of course recollect that the *amount* is the sum of both principal and interest. The principal being \$480, and the amount \$660. \$180 of this amount must be interest; now interest is required as the answer to the question, therefore, we place \$180 on the right of the line, and observe that this is the interest of \$480, and we wish to find the interest of \$100, which will be less than the interest of \$480; again, this \$180 is the amount of interest accumulated in 7 years and 6 months, and we wish to find the interest for 1 year, which will be a less sum than that for 7 years and 6 months.

117. At what per cent. a year will \$740 amount to \$969.40 in 5 years and 2 months?

$$
\begin{array}{r|l}
 & \$ \\
\cancel{5} & \cancel{1147} \text{ interest}=229.40 \\
 & \cancel{37} \\
\cancel{740} & \cancel{100} \\
5 \text{ yrs. } 2 \text{ mos}=\cancel{31} & 6 \\
\hline
\text{Ans.} & 6 \text{ per cent a yr.}
\end{array}
$$

118. At what rate per cent. a year will \$870 amount to \$1160 in 5 years and 4 months? Ans. $6\frac{1}{4}$ per cent. (0.)

119. At what rate per cent. a year will \$96 amount to \$120 in 3 years and 9 months? Ans. $6\frac{2}{3}$ per cent. (0.)

120. At what rate per cent. a year will \$495 amount to \$613.80 in 2 years and 8 months. Ans. 9 per cent. (1.)

121. At what rate per cent. a year will \$352 amount to \$512.16 in 6 years and 6 months?

Ans. 7 per cent. (3.)

122. At what rate will \$600 amount to \$856.50 in 9 years and 6 months? Ans. $4\frac{1}{2}$ per cent. (0.)

123. At what rate per cent. a year will \$264 amount to \$281.60 in 1 year and 3 months?

Ans. $5\frac{1}{3}$ per cent. (0.)

124. At what rate per cent. a year will \$89.10 amount to \$108.90 in 3 years and 4 months?

Ans. $6\frac{2}{3}$ per cent. (0.)

125. At what rate per cent. a year will \$197.60 amount to \$222.30 in 15 months? Ans. 10 per cent. (0.)

126. At what rate will \$648 amount to \$1012.50 in 6 years and 9 months? Ans. $8\frac{1}{3}$ per cent. (3.)

127. At what rate per cent, a year will \$1200 amount to \$1740 in 7 years and 6 months?

Ans. 6 per cent. (0.)

128. At what rate per cent. a year will \$340.80 amount to 589.30 in 9 years and 4 months?

Ans. $7\frac{13}{16}$ per cent. (8.)

129. In what time will \$500 amount to \$860 at 6 per cent. a year?

$$
\begin{array}{r|l}
 & \text{yr.} \\
 & 1 \\
\cancel{5}\cancel{0}\cancel{0} & \cancel{1}\cancel{0}\cancel{0} \\
\cancel{6} & \cancel{3}\cancel{6}\cancel{0} \\
\hline
\text{Ans.} & 12 \text{ years.}
\end{array}
$$

In this question, time is required as the answer, and we have 1 year, \$100 principal, (since per cent. means for 100,) and \$6 interest, \$500 principal, and \$360 interest, from which to make a statement that will give the time required; then we write 1 year on the right, at the head of the line, and observe, 1 year is the time required by the question, for \$100 to gain interest, (no matter what sum,) and \$500 will require less time to gain a like sum, hence the *less* of those two numbers is placed on the right of the line; again 1 year is the time required to gain \$6 interest, and a longer time will be required to gain \$360 interest, therefore the greater number (360) is placed on the right of the line.

130. In what time will \$640 amount to \$1018 at 7 per cent. a year?

$$\begin{array}{r|l} \multicolumn{2}{c}{\text{Year.}} \\ & 1 \\ \cancel{640} & 10\cancel{0} \\ \cancel{7} & \cancel{378} = 1018-640 \\ \cancel{32} & \cancel{54} \\ \cancel{4} & 270 \\ & \cancel{14} \\ & \cancel{21} \\ \hline \end{array}$$

Ans. 8 yrs. $5\frac{1}{4}$ mos.

We divide 32 (and 12 times 14) by 8 and obtain 4 and 21.

131. In what time will $171.40 amount to $231.39 at 7 per cent. a year?

$$\begin{array}{r|l} & 1 \text{ yr.} \\ \cancel{857} & 5 \text{ yrs. Ans.} \\ \cancel{7} & \cancel{100} \\ & \cancel{59.99} = 231.39-171.40 \end{array}$$

In this solution we cancel 100, and then consider the 99 as whole numbers.

132. In what time will $189 amount to $252 at 8 per cent. a year? Ans. 4 yrs. and 2 mos. (0.)

133. In what time will $280 amount to $371 at 5 per cent. a year? Ans. 6 yrs. 6 mos. (0.)

134. In what time will $396 amount to $1144, at $8\frac{1}{3}$ per cent. a year? Ans. 22 yrs. 8 mos. (2.)

135. In what time will $4810.25 amount to $5002.66, at 6 per cent. a year? Ans. 8 months. (0.)

136. In what time will $684 amount to $931, at $6\frac{1}{2}$ per cent. a year? Ans. 5 yrs. $6\frac{2}{3}$ mos. (4.)

137. In what time will $675.25 amount to $864.32, at 4 per cent. a year? Ans. 7 years. (0.)

138. If I lend $275, at $3\frac{1}{2}$ per cent. a year, and on payment being made receive for principal and interest $$342.37\frac{1}{2}$, what is the time? Ans. 7 years. (0.)

139. In what time will $540 amount to $734.40, at 4 per cent. a year? Ans. 9 years. (0.)

140. In what time will $37.43 amount to $74.86, at 6 per cent. a year? Ans. $16\frac{2}{3}$ years. (0.)

In the last question observe the interest amounts to the same as the principal, that is, the principal is doubled, hence any sum will double itself, at 6 per cent. in $16\frac{2}{3}$ years.

141. In what time will \$87.39 amount to \$174.78, at 5 per cent.; or in what time will any sum double itself at 5 per cent.? Ans. 20 years. (0.)

NOTE.—To find the time required for any sum to double we simply divide 100 by the rate.

142. In what time will any sum double at 7 per cent. a year, at 8 per cent., 4 per cent., 9 per cent., 10 per cent., 11 per cent., 12 per cent.?

PARTIAL PAYMENTS.

The United States Court has established a general rule for the computation of interest, when partial payments have been made, expressed by Chancellor Kent as follows:

The rule for casting interest, when partial payments have been made, is to apply the payment in the first place to the discharge of the interest then due. If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal until the period when the payments taken together exceed the interest due, and then the surplus is to be applied towards discharging the principal, and interest is to be computed on the balance as aforesaid.

1. \$300. Philadelphia, May 1, 1842.

For value received, I promise to pay James Thornton, or order, the sum of three hundred dollars on demand, with lawful interest. THOMAS CLARK.

The following endorsements were made on this note:

1842, October 16, received - - - \$60.00.

1843, March 21,	do. - - - - -	17.50.
1843, August 27,	do. - - - - -	28.40.
1844, April 1,	do. - - - - -	132.25.

What was the balance due September 19, 1844?
First find the time.

(1.)

year.	mo.	d.
42	10	16
42	5	1
	5	15

(2.)

year.	mo.	d.
43	3	21
42	10	16
	5	5

(3.)

year.	mo.	d.
43	8	27
43	3	21
	5	6

(4.)

year.	mo.	d.
44	4	1
43	8	27
	7	4

(5.)

year.	mo.	d.
44	9	19
44	4	1
	5	18

Amount of the note, - - - - - -	$300.00
Interest to October 16, 1842, (5 mos. 15 d.) -	8.25
First amount, - - - - - - -	308.25
First payment, - - - - - - -	60.00
Balance forming a new principal, - - - -	248.25
Interest from October 16, 1842, to March 21, 1843, (5 mos. 5 d.) - - - - - - -	6.41
Second amount, - - - - - - -	254.66
Second payment, - - - - - - -	17.50
Balance forming a new principal, - - - -	237.16
Interest from March 21, 1843, to August 27, 1843, (5 mos. 6 d.) - - - - - - -	6.17
Third amount, - - - - - - -	243.33
Third payment, - - - - - - -	28.40
Balance due, - - - - - - -	214.93
Interest from August 27, 1843, to April 1, 1844, (7 mos. 4 d.) - - - - - - -	7.66
Fourth amount, - - - - - - -	222.59
Fourth payment, - - - - -	132.25
Balance due, - - - - - - -	90.34

Balance due, brought forward, - - -	$90.34
Interest from April 1, 1844, to September 19, 1844, (5 mos. 18 d.) - - - - - - -	2.52
Balance due September 19, 1844, Ans.	$92.86

2. $1260. Harrisburg, June 7, 1843.

For value received, I promise to pay to James Taylor, or order, the sum of one thousand two hundred and sixty dollars, on demand, with lawful interest.

HENRY DICKINSON.

The following endorsements were made on this note:

1844, received August 19, - - - - -	$145.00
1846, received March 4, - - - - -	36.40
1848, received September 13, - - - -	460.56

What was the balance due July 3, 1849?

To find the time.

(6.)

year.	mo.	d.
44	8	19
43	6	7
1	2	12

(7.)

year.	mo.	d.
46	3	4
44	8	19
1	6	15

(8.)

year.	mo.	d.
48	9	13
46	3	4
2	6	9

(9.)

year.	mo.	d.
49	7	3
48	9	13
	9	20

Amount of the note, - - - - - -	$1260.00
Interest from June 7, 1843, to August 19, 1844, (1 yr. 2 mos. 12 d.) - - - - - -	90.72
First amount, - - - - - - - -	1350.72
First payment, - - - - - - - -	145.00
Balance forming a new principal, - - -	1205.72
Interest from August 19, 1844, to March 4, 1846, (1 yr. 6 mos. 15 d.) - - - - - -	111.53
Second payment, less than the interest due, -	36.40
Leaving interest unpaid, - - - - - -	75.13

Leaving interest unpaid, brought forward, -	$75.13
Interest from March 4, 1846, to September 13, 1848, (2 yrs. 6 mos. 9 d.) - - - - - -	182.67
Whole amount of interest, - - - - -	257.80
Second amount, - - - - - - -	1463.52
Third payment, - - - - - - -	460.50
Balance forming a new principal, - - -	1003.02
Interest from September 13, 1848, to July 3, 1849, (9 mos. 20 d.) - - - - - - -	48.48
Balance due July 3, 1849, Ans.	$1051.50

NOTE.—As we do not use the decimals beyond the second place, (or cents,) if the third place decimal in the calculation exceeds 5, increase the cents by 1.

3. $2480. Pittsburg, January 8, 1839.

For value received, I promise to pay to Caleb Temple, or order, the sum of two thousand four hundred and eighty dollars, on demand, with lawful interest.

JOSEPH EDWARDS.

Payments were made on this note as follows:

May 20, 1841, - - - - - -	$640.00
September 25, 1843, - - - -	850.00
July 10, 1844, - - - - - -	1234.60

What was the balance due May 25, 1846?

Ans. $550.36

NOTE.—The pupil should find the time for each payment previous to making any further calculations.

4. $1890. Columbus, March 13, 1843.

For value received, I promise to pay to Henry A. Warrener, or order, the sum of one thousand eight hundred and ninety dollars, on demand, with lawful interest.

ENOCH WOOLMAN.

The following payments were made and endorsed on this note:

April 7, 1844, received, - - - -	$482.25
September 19, 1845, received, - - -	746.80
June 13, 1846, received, - - - -	568.31

The balance was paid Oct. 9, 1847; how much was it?
Ans. $417.55.

5 $835.50. Havana, July 4, 1842.
For value received, I promise to pay to James Hambleton, or order, the sum of eight hundred and thirty-five dollars and fifty cents, on demand, with lawful interest.
WILLIAM TAYLOR.

On the back of this note were the following endorsements:

March 13, 1843, received, - - -	$124.68
July 21, 1844, received, - - -	36.25
September 6, 1845, received, - - -	248.31
May 3, 1846, received, - - - -	420.00

The balance was paid August 28, 1847.
What was the balance? Ans. $188.28.

6. $1440. Philadelphia, September 16, 1841.
For value received, we promise to pay to Joseph Tatum of Mantua Grove, the sum of one thousand four hundred and forty dollars, on demand, with interest, at the rate of 6 per cent. a year. PIERCE & CALLAN.

Partial payments were made and endorsed on this note as follows, viz.

September 3, 1842, received, - - -	$327.50
July 21, 1844, received, - - -	468.25
March 1, 1845, received, - - -	118.75
August 19, 1846, received, - - -	648.60

The balance was paid January 1, 1847; required the balance. Ans. $199.47.

7. On a bond of $2620, dated June 28, 1840, with lawful interest, there were endorsed the following payments, viz.

August 25, 1841, received, - - -	$865.40
November 20, 1842, received, - - -	47.50
May 5, 1844, received, - - - -	964.70
October 14, 1845, received, - - -	687.41
April 29, 1846, received, - - -	426.37

What was the balance which was paid July 14, 1847?
Ans. $270.52.

COMPOUND INTEREST.

Compound interest is that which is paid not only for the use of principal, but also for the use of interest after it becomes due.

RULE.

Find the amount of the principal to the time of the first interest becoming due, and call this amount a new principal, find the *amount* of this *new* principal, which consider a *second* new principal, and find *its* amount as before, and so on for the number of payments required.

Subtract the given principal from the last amount, and the remainder is the compound interest.

1. What is the compound interest of $600 for 6 years, at 6 per cent. a year?

Principal	$600	×106
	36|00	
New principal	636	×106
	38 16	
2d new principal	674.16	×106
	40.4496	
3d new principal	714.6096	×106
	42.876576	
	757.486~~176~~	×106
	45.44916	
	802.93533	×106
	48.1761198	
Last amount	851.1114498	
Given principal	600.	
Ans.	$251.11.1	

If we consider the principal to consist of 100 parts, the interest will be 6 of those parts, and the amount, or sum of both principal and interest will be 106 parts.

2. What is the compound interest of $450 for 3 years, at 6 per cent. a year ? Ans. $85.95.72.

3. What is the compound interest of $680 for 4 years, at 5 per cent. a year? Ans. $146.544+

4. What is the compound interest of $500 for 5 years, at 4 per cent. a year? Ans. $108.326+

5. What is the compound interest of $400 for 4 years, at 7 per cent. a year? Ans. $124.318+

6. What is the compound interest of $700 for 7 years, at 6 per cent a year? Ans. $352.54+

7. What is the compound interest of $1000 for $2\frac{1}{2}$ years, at 6 per cent. a year, payable half yearly?

Ans. $159.274+

This question is the same as if it *were* what is the compound interest of $1000 for 5 years, at 3 per cent. a year?

DISCOUNT.

Discount is an allowance made for the payment of money before it becomes due.

The *present worth* of a debt, payable at a subsequent period, without interest, is such a sum of money as being put on interest would amount to the debt at the *time* when the debt is payable.

Illustration.—Suppose I purchase a horse, for which I agree to pay $106 at the expiration of one year from the time I make the purchase, but am willing to pay at present, if a reasonable deduction be made in the price. Now if money is worth 6 per cent. a year, and I pay $100 at present, that sum will amount to $106, by the time the payment was to have been made, according to the contract—$6 then is the *discount*, and $100 is the present worth of the price of the horse.

1. Bought a house for which I am to pay $590 in 3 years from the time of making the purchase, or if I choose to pay the cash, the present worth of $590, estimating money worth 6 per cent. what is the cash price of the house? Observe that $100 at this rate and time would yield $18 interest, or

would amount to $118, therefore $100 is the present worth of $118 payable three years hence, money being worth 6 per cent. a year.

Then to state the question, we say, $100 is the present worth of $118, and the present worth of $590 will be more, therefore we place $590, the greater of these two numbers, on the right of the line, and we find the present worth of $590 to be $500, which being subtracted from $590 gives the discount $90.

$$\begin{array}{r|r} & \$ \\ & 100 \\ \cancel{118} & \cancel{590} \\ & 5 \\ \hline \text{Ans.} & \$500 \end{array}$$

RULE.

Take the amount of $100 dollars for the given time and rate, as a divisor, which place on the left of the line; and $100 and the given sum or amount on the right. Then perform the operations indicated by your statement; and the result will be the present worth, bearing the same ratio to $100 which the *given amount* bears to the *amount* of $100.

2. What is the present worth of $920, due in $2\frac{1}{2}$ years, allowing money worth 6 per cent. a year?

$$\begin{array}{r|r} & \cancel{100} \\ \cancel{115} & \cancel{920} \\ \hline \text{Ans.} & \$800 \end{array}$$

$2\frac{1}{2}$ times 6=15, which added to 100=115

3. What is the present value of a note for $760.76 payable in 9 months, when the interest is 6 per cent. a year?

$$\begin{array}{r|r} & 100 \\ & \cancel{760.76} \\ \cancel{209} & 2 \quad \cancel{40.04} \\ \cancel{19} & 3.64 \\ \hline \text{Ans.} & \$7.28 \end{array}$$

$\frac{3}{4}$ of $6=4\frac{1}{2}$ which added to $100=104\frac{1}{2}=\frac{209}{2}$ (11 measures.)

4. What is the present worth of $1296 payable in 16 months, when interest is 6 per cent. a year?

Ans. $1200. (0.)

5. What is the present worth of \$115.32 payable in 4 years, money being worth 6 per cent. a year?

Ans. \$93. (2.)

6. What is the present worth of a note for \$852.55 payable in 15 months, estimating interest at 5 per cent. a year?

Ans. \$802.40. (7.)

7. Purchased goods to the amount of \$280.90 on a credit of 9 months; if money is worth 8 per cent. a year, how much cash will pay the bill? Ans. \$265. (4.)

8. A offers to sell B a valuable farm, on a credit of 12 months, for \$9548.45, or for present payment will allow a discount at the rate of $9\frac{1}{4}$ per cent. a year, what is the cash price of the farm? Ans. \$8740. (12.)

9. When money is worth 7 per cent. a year, what is the present worth of \$488.41 payable in 18 months?

Ans. \$442. (4.)

10. What is the present worth of \$684.37 payable in 2 years and 2 months, allowing interest at 6 per cent. a year?

Ans. $\$605.63.7\frac{19}{113}$ (7.)

11. A owes B \$1260, of which \$700 are payable in 8 months, and the remainder in 15 months, what is the present worth of this debt, allowing interest to be 6 per cent. a year?

Ans. \$1194.006+ (17 figures in both statements.)

NOTE.—When payments are due at different periods, find the present worth of each payment separately; their sum will be the answer.

The discount is found by subtracting the present worth from the given debt or amount; but it is more convenient to find the discount by a direct statement.

12. What is the discount on \$1249.30 for 8 months, at the rate of 5 per cent. a year?

$$\begin{array}{r|l} \cancel{3} & \cancel{1}\cancel{0} \\ 31\cancel{0} & 1249.30 \\ & \cancel{3} \end{array}$$

8 months being $\frac{2}{3}$ of a year the discount of \$100 is

(Ans. \$40.30=discount. [$\frac{2}{3}$ of $5=\frac{10}{3}$.

The discount of any sum will have the same ratio to the discount of \$100 which the amount of that sum has to the amount of \$100 for a given time and rate.

13. What is the discount of \$493.79 for 1 year and 8 months, at 7 per cent. a year?

$$\begin{array}{r|l}
\cancel{3} & \cancel{35}\quad 7 \\
\cancel{335} & \cancel{3} \\
67 & 493.79(737 \\
 & 24 \\
 & 46 \\
\hline
\text{Ans.} & 51.59 = \text{discount.}
\end{array}$$

1 yr. 8 mos. $= \frac{5}{3}$ yr.

$\frac{5}{3}$ of $7 = \frac{35}{3}$ discount of \$100

14. What is the discount of \$1024 for 2 years, at 6 per cent. a year? Ans. $109.71\frac{3}{7}$. (8.)

15. What is the discount of \$198.60 for 9 months, when interest is 5 per cent. a year? Ans. $7.178\frac{26}{83}$. (12.)

16. What is the discount of \$401.25, due 1 year hence, the rate of interest being 7 per cent. a year? Ans. \$26.25. (0.)

17. What is the discount of \$874.51 payable in 1 year and 5 months, the rate of interest being 6 per cent. a year? Ans. \$68.51. (4.)

18. What is the discount of \$1174.03 for 7 months, when interest is $4\frac{1}{2}$ per cent. a year? Ans. \$30.03. (9.)

19. What is the discount of \$703.56 for 5 months, when interest is $7\frac{1}{2}$ per cent. a year? Ans. \$21.32. (0.)

20. What is the discount of \$476.29 for 11 months, if money is worth 9 per cent. a year? Ans. \$36.299+ (18.)

21. What is the discount of \$1348.27 for 4 months and 15 days, when money is worth 8 per cent. a year? Ans. \$39.27. (7.)

22. What is the discount of \$768.43 for 10 months and 15 days, at 6 per cent. a year? Ans. \$38.33+ (16.)

23. What is the difference between the interest of \$7290 for 7 years, at 5 per cent. a year, and the discount of the same sum for the same time, and at the same rate? Ans. \$661.50.

24. What is the difference between the interest of \$486.72 for 8 years at 7 per cent. and the discount of the same sum for the same time and at the same rate. Ans. $\$97.84.3\frac{1}{5}$.

25. Bought a farm for \$7368, and on the same day sold it for \$8985, on a credit of 9 months, how much did I make

by this transaction, allowing money to be worth 6 per cent. a year? Ans. \$1230.086+ (26.)

26. A merchant gave his note for \$1831.75 payable in 8 months, but the holder of the note being pressed for money, the merchant paid it in 3 months; allowing money to be worth 6 per cent. a year, what sum was requisite to redeem the note? Ans. \$1787.07$\frac{13}{41}$. (9.)

COMMISSION.

Commission is the compensation made to factors, brokers, and other agents, for their services in buying and selling. It is computed at so much per cent. on money employed in the transaction.

A *factor* is an agent employed by merchants residing in other places, to buy and sell, and to negotiate bills of exchange, or to transact other business on their account.

1. What is my commission at 2½ per cent. for purchasing goods to the amount of \$962.

$$\begin{array}{r|l} & \$ \\ 2 & 9.62 \\ \hline & 481 \\ & 5 \\ \hline \text{Ans.} & 24.05 \end{array}$$

Pointing off 2 figures to the right, being equivalent to dividing by 100, we have only to find $\frac{5}{2}$ of 9.62.

2. How much will an agent receive for purchasing 760 barrels of flour, at \$4.75 a barrel, if he be allowed a commission of 1¼ per cent. on the disbursement?

$$\begin{array}{r|l} & \$ \\ \cancel{4} & 19 \\ & \cancel{760} \\ 4 & 5 \\ \hline & 36.10 \\ \hline \text{Ans.} & \$45.12\tfrac{1}{2} \end{array}$$

This solution is given that the learner may perceive there is no necessity for calculating the value of the 760 barrels of flour, in order to find the commission, but simply to make the statement; then pointing off 2 figures as decimals, we have the expression for 1 per cent. $\frac{5}{4}$ of which expresses the commission. In

the solution we cancel 760 by 4, then square 19, (mentally,) placing the cipher on the right; we have yet to multiply by 5, and divide the product by 4, which we do at one operation, viz., suppose a cipher annexed to the multiplicand, and divide by 8, hence you perceive the solution requires but 4 figures in the work.

3. If I allow my factor in London a commission of $2\frac{1}{2}$ per cent., for purchasing goods to the amount of £599 12s. 6d., what will his commission amount to in Federal money?

Ans. $66.62\frac{1}{2}$. (3.)

4. Suppose I purchase in Cincinnati 784 barrels of pork, at $13.25 a barrel, for a merchant in Philadelphia, who allows me a commission of $\frac{3}{4}$ of 1 per cent; how much will I receive?

Ans. $77.91. (2.)

5. I have remitted to my factor a certain sum of money, which he is to lay out for me in iron, and having reserved to himself $2\frac{1}{2}$ per cent. on the purchase, which amounted to $90, he buys for me the iron at $95 a ton, required the quantity of iron purchased.

Ans. 37 tons, 17 cwt. 3 qrs. $16\frac{4}{19}$ lbs. (5.)

NOTE.—As he is to take out his commission previous to making the purchase, according to the principles of discount, the purchase money after the commission is deducted will be to his commission as 100 to $2\frac{1}{2}$.

6. Sent a cargo of flour to Liverpool, which my factor sold for £987 18s. 6d.; he invested this sum in broadcloth, at £1 3s. 8d. per yard, his commission for selling the flour is $2\frac{1}{4}$ per cent., and for purchasing the broadcloth $1\frac{7}{8}$ per cent., and he is to receive his commission for selling and buying out of the proceeds of the flour. Required the number of yards of broadcloth that I should receive.

Ans. $801\frac{3576}{57865}$ yds. (21.)

NOTE.—In this question his commission for selling will be $2\frac{1}{4}$ per cent. of the price of the flour, but as he deducts his commission for selling previous to making the purchase, his commission in this case will be discount.

The last two questions are taken from Mr. Greenleaf's Arithmetic, in order to contrast his solution with our own.

Greenleaf's solution of the sixth statement.

12 | 6
20 | 18.5

987.925
.0225

4539625
1975850
1975850

£22.2283125
20

4.56625|00s.
12

6.795|00d.

1000 $=\frac{159}{200}$

£	s.	d.
1	3	8

20

23
12

284
815

1420
284
2272

231460d.

£	s.	d.
987	18	6
22	4	$6\frac{159}{200}$
965	13	$11\frac{41}{200}\times 100$

815)96669 13 $4\frac{1}{2}$
8

772557 7 0(947 £ 18 s. $5\frac{449}{815}$d.
7335

3905
3260

6457
5705

752
20

15047
815

6897
6520

377
12

4524
4075

449

947
20

18958
12

227501
815

1137954
227501
1820008

231460)185413764(801 $\frac{3576}{57865}$ yds. Ans.
185168

245764
231460

4)$\frac{14304}{231460}=\frac{3576}{57865}$.

Our solution.

yard.	
	1
$\cancel{40}$	39517×3711
$\cancel{4}$	391
815	$\cancel{8}$
	3
71	$\cancel{60}$
57865	46353441(801$\frac{35776}{57865}$ yds. Ans.
	614

In making our statement, 18s. 6d. being equal to £$\frac{37}{40}$, we throw the £987 18s. 6d. into an improper fraction, (mentally,) and then observe, since $2\frac{1}{4}$ per cent. are to be taken from it, (that is, $2\frac{1}{4}$ parts from 100 parts,) $97\frac{3}{4}$ parts will remain,=£$\frac{391}{4}$. Now to satisfy the other conditions we state the terms so as to obtain the present worth of the sum indicated by the first part of the statement, discount being $1\frac{7}{8}$ per cent. In this solution 21 figures completes the work after the statement is made; Mr. Greenleaf's solution requires 305 figures, omitting those in the first statement, making a difference of 284.

7. If an agent sell 1296 tons of coal at \$4.62½ cents a ton, and receives 12½ per cent. of the proceeds for his services, what is the amount of his commission?

Ans. \$749.25. (2.)

8. A commission merchant sold goods in one year to the amount of \$24,680.60, on which he charged 5½ per cent. for his services, what was the amount of his year's labour?

Ans. \$1357.433. (0.)

9. Sold 1680 cords of wood at an average price of \$3.75 a cord, on which my commission was 4¾ per cent. what did it amount to? Ans. \$299.25. (6.)

10. My factor at New Orleans advises me that he has purchased on my account 37 bales of cotton at \$107.75 per bale, what is his commission at ⅜ of 1 per cent.?

Ans. \$14.95$\frac{1}{32}$. (7.)

BANKING.

A bank is an incorporated institution created for the purpose of loaning money and dealing in exchange.

It is owned in shares by a company of individuals called stockholders, and its operations are carried on by a president and board of directors.

Banks are authorized by law to issue notes or bills of their own, intended to be a circulating medium of exchange or currency instead of gold and silver.

The interest on a sum of money borrowed of a bank is paid at the time when the money is taken out, the borrower receiving as much less than the sum he promises to pay as would be equal to the *interest* of that *sum*, from the time he borrows the money, until the time it is to be paid.

This interest is called discount, that is, bank discount, means interest paid in advance.

Banks take interest for four days more than the number intervening between the day when the note is discounted and the day on which the time specified in it expires, reckoning 30 days to the month, hence we must compute the interest on the sum borrowed for four days more than the time specified in the note.

1. What is the bank discount on \$450 payable 60 days hence, at 6 per cent. a year?

$$
\begin{array}{r|l}
 & \$ \\
 & \cancel{4}.\cancel{5}\cancel{0} \\
 & \quad 15 \\
\cancel{3}\cancel{0} & \cancel{6}\cancel{4} \\
 & \quad 32 \\
\cancel{1}\cancel{2} & \cancel{6} \\
\hline
\text{Ans.} & \$4.80
\end{array}
$$

2. What is the bank discount on \$960 for 90 days, at 6 per cent. a year?

$$
\begin{array}{r|l}
 & \$ \\
 & \cancel{9}.\cancel{6}\cancel{0} \\
\cancel{3}\cancel{0} & \quad 94 \\
\cancel{1}\cancel{2} & \cancel{6} \\
\hline
\text{Ans.} & \$15.04
\end{array}
$$

3. What is the bank discount on $480 for 30 days, at 6 per cent. a year? Ans. $2.72. (0.)

4. What is the bank discount on $250 for 120 days, at 6 per cent. a year? Ans. 5.16\frac{2}{3}$. (2.)

5. What is the bank discount on $780 for 180 days, at 7 per cent. a year? Ans. 27.90\frac{2}{3}$. (5.)

The borrower is allowed 3 days more than the time specified in the note, (called days of grace,) interest being taken for these three days, and also for the day on which the money was borrowed.

6. What is the bank discount on $1260 for 140 days, at 5$\frac{1}{2}$ per cent. a year? Ans. $27.72. (0.)

7. What is the bank discount on $347 for 48 days, at 4$\frac{1}{2}$ per cent. a year? Ans. 2.25\frac{1}{2}$. (4.)

8. What is the bank discount on $536 for 41 days, at 7$\frac{1}{2}$ per cent. a year? Ans. 5.02\frac{1}{2}$. (4.)

9. What is the bank discount on $626 for 79 days, at 5 per cent. a year? Ans. $7.216+ (9.)

10. What is the bank discount on $833 for 117 days, at 6 per cent. a year? Ans. $16.798+ (6.)

11. What is the bank discount on $1760 for 90 days, at 4 per cent. a year? Ans. $18.38+ (5.)

12. What is the bank discount on $24,760 for 120 days, at 3 per cent. a year? Ans. $255.85+ (5.)

13. What is the bank discount on $84 for 60 days, at 8 per cent. a year? Ans. $1.19+ (5.)

14. What is the bank discount on $1358 for 87 days, at 6 per cent. a year? Ans. $20.596+ (6.)

15. What is the bank discount on $1670 for 180 days, at 5$\frac{1}{2}$ per cent. a year? Ans. $46.94+ (8.)

16. What is the bank discount on $75 for 90 days, at 6 per cent. a year? Ans. 1.17\frac{1}{2}$. (0.)

17. How much do I receive on a note of $360 payable in 90 days, the bank discount being 6 per cent.? Ans. $354.36. (3.)

I will receive what will remain of $360 after the interest of this sum for 90 days is deducted.

18. A note for $276.80 payable in 5 months, or 150 days is discounted at bank, when interest is 6 per cent. a year, what sum do I receive? Ans. $269.70. (9.)

X

19. If a note for \$2640 payable in 4½ months be discounted at bank, at 6 per cent. a year, what sum do I receive? Ans. \$2578.84. (4.)

20. A farmer sold wheat to the amount of \$981, for which he received a note payable in 6 months, how much will he receive at bank for this note, money being worth 6 per cent. a year? Ans. \$950.916. (11.)

21. Had a note of \$3900 payable 60 days after date, discounted at bank, how much did I receive, interest being 6 per cent. a year? Ans. \$3858.40. (4.)

22. Bought 760 barrels of flour, at \$4.50 a barrel, and sold it the same day for \$4.70 a barrel, on a credit of 4 months, taking a note which I had discounted at bank, at 6 per cent. a year, how much did I make by the transaction? Ans. \$78.178⅔. (30.)

23. A merchant bought a cargo of oil for \$7960, for which he gave his note payable in 8 months. If this note be discounted at bank at the rate of 6 per cent. a year, what sum will be received for it? Ans. \$7636.29⅓. (13.)

INSURANCE AND POLICIES.

Insurance is a contract by which one party engages for a stipulated sum, or premium per cent. to make up a loss which another may sustain.

The article of agreement between the parties is called a *policy*.

1. What would be the expense of insuring a house valued at \$4760, at 1¾ per cent.?

$$\begin{array}{r|r} & \$ \\ & \cancel{4}\cancel{7}.\cancel{6}\cancel{0} \\ & 11.90 \\ \cancel{4} & 7 \\ \hline \text{Ans.} & \$83.30 \end{array}$$

2. What is the insurance of an East India ship and cargo valued at \$17,984, at 15¼ per cent.?

Ans. \$2742.56. (4.)

3. What must be paid for insurance on a steamboat and cargo, the boat being valued at $12,600, and the cargo at $14,400, the cost of the policy being $1, and the premium being $4\frac{1}{2}$ per cent.? Ans. $1216. (3.)

4. What would be the premium for insuring a cotton factory valued at $9746, at $2\frac{1}{2}$ per cent.?

Ans. $243.65. (0.)

5. What must be paid for insuring a ship at $21,000, and a house valued at $1200, there being two policies at $1 each, and the premium on the ship being $12\frac{1}{2}$ per cent. on $\frac{4}{7}$ of the estimated value, and on the house $1\frac{1}{2}$ per cent. for the whole value? Ans. $1520. (0.)

6. A company owning a steam saw-mill, valued at $27,680, procured an insurance on it for 4 years, upon these conditions: for the first year they were to pay $3\frac{1}{2}$ per cent., for the 2d year 3 per cent., for the 3d year $2\frac{1}{2}$ per cent., and for the 4th year 2 per cent., what did they pay in the 4 years?

Ans. $3044.80. (0.)

MISCELLANEOUS QUESTIONS.

1. How many pounds of coffee at 17 cents a pound are equal in value to 3978 pounds of tea at $62\frac{1}{2}$ cents a pound?

Ans. 14625 lbs. (3.)

2. A merchant bought a ship for $11,475, and sold her for $13,770. How much did he gain per cent., that is, how much does he gain on $100?

NOTE.—The number to which 100 will increase must be greater than 100, in the same ratio that the number to which $11,475 *has* increased (13,770) is greater than 11,475.

Hence, we state the question,

$$\text{Thus,}\quad \begin{array}{r|l} & \cancel{100} \\ \cancel{11475} & \quad 4 \\ \cancel{459} & \cancel{13770} \\ \hline & \ 120 \end{array}$$

Ans. 20 per cent.

And we find 100 to increase to 120, therefore the gain is 20 per cent.

3 Bought a horse for \$75, and sold him for \$82.50, what did I gain per cent.? Ans. 10 per cent. (0.)

4. A merchant bought a lot of flour, at \$4.37$\frac{1}{2}$ a barrel, and sold it at \$5.15 a barrel, what was his gain per cent.?

Ans. 17$\frac{5}{7}$ per cent. (0.)

5. A speculator bought a lot of ground for \$2754, and by dividing into several small lots, sold the same for \$4626.72, what was his gain per cent.? Ans. 68 per cent. (8.)

6. A man bought a chaise for \$154, and sold the same for \$198, what was the gain per cent.?

Ans. 28$\frac{4}{7}$ per cent. (0.)

7. A stationer sold quills at \$1.83$\frac{1}{3}$ per thousand, and thereby gained $\frac{1}{4}$ of the first cost, but quills growing scarce he afterwards raised them to \$2.04 per thousand, what did he gain per cent. by the last sale?

Ans. 39$\frac{1}{11}$ per cent. (3.)

NOTE.—The learner should constantly bear in mind that the denominator of a fraction shows the number of parts to be used as a starting point to reason upon. In this question one part having been gained; \$1.83$\frac{1}{3}$ must be 5 parts, and the first cost of the quills 4 parts. Then, as we wish to find what will be gained on 100, we say that the number to which 100 will increase, must be greater than 100, in the same ratio that the number to which 4 parts (that is $\frac{4}{5}$) of \$1.83$\frac{1}{3}$ has been increased to is greater than ($\frac{4}{5}$ of 183$\frac{1}{3}$.)

Hence, we state the question,

Thus,

$$\begin{array}{r|l} & 10\cancel{0} \\ \cancel{5}\cancel{5}\cancel{0} & \cancel{3} \\ \cancel{4} & \cancel{5} \\ & \cancel{2}\cancel{0}\cancel{4} \\ & 153 \\ \hline \text{Ans.} & 1.39\frac{1}{11} \end{array}$$

We might have written 51 on the right and 11 on the left of the line, but this would be useless; 3 figures are all that need be written, and even these might be dispensed with, if we wish to make the solution of problems an intellectual rather than a mechanical process.

8. If by selling goods at 50s. per cwt. I gain 20 per cent. what do I gain or lose per cent. by selling the same at 45s. per cwt. Ans. gain 8 per cent. (0.)

20 being $\frac{1}{5}$ of 100, the cost of the goods is 5 parts, and 50 is 6 parts, then say, the number to which 100 will increase, or decrease, must be greater or less than 100, in the same ratio that 45s. is greater or less than $\frac{5}{6}$ of 50.

9. Sold goods for $166, and by so doing lost 17 per cent. whereas I ought in dealing to have cleared 20 per cent. how much ought the goods to have been sold for?

Ans. $240.

NOTE.—17 parts lost out of 100, 83 parts remain. This question should be solved mentally, without a statement.

10. Sold goods for $63, and by so doing lost 17 per cent. whereas I ought in dealing to have cleared 20 per cent. what was the amount of loss sustained?

Ans. 28.08\frac{36}{83}$. (7.)

NOTE.—According to this question, $100 worth of goods are sold for $83 instead of $120. Now in order to make up the loss sustained by selling thus, we must gain $37 on $83; and by gaining on $63 in the same ratio, we find the loss sustained by selling for $63.

11. A merchant sold a quantity of coffee at 15 cents per pound, and by so doing lost 10 per cent., soon after he sold another parcel to the amount of $525, and gained 40 per cent.; how many pounds were there in the last parcel?

Ans. 2250 lbs. (0.)

12. Bought a quantity of goods for $480, and after keeping them 5 months sold them for $576, how much per cent. was gained by this transaction?

Ans. 17$\frac{3}{41}$. per cent. (5.)

NOTE.—6 per cent. a year will be 2$\frac{1}{2}$ per cent. for 5 months, that is, 2$\frac{1}{2}$ parts on 100 parts $= \frac{1}{40}$, hence $480 will amount in 5 months to ($\frac{41}{40}$ of $480;) now this $\frac{41}{40}$ of $480 must increase to $576, and 100 is to increase in the same ratio.

13. Bought 85 yards of broad cloth for $297.50, and sold it at 4.87\frac{1}{2}$ a yard, what was the gain per cent.?

Ans. 39$\frac{2}{7}$ per cent. (7.)

It is not necessary to find the cost per yard, but simply to indicate it,

Thus,

$$\begin{array}{r|l} & \$ \\ & \cancel{100} \\ \cancel{595} & \cancel{2} \\ 7 & \cancel{85} \\ \cancel{8} & 39 \\ \hline & 975 \end{array}$$

Ans. $\overline{139\frac{4}{7}}=39\frac{4}{7}$ per cent.

14. The interest on a certain note for 1 year and 9 months was \$49.87½, what was the principal, supposing the rate of interest to be 6 per cent. a year? Ans. \$475. (2.)

Remark.—\$100 gains \$6, and a greater sum will be required to gain \$49.87½; again, \$100 gains some interest in 1 year, and a less sum will effect the same having 1 year and 9 months to gain it in.

$$\begin{array}{r|r} \cancel{6} & \cancel{100} \\ \cancel{8} & \cancel{399} \\ \cancel{7} & 19 \\ & \cancel{4} \\ \hline \end{array}$$

Ans. \$475

15. What principal at 5 per cent. a year will gain \$35 in 16 months and 24 days? Ans. \$500. (0.)

16. A trader bought some goods for £18 and sold the same immediately for £25, on a credit of 4 months, what is gained at this rate per cent. a year?

Ans. £116 13s. 4d. on £100. (0.)

£7 is gained on £18—£100 will gain more.

£7 is the gain for 4 months—the gain for 12 months will be more.

17. A usurer put out \$75 on interest, and at the end of 8 months received for principal and interest \$79, at what rate per cent. a year did he receive interest?

Ans. 8 per cent.? (0.)

18. R gave 189 yards of linen at 6s. 8d. per yard, to C for 42 yards of cloth, what was the cloth per yard?

Ans. 30s. (0.)

19. A man purchased 7 pieces of muslin at \$13.75 per piece, but finding it somewhat damaged, he paid \$3.12½ a piece for having it dyed; how much per piece must it be sold for to gain 12 per cent. on the whole cost?

Ans. \$18.90. (0.)

20. A merchant found that by selling cloth from a certain

piece at \$2.75 a yard he gained 10 per cent., he then raised the price to \$3.25, what was his gain per cent. by selling at the last price? Ans. 30 per cent. (0.)

21. If I purchase 16 pieces of cloth at \$14 a piece, and sell 5 pieces at \$17 a piece, and 6 at \$15 a piece, at what price per piece must I sell the remainder to gain 12 per cent. on the whole? Ans. \$15.17$\frac{3}{5}$. (9.)

22. The sales of certain goods amount to \$1873.40, what sum is to be received for them, allowing $2\frac{1}{2}$ per cent. commission for selling, and $\frac{1}{4}$ per cent. on the money to be transmitted, to be deducted from the proceeds of the goods, after the commission is taken out?

Ans. \$1822$\frac{4}{401}$. (15.) One statement.

23. Bought a hogshead of molasses for \$112, but a number of gallons having leaked out, I sell the remainder for \$2.21$\frac{2}{3}$ per gallon, and by so doing lose 5 per cent.; how many gallons leaked out? Ans. 15 gallons. (0.)

NOTE.—$\frac{19}{20}$ of \$112 will be the amount the number of gallons remaining must be sold for.

24. If I sell cloth at \$5.60 per yard, and thereby lose 7 per cent., what should I gain or lose per cent, by selling it at \$6.25 per yard? Ans. $3\frac{89}{112}$ per cent. gain. (6.)

25. How many bushels of potatoes, at $37\frac{1}{2}$ cents a bushel, must be given for $387\frac{1}{2}$ pounds of beef, at 7 cents a pound?

Ans. $72\frac{1}{3}$ bushels. (0.)

26. A factor purchased 17 bales of cotton, each containing 4 cwt. 1 quarter, 5 pounds, at $11\frac{1}{2}$ cents a pound, what will his commission amount to, at $2\frac{1}{2}$ per cent.?

Ans. \$23.50$\frac{71}{80}$. (9.)

27. If \$875 in 7 years, 8 months, amounts to \$1478.75 at simple interest, what is the rate per cent. a year?

Ans. 9 per cent. (0.)

28. A merchant purchased goods for \$750 ready money, and sold them again for \$900, payable in 9 months; what did he gain, allowing money worth 6 per cent. a year?

Ans. \$111.248+ (15.)

29. If $27\frac{1}{2}$ cwt. of sugar be sold at \$12.50 per cwt., and there is gained 17 per cent., what was the first cost of the whole? Ans. \$293.80$\frac{40}{117}$. (16.)

PARTNERSHIP.

Partnership is the association of two or more persons in business, with an agreement to share the profits and losses in proportion to the amount of capital stock contributed by each. The company thus associated is called a firm.

1. Thompson & Williams are partners; Thompson has \$8400 invested in the joint stock, and Williams has \$5600, their business in one year is found to yield a profit of \$2800; what should be each one's share of this profit, as the whole amount of capital invested is \$14,000, and the gain \$2800, and as each one's share of the gain is to be in proportion to his capital?

Thompson's share will be $\frac{8400}{14000}=\frac{3}{5}$ of 2800, and Williams's $\frac{5600}{14000}=\frac{2}{5}$ of 2800.

$$\begin{array}{r|l} & \$ \\ & 28\cancel{0}\cancel{0} \\ \text{Or, } \cancel{1}\cancel{4}\cancel{0}\cancel{0}\cancel{0} & \cancel{8}\cancel{4}\cancel{0}\cancel{0} \\ \hline \text{Thompson's share} & \$1680 \end{array} \qquad \begin{array}{r|l} & \$ \\ & \cancel{2}\cancel{8}\cancel{0}\cancel{0} \\ \cancel{1}\cancel{4}\cancel{0}\cancel{0}\cancel{0} & 560\cancel{0} \\ \hline \text{Williams's share} & \$1120 \end{array}$$

Hence we see that each partner's share of the profits will have the same ratio to the whole gain that his *share* of the joint stock has to the whole stock.

2. Clark & Davis trade in company; Clark puts in \$950, and Davis \$640, they gain \$530; what is each one's share of the gain?

$$\text{Ans.}\begin{cases} \begin{array}{r|l} & \$ \\ & \cancel{5}\cancel{3}\cancel{0} \\ \cancel{1}\cancel{5}\cancel{9}\cancel{0} & 640 \\ \hline \text{Davis' share} & \$213.33\frac{1}{3} \\ \hline \cancel{1}\cancel{5}\cancel{9}\cancel{0} & \cancel{5}\cancel{3}\cancel{0} \\ & 950 \\ \hline \text{Clark's share} & \$316.66\frac{2}{3} \end{array} \end{cases}$$

3. Adams & Bender purchased goods to the amount of \$1760, of which Adams paid \$780, and Bender \$980. They gained \$550; what was each one's share of the gain?

Ans. $\begin{cases} \text{Adams's } \$243.75. \\ \text{Bender's } \ \ 306.25. \end{cases}$ (0.)

4. Three merchants traded together; A's stock was $1780, B's $2460, and C's $2960. They gain $840; what is each one's share of this gain?

Ans. { A's $207.66⅔, B's $287., C's $345.33⅓ } (0.)

5. A, B, and C freighted a vessel with 3680 barrels of flour, of which 1260 barrels belonged to A, 1620 to B, and 800 barrels to B; by reason of a storm at sea they were obliged to throw 184 barrels overboard; how much of this loss must each sustain?

Ans. { A 63 barrels., B 81 barrels., C 40 barrels. } (0.)

6. Anderson, Baker & Jones traded in company; A put in $5000, B put in $6500, and J put in $7500; they gain 40 per cent. on their capital, but receive the whole amount of their gains in bills, on which they are obliged to allow a deduction of 10 per cent.; how much was each man's net gain?

Ans. { A's $1800., B's $2340., J's $2700. } (0.)

7. Three merchants trading together lost goods to the value of $1920; now suppose A's stock was $2880, B's $11,520, and C's $4800, what share of this loss must each sustain?

Ans. { A $288., B $1152., C $480. } (0.)

8. A bankrupt is indebted to A $384.60, to B $786.75, to C $850.65, and his property is found to be worth $1348. If the whole of this property be divided among his creditors, how much will each receive?

Ans. { A $256.40. (5.), B $524.50. (3.), C $567.10. (3.) }

FELLOWSHIP HAVING REFERENCE TO TIME.

9. Two men hired a pasture for $16.80; A had 8 cows in the pasture for 4 months, and B had 16 cows in for 3 months; how much of the price of the pasture should each pay?

The pasturing of 8 cows for 4 months would be worth as much as the pasturing of 32 cows for 1 month, and the pas-

turing of 16 cows for 3 months would be worth as much as the pasturing of 48 cows for 1 month, hence the question is the same as if A had kept 32 cows, and B 48 cows in the pasture during 1 month, or during the season, and they must pay accordingly.

$$32+48=80 \text{ cows} \qquad \begin{array}{r|l} & \$ \\ & \cancel{1}\cancel{6}.\cancel{8}\cancel{0} \\ \cancel{8}\cancel{0} & \quad 32 \\ \hline \text{Ans. A paid} & \$6.72 \end{array} \qquad \begin{array}{r|l} & \$ \\ & \cancel{1}\cancel{6}.\cancel{8}\cancel{0} \\ \cancel{8}\cancel{0} & \quad 48 \\ \hline \text{B paid} & \$10.08 \end{array}$$

RULE.

Multiply each man's stock by the time it is continued in trade, then the share of each will have the same ratio to the whole expense, gain or loss, that each particular product has to the sum of all the products.

10. A, B, and C, traded together; A put in $70 for 10 months, B $180 for 5 months, C $200 for 3 months. They gain $132; what was each man's share of this gain?

$$\begin{array}{r} 70\times10=700 \\ 180\times\ 5=900 \\ 200\times\ 3=600 \\ \hline 2200 \end{array} \qquad \begin{array}{r|l} & \$ \\ & \cancel{1}\cancel{3}\cancel{2} \\ \cancel{2}\cancel{2}\cancel{0}\cancel{0} & 7\cancel{0}\cancel{0} \\ \hline \text{Ans. A} & \$42 \end{array} \qquad \begin{array}{r|l} & \$ \\ & \cancel{1}\cancel{3}\cancel{2} \\ \cancel{2}\cancel{2}\cancel{0}\cancel{0} & 9\cancel{0}\cancel{0} \\ \hline \text{B} & \$54 \end{array} \qquad \begin{array}{r|l} & \$ \\ & \cancel{1}\cancel{3}\cancel{2} \\ \cancel{2}\cancel{2}\cancel{0}\cancel{0} & 6\cancel{0}\cancel{0} \\ \hline \text{C} & \$36 \end{array}$$

11. A, B, and C formed a joint stock of $13,176, by which they gained $936; A's money was in trade 4 months, B's 5 months, and C's 13 months; what was each one's share of the gain, A's stock being $4680, B's $5616, and C's $2880?

$$\text{Ans.} \left\{ \begin{array}{l} \text{A's gain \$208.} \\ \text{B's gain \$312.} \\ \text{C's gain \$416.} \end{array} \right\} \text{(0.)}$$

12. Taylor, Martin & Smith trade in company; Taylor's capital, which was $345.75, was in trade 6 months, Martin's capital was $378, and was 5 months in trade, Smith's capital was $362.25, and was 8 months in trade. They gain $320.25; what was each man's share of the gain?

$$\text{Ans.} \left\{ \begin{array}{ll} \text{Taylor's} & \$96.81. \\ \text{Martin's} & \$88.20. \\ \text{Smith's} & \$135.24. \end{array} \right\} \text{(6.)}$$

13. Three men, P, S, and T, undertook to construct an embankment, to prevent the overflowing of some meadow, on the margin of the Delaware, for which they were to receive \$1875; P employed 18 workmen during 24 days, S employed 27 workmen 21 days, and T employed 16 men 36 days; how should the \$1875 be divided among the three undertakers?

Ans. { P \$514.28$\frac{4}{7}$. S \$675.00. T \$685.71$\frac{3}{7}$. }

(7.)
(0.)
(7.)

14. A, B, and C, made a stock for 12 months; A put in at first \$873.60, and 4 months after he put in \$96 more; B at first put in \$979.20, and at the end of 7 months he took out \$206.40; C put in at first \$355.20, and 3 months after he put in \$206.40, and five months after he put in \$240 more. At the end of 12 months their gain is found to be \$3446.40; what is each man's share of the gain?

Ans. { A's \$1334.825+ B's \$1271.614+ C's \$839.96+ }

(47.)
(47.)
(33.)

DUODECIMALS.

Duodecimals are fractions of a foot. The word is derived from the Latin word *duodecim*, meaning 12, a foot being divided into 12 equal parts, called inches, or primes, and the inches are again divided into 12 equal parts called seconds, &c.

1. If a room be 16 feet long, and 12 feet wide, how many square feet of boards will lay the floor, allowing the boards already dressed and fitted to each other?

If we might suppose the room 16 feet long, and 1 foot wide, 16 square feet would make the floor. If it were 2 feet wide, twice 16 feet would be required. If 3 feet wide, 3 times 16 feet, and if 12 feet wide, 12 times 16 square feet, that is, if we multiply the number denoting the *feet* in the *length* of the room by the number denoting the *feet* in the

width of the room, the product will be the number of square feet. Ans. $16 \times 12 = 192$ sq. feet.

2. How many square feet in a floor 16 feet 8 inches long, and 12 feet 6 inches wide?

$$
\begin{array}{r|l}
 & \text{Feet.} \\
3 & \cancel{50} \\
\cancel{2} & \;25 \\
\hline
 & 625 \\
\hline
\text{Ans.} & 208\tfrac{1}{3} \text{ feet.}
\end{array}
$$

3. What will be the cost of 12 boards, each 14 feet 8 inches long by 1 foot 3 inches wide, at 3 cents a foot?

$$
\begin{array}{r|l}
 & \text{Feet.} \\
\cancel{3} & 44 \\
\cancel{4} & \;5 \\
 & \cancel{12} \\
 & \;3 \\
\hline
\text{Ans.} & \$6.60
\end{array}
\qquad
\begin{array}{l}
14 \text{ ft. } 8 \text{ in.} = 14\tfrac{2}{3} = \tfrac{44}{3} \\
1 \text{ ft. } 3 \text{ in.} = 1\tfrac{1}{4} = \tfrac{5}{4}.
\end{array}
$$

4. What will the expense be for boards to make a fence 87 feet 6 inches long, and 5 feet 4 inches high, at $2\frac{1}{2}$ cents a foot? Ans. $\$11.66\frac{2}{3}$. (0.)

5. What must I pay for boards sufficient to cover the sides of a building 37 feet 6 inches long, 23 feet 4 inches wide, and 16 feet 8 inches high, at $2\frac{1}{2}$ cents a foot.
Ans. $\$50.69\frac{4}{9}$. (5.)

6. What will the plastering of the ceiling of 4 rooms, each 29 feet 4 inches long, and 22 feet 6 inches wide come to at 12 cents a square yard? Ans. \$35.20. (0.)

7. If a ceiling be 59 feet 9 inches long, and 24 feet 6 inches broad, what will the plastering of it come to at 9 cents a square yard? Ans. $\$14.63\frac{7}{8}$. (5.)

8. What will the paving of a court yard come to at $12\frac{1}{2}$ cents per square yard, the length 97 feet 6 inches, and the breadth 58 feet 10 inches? Ans. $\$79.67\frac{1}{72}$. (18.)

9. A workman plastered 3 rooms at 12 cents a square yard. The ceiling of each room was 20 feet by 16 feet 6

inches, and the walls were 9 feet 6 inches high. What did the plastering amount to, supposing 9 square feet to have been deducted for doors, windows, &c.

Ans. \$39.74. (7.) 2 statements.

10. The walls of a hall 54 feet 8 inches long, 47 feet 10 inches wide, and 16 feet 8 inches high, is to be papered at $4\frac{1}{2}$ cents a square yard, a deduction being made for 1 door 7 feet 9 inches high by 4 feet 4 inches wide, 6 windows 7 feet 2 inches high by 4 feet 3 inches wide, and a mop board 10 inches wide around the hall. What will be the expense for papering? Ans. \$$15.16\frac{5}{9}$. (55.) 4 statements.

11. How many feet of board measure, in a plank 17 feet 6 inches long, 1 foot 3 inches wide, and $3\frac{1}{2}$ inches thick?

NOTE.—By board measure we are to understand 1 inch as the standard thickness,—a plank, therefore, $3\frac{1}{2}$ inches thick will contain $3\frac{1}{2}$ times as many square feet, of board measure, as a board 1 inch thick of the same length and width.

Hence we state the question,

$$\text{Thus}\quad \begin{array}{r|l} 2 & 35 \\ 4 & 5 \\ 2 & 7 \\ \hline & 1225 \end{array}$$

Ans. $76\frac{9}{16}$ square feet.

12. How many feet of board measure in 4 planks, each 18 feet 9 inches long, by 1 foot 4 inches wide, $2\frac{1}{2}$ inches thick? Ans. 250 ft. (0.)

13. What must I pay for 9 planks, each 34 feet 4 inches long, by 2 feet 3 inches in width, and $3\frac{1}{4}$ inches thick, at $2\frac{3}{4}$ cents a foot, board measure? Ans. \$$62.13\frac{51}{64}$. (20.)

14. How many feet, board measure, will be required to lay a barn floor, 76 feet 10 inches long, by 58 feet 4 inches wide, with plank $2\frac{3}{4}$ inches thick, allowing 5 per cent. for waste? Ans. $12{,}974\frac{17}{342}$ ft. (29.)

15. A bridge 96 feet long, and 18 feet 8 inches wide, is to be laid anew with plank $3\frac{1}{4}$ inches thick, what will be the expense of the lumber, at $2\frac{1}{2}$ cents a square foot, board measure? Ans. \$131.04. (0.)

16. How many solid or cubic feet, in a piece of scantling 12 feet long, 7½ inches wide, and 8 inches deep.

Remark.—If this piece of scantling were 1 foot wide and 1 foot deep, it would then contain 12 cubic feet, and if it were 1 foot wide and only 8 inches deep, it would contain but ⅔ of 12 cubic feet, (8 inches being ⅔ of a foot,)—now since it would contain ⅔ of 12 feet, by being 8 inches deep and 1 foot wide, it will contain but ⅝ as much by being 7½ inches wide, (as 7½ inches is ⅝ of a foot,) hence, we state the question,

$$\text{Thus,}\qquad \begin{array}{r|l} & \cancel{12} \\ \cancel{8} & 5 \text{ cubic feet. Ans.} \\ \cancel{3} & \cancel{2} \end{array}$$

Again, suppose we look at it thus, a cubic foot being 12 inches long, 12 inches wide, and 12 inches deep, contains 1728 cubic inches, which is the result of multiplying together the numbers representing the inches, in the length, width, and depth or thickness. If, in like manner, we multiply together the numbers representing the inches in the length, width, and depth or thickness of the scantling, we will have as the result, the number of cubic inches it contains. Thus, 12 ft.=144 in. ×7½×8=8640 cubic inches, which is 5 times 1728, therefore the scantling contains 5 cubic feet. Hence, if the numbers representing the length, width, and depth, be multiplied together, the result will be the cubic contents in the same denomination as the factors.

17. How many cubic feet does a slab of marble, 4 feet 4⅜ inches long, 3 feet 7 inches wide, and 1 foot 4 inches thick, contain?

$$\begin{array}{r|l} 8 & 35 \\ \cancel{12} & 43 \\ 3 & \cancel{4} \\ \hline 72 & 1505(20\tfrac{65}{72} \text{ cubic ft. Ans.} \end{array}$$

18. What is a pile of wood 28 feet long, 8 feet wide, and 7 feet 6 inches high, worth, at $4.00 a cord

$$\begin{array}{r|l} & \cancel{28} \\ & \cancel{8} \\ 2 & 15 \\ \text{A cord } \cancel{128} \text{ ft.} & \cancel{4} \\ \hline \text{Ans.} & \$52.50 \end{array}$$

19. How many cords of wood in a pile 47 feet long, 32 feet wide, and 12 feet high? Ans. 141 cords. (0.)

20. What is the value of a pile of wood 97 feet 9 inches long, 24 feet wide, and 5 feet 4 inches high, at the rate of $2.50 a cord? Ans. $244.37½. (0.)

21. A tanner has a bark house 96 feet long, 37 feet 6 inches wide, and 18 feet 8 inches high to the beam, to which he wished to fill it with bark, at an average price of $7.25 a cord, what sum will it cost? Ans. $3806.25. (8.)

22. If a pile of stones be 54 feet long, its average width 16 feet 6 inches, and height 7 feet 3 inches; how many perches does it contain, a perch of stone being 24¾ cubic feet. Ans. 261 perches. (0.)

23. What is the value of a pile of stone, the average length, width, and height, being 25 feet, 12½ feet, and 5½ feet, in order, at 75 cents a perch? Ans. $52.08⅓. (0.)

24. A man has a cellar 28 feet long, 17 feet 6 inches wide, and 7 feet 4 inches deep, to be walled with stone, the wall to be 15 inches thick; what will the stone cost at 72 cents a perch? Ans. $22.96⅔. (4.)

NOTE.—You must deduct 4 times the thickness of the wall from twice the sum of the length and width, for if the side walls are supposed to extend the whole length of the cellar, the length of each end wall will be twice the thickness of the wall, less than the width of the cellar.

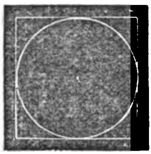
25. A agrees to dig a cellar for B, 34 feet 8 inches long, 27 feet 6 inches wide, and 6 feet 9 inches deep, at 9 cents per cubic yard, what will it amount to? Ans. $21.45. (0.)

26. A farmer has a bin 19 feet 6 inches long, 3 feet 10 inches wide, and 4 feet 8 inches deep, filled with wheat, which he wishes to sell at $1.08 a bushel, what is the whole worth at this rate? Ans. $302.73¾. (18.)

NOTE.—The standard gallon, dry measure, contains 268⅘ cubic inches, and of course the bushel 8 times 268⅘ cubic inches.

27. How many hogsheads of water will a vat 9 feet 9 inches deep, 7 feet 7 inches long, and 4 feet 6 inches wide hold? The gallon, wine measure (by which water is measured) contains 231 cubic inches. Ans. 39$\frac{32}{77}$ hogsheads. (11.)

28. A bushel measure is 18½ inches in diameter, and 8 inches deep, how many cubic inches does it contain?



Note.—In order to solve this question, take, instead of the bushel measure, a box 8 inches deep, and 18½ inches across each side, measured in the clear, and suppose the annexed figures, one to represent the bottom of the box, and the other that of the measure; then, if you multiply together the length width, (each 18½ inches) and depth of the box, the result will be the number of cubic inches contained by the box; but it is obvious that the bushel measure will not contain so much, since a quantity equal to that contained in the four corners of the box will remain after the bushel is filled. Now it has been ascertained that the area of a circle, whose diameter is 1 will be .7854, a little more than ¾, whilst the area of a square whose side is 1 foot or 1 inch, &c., will be 1 cubic foot, 1 cubic inch, &c. Hence if you multiply the contents of the box by the decimal .7854, you find the cubic inches in a bushel to be 2150.4252.

29. How many gallons of water will a tub that is 3 feet 8 inches in diameter, and 2 feet 3 inches deep hold?

Ans. 177.7248 gals. (10.)

30. How many gallons of water will a cistern 8 feet 3 inches deep, and 3 feet 6 inches in diameter hold?

Ans. 593.7624 gals. (3.)

INVOLUTION.

Involution is the raising of powers from any given number as a root.

A *power* is the number resulting from the multiplication of any given number called a root, a certain number of times in regular succession by itself; for example let=3 be multiplied into itself.

Thus, $3^1=3$ the root or first power.
$3^2=3\times3=9$ is the 2d power or square.
$3^3=3\times3\times3=27$ is the 3d power or cube of 3.
$3^4=3\times3\times3\times3=81$ is the 4th power of 3.
$3^5=3\times3\times3\times3\times3=243$ is the 5th power of 3.

The number denoting the power is called the index or exponent of the power—thus—3^5 denotes the fifth power of 3, and 5 is the index or exponent.

MENTAL OPERATIONS.

1. What is 4^2?
2. What is 6^3?
3. What is 9^2?
4. What is 24^2?
5. What is 7^3?
6. What is 5^4?
7. What is 6^4?
8. What is 7^4?
9. What is 8^4?
10. What is 9^4?
11. What is 10^4?
12. What is $(3\frac{1}{2})^3$? Ans. $(\frac{7}{2})^3=\frac{343}{8}=42\frac{7}{8}$
13. What is $(2\frac{1}{3})^2$? Ans. $(\frac{7}{3})^2=\frac{49}{9}=5\frac{4}{9}$.
14. What is $(4\frac{2}{3})^3$? Ans. $(101\frac{17}{27}.)$
15. What is $(5\frac{3}{5})^2$? Ans. $31\frac{9}{25}$ (0.)
16. What is $(2\frac{1}{2})^4$? Ans. $39\frac{1}{16}$ (0.)
17. What is $(2\frac{3}{4})^4$? Ans. $228\frac{49}{64}$. (9.)
18. What is $(4\frac{1}{2})^3$? Ans. $91\frac{1}{8}$. (0.)
19. What is $(17\frac{1}{2})^3$? Ans. $5359\frac{3}{8}$. (5.)
20. What is $(25)^4$? Ans. 390625. (3.)

NOTE.—The square of the second power will be the fourth power.

21. Multiply the second power of 4 by the second power of 4—thus, $4^2\times4^2=4^4=(4\times4)\times(4\times4)=256$.

22. Multiply the third power of 4 by the second power of 4—thus $4^3\times4^2=4^5=(4\times4\times4)\times(4\times4)=1024$.

23. Multiply the fourth power of 4 by the fourth power of 4—thus, $4^4\times4^4=4^8=(4\times4\times4\times4)\times(4\times4\times4\times4)=65536$.

24. Multiply the fifth power of 3 by the fourth power of 3, thus, $3^5\times3^4=3^9=(3\times3\times3\times3\times3)\times(3\times3\times3\times3)=19683$.

Hence, we see that the second power multiplied by the second power produces the fourth power; the third power multiplied by the third power produces the sixth power; the fourth power multiplied by the fourth power produces the eighth power, &c. &c.

Let the answers to as many of the following questions as possible be given by a purely mental operation.

25. What is the second power of 29, of 14, of 13, of 47, of 16?

26. What is the third power of 3, of 2, of 4, of 5, of 6, of 7, of 8?

27. What is the fourth power of 1, of 2, of 3, of 4, of 5, of 6, of 7, of 8?

28. What is the fifth power of 1, of 2, of 3, of 4, of 5, of 6, of 7, of 8, of 9?

29. What is the sixth power of 1, of 2, of 3, of 4, of 5, of 6, of 7, of 8, of 9?

30. What is the seventh power of 1, of 2, of 3, of 4, of 5, of 6, of 7?

31. Multiply the second power of 3 by the fourth power of 3, 4^2 by 4^3, 2^4 by 2^4, 5^3 by 5^5, 6^2 by 6^3, 8^3 by 8^2, 9^3 by 9^2 7^4 by 7^3.

32. If the third power of 4 be multiplied by the fourth power of 4, what power of 4 will the product be?

Ans. The seventh power of 4, that is, the sum of the exponents expresses the power.

33. What power of 6 will $6^4 \times 6^5$ give? Ans. 6^9.

34. What power of 8 is $8^3 \times 8^5$ equal to? Ans. 8^8.

35. What power of 7 is $7^5 \times 7^6$, of 4 is $4^7 \times 4^6$, of 9 is $9^8 \times 9^4$, of 5 is $5^7 \times 5^3$?

36. What power of 3 is $3^3 \times 3^4 \times 3^2$, of 8 is $8^4 \times 8^2 \times 8^3$, of 2 is $2^7 \times 2^8$? Since the *sum* of the exponents expresses the *power* resulting from the multiplication of *different* powers of a number together, and division is the opposite of multiplication, the difference of the exponents will express the power resulting from the division of one power by another. Thus, if we divide the fifth power of 8 by the third power of 8 we get the second power.

$$\frac{8^5}{8^3}=8^2 \text{ or } 64 \quad \text{or thus,} \quad \begin{array}{c|c} \cancel{8} & \cancel{8} \\ \cancel{8} & \cancel{8} \\ \cancel{8} & \cancel{8} \\ & 8 \\ & 8 \\ \hline \multicolumn{2}{c}{64} \end{array}$$

37. Divide 9^6 by 9^3, 8^7 by 8^4, 6^8 by 6^5.

38. Divide 17^4 by 17^2, 14^5 by 14^3, 13^6 by 13^2.

39. Divide $8^7 \times 8^4$ by 8^5, $4^3 \times 4^2 \times 4^4$ by 4^5.

40. Divide 4^3 by 4^8, 2^9 by 2^5, 3^7 by 3^7.

41. What is 8^0—observe that $8=8^1$, and since 8^2 divided by $8^1=8^1=\frac{\cancel{8}\,|\,\cancel{8}^2}{8}$, it follows that 8^1 divided by $8^1=\left(\frac{8^1|8^1}{1}\right)=8^0$.
Therefore $8^0=1$, $(26)^0=1$, $(36)^0=1$, that is, any number whose exponent is 0, is equal to 1, for the meaning of the the expression *is* the division of the number by itself.

42. Multiply 4^3 by 7^2. Ans. $4\times4\times4\times(7\times7)=3136$.

43. Divide 8^4 by 4^5.

$$\begin{array}{r|l} \cancel{4} & \cancel{8} \\ \cancel{4} & \cancel{8} \\ \cancel{4} & \cancel{8} \\ \cancel{4} & \cancel{8} \\ \cancel{4} & \\ \hline & 4 \end{array}$$

Ans. 4

44. Multiply $(12)^3$ by $(16)^2$.	Ans. 442368.	(0.)
45. Divide $(12)^5$ by $(16)^3$.	Ans. $60\frac{3}{4}$.	(3.)
46. Multiply $(1.7)^2$ by $(1.4)^2$.	Ans. 5.6644.	(0.)
47. Multiply $(4\frac{2}{3})^3$ by $(6\frac{3}{4})^2$.	Ans. $4630\frac{1}{2}$.	(7.)
48. Multiply $(8\frac{1}{2})^4$ by $(\frac{4}{17})^3$.	Ans. 68.	(0.)
49. Divide $(6.4)^3$ by $(1.6)^3$.	Ans. 64.	(0.)
50. Divide $(12.96)^2$ by $(36)^3$.	Ans. .0036.	(0.)
51. Divide $(1.225)^2$ by $(35)^3$.	Ans. .000035.	(0.)
52. Divide $(17\frac{1}{2})^3$ by $(8\frac{3}{4})^3$.	Ans. 8.	(0.)
53. Divide $(2.56)^2$ by $(48)^2$.	Ans. $.0028\frac{4}{9}$.	(0.)
54. Multiply $(1.75)^3$ by $(49)^2$.	Ans. $12867\frac{55}{64}$.	(21.)

EVOLUTION, OR THE EXTRACTION OF ROOTS.

Evolution, from the Latin (*evolutio*, the act of unfolding, or unrolling,) is the *reverse* of *involution*, and teaches to find the roots of any given powers.

The root of any number, is such a number as being multiplied into itself a certain number of times will exactly produce that number; thus, 6 is a root of 36, because when multiplied by itself it will exactly produce 36: 5 is a root of 125, because when used 3 times as a factor it will exactly produce 125: 6 is the second or square root of 36, because

it will produce 36, by being used twice as a factor,—5 is the third or cube root of 125, because it will exactly produce 125, by being used three times as a factor, hence the root is denominated the square, cube, &c., or, more properly, the 2d, 3d, &c. root, according to the number of times it must be used as a factor to produce the power whose root is required.

SQUARE ROOT.

The extraction of the square root is the finding of a number whose second power is the number whose root is required.

MENTAL EXERCISES.

1. What is the square root of 25? Ans. 5. Why? Say, because the second power of 5 is 25.
2. What is the square root of 81? Why?
3. What is the square root of 64? Why?
4. What is the square root of 144? Why?
5. What is the square root of 169? Why?
6. What is the square root of 256? Why;
7. What is the square root of 576? Why?
8. What is the square root of 400? Why?
9. What is the square root of 196? Why?
10. What is the square root of 324? Why?
11. What is the square root of 121? Why?
12. What is the square root of 841? Why?
13. What is the square root of 1521? Why?
14. What is the square root of 1296? Why?
15. What is the square root of 5625? Why?
16. What is the square root of 4096? Why?
17. What is the square root of 225? Why?
18. What is the square root of 289? Why?
19. What is the square root of 361? Why?
20. What is the square root of 484? Why?
21. What is the square root of 1024? Why?

22. What is the square root of 1444? Why?
23. What is the square root of 1156? Why?
24. What is the square root of 961? Why?
25. What is the square root of 1225? Why?
26. What is the square root of 676? Why?
27. What is the square root of 9801? Why?
28. What is the square root of 3481? Why?
29. What is the square root of 6084? Why?
30. What is the square root of 529? Why?
31. What is the square root of 625? Why?
32. What is the square root of 2401? Why?
33. What is the square root of 3025? Why?
34. What is the square root of 4225? Why?
35. What is the square root of 1936? Why?
36. What is the square root of 2025? Why?
37. What is the square root of 6561? Why?
38. What is the square root of 7225? Why?
39. What is the square root of 1681? Why?

The second power or square of any number will contain either twice as many figures as the root, or one less than twice as many.

Thus the second power of 32 is 1024, 4 figures—twice the number in the root.

The square of 31 is 961—3 figures, one less that twice the number in the root.

The square of 317 is 100489—6 figures, twice the number in the root.

The square of 316 is 99856—5 figures, one less than twice the number in the root.

The square of 3163 is 10004569—8 figures, twice the number in the root.

The square of 3162 is 9998244—7 figures, one less than twice the number in the root.

Now it is obvious that the *square* of any number of two figures greater than 31, must contain 4 figures, and that the square of any number of 2 figures less than 32, will contain but 3 figures.

Also that the square of any number of 3 figures greater than 316, must contain 6 figures, and the square of any number of 3 figures less than 317, will contain but 5 figures.

It is likewise evident from the above examples, that the square, or 2d power of any number of 4 figures greater than 3162, must contain 8 figures, whilst the square of any number of 4 figures less than 3163, will contain but 7 figures. Another fact may be inferred from the above examples, viz., If a number contains 4 figures, its root must be greater than 31, and if but 3 figures, its root will be less than 32. Likewise, if a number contains 6 figures, its root must be greater than 316; and if it contains 5 figures, its root will be less than 317.

And again, if a number contains 8 figures, its root must be greater than 3162, whilst if it contains but 7 figures, its root will be less than 3163, &c.

We may remark further, since the power, or square always contains either *twice*, or *one less* than twice as many figures as its root, it follows that the *root must* contain half as many figures as the power, when the power is an *even number*, or half as many as the number of figures in the power increased by one, when the power contains an odd number of figures.

If we multiply any number of two figures by itself and write each product in a separate column, the result thus presented will enable us to understand more fully the formation of the 2d power, or square, and to comprehend the process of resolving it into its factors, or finding its root.

Take for example 47, and separate the units and tens by a line.

Thus,

	4	7
	4	7
	28	49
16	28	

Two products appear in the second column, but this is only the same product written twice, 7 times 4, and 4 times 7 being the same. The whole product we see contains the square of the tens figure $(4)=16$, twice the product of the units and tens, $(7\times4\times2)=28\times2)$ together, with the square of the units figure $(7)^2=49$. Now if the square were always presented in this form the process of finding the square root would be a very simple process, as we would only need to take the root of the numbers in the right and left-hand columns. Thus the square root of 49 is 7, and the square

root of 16 in the left-hand column is 4 tens, these placed according to their numerical values express the root (47.)

It is important that the learner should fully understand and remember the *formation* of the square or 2d power, hence we will repeat:—The *square* of any number of 2 figures will contain, first, the *square* of the *tens* figure; secondly, *twice* the *product* of the *units* and *tens*; and thirdly, the *square* of the *units*.

EXAMPLES FOR MENTAL EXERCISE.

What numbers should occupy the middle column in the following statements?

1. | 25 | | 9 | Say, twice 15. Why? Because the square root of 25 is 5, (tens,) and the square root of 9 is 3, (units,) and the middle column must contain twice the product of the units and tens; therefore it must contain twice the product of 3 and 5, or twice 15.

2. What number in the middle column in this statement? | 49 | | 64 | Ans. Why?

3. What number in the middle column? | 81 | | 36 | Ans. Why?

4. What number in the middle column? | 36 | | 16 | Ans. Why?

5. What number in the middle column? | 25 | | 25 | Ans. Why?

6. What number in the middle column? | 64 | | 49 | Ans. Why?

7. What number in the middle column? | 16 | | 25 | Ans. Why?

8. What number in the middle column? | 64 | | 64 | Ans. Why?

9. What number in the middle column? | 81 | | 81 | Ans. Why?

10. What is this square fully expressed? | 36 | | 64 |

The number in the middle column must be tens, because the product of the units and tens simply expresses what the tens figure amounts to when repeated as many times as the units figure contains 1, and the number in the left-hand column must be hundreds, because the result of ten multi-

plied by ten, is 100, and therefore any number of tens multiplied by tens must be hundreds. Hence the square in the last example when fully expressed will be

64	or, 64
96	960
36	3600
4624	4624

11. What is this square fully expressed? | 16 | | 25 |
12. What is this square fully expressed? | 36 | | 16 |
13. Express this square in full, | 49 | | 16 | without expressing the component parts. Thus, 5476.
14. Express this | 25 | | 64 | in full without expressing the component parts.
15. Express this | 81 | | 25 | without the parts.
16. Express this | 36 | | 36 | in full.
17. Express this | 25 | | 36 | in full.

Express the first ten examples in like manner.

18. What number should occupy the right-hand column to complete this square.

16	28 28	

4 being the *root* of 16, the number required is such that its root multiplied by 4 will produce 28, this root we see at once must be 7; therefore the number required in the right-hand column is 49, the square of 7.

19. What number will complete this square?

36	30 30	

25

20. What number will complete this square?

49	21 21	

9

21. What number will complete this square?

64	80	

This example differs from the three preceding ones only by the number in the middle column being expressed entire,

instead of writing it in two parts. Now since the *root* of the number in the left-hand column, divided into *one* of those numbers in the middle *column*, will give the root of the number which should occupy the right-hand column, (that is, the units figure of the square root,) it follows that twice the root of the left-hand number, divided into the whole number in the middle column, will give this root, or the units figure of the square root required also.

22. What number will complete this square?

| 36 | 84 | | 49 root 67

23. What number will complete this square?

| 25 | 70 | | 49

24. What number will complete this square?

| 81 | 90 | |

25. What number will complete this square?

| 4 | 36 | | 81 — 24

26. What number will complete this square?

| 9 | 54 | | 81 — 24

27. What is the square root of this square when completed? | 16 | 56 | |

The square root of 16 is 4, twice 4 are 8; now 8 will go 7 times into 56, hence 7 is the units figure and 4 the tens, 47 therefore is the root required.

28. What is the square root of | 36 | 48 | | when completed?

29. What is the square root of | 49 | 70 | | when completed?

30. What is the square root of | 25 | 80 | | when completed?

31. What is the square root of | 9 | 48 | | when completed?

32. What is the square root of | 1 | 12 | | when completed?

33. What is the square root of | 1 | 14 | | when completed?

34. What is the square root of | 9 | 30 | | when completed?

35. What is the square root of | 4 | 28 | | when completed?

36. What is the square root of | 4 | 32 | | when completed?

37. What is the square root of | 64 | 16 | | when completed?

38. What is the square root of | 49 | 28 | | when completed?

39. What is the square root of | 36 | 12 | | when completed?

40. What is the square root of | 81 | 54 | | when completed?

We will now express the square entire, and it will be seen that the root may be ascertained by the same process slightly modified.

What is the square root of 441? The root of the left-hand or hundreds figure is 2, twice 2 are 4, 4 will go 1 time into the tens figure (4,) hence 2 is the tens figure of the root, and 1 the units, 21 is therefore the root.

What is the square root of 10.24? Here the root must contain 2 figures, since the power contains 4, (an even number,) the tens of the root must be derived from the 10, and the units from what remains after the square of the tens figure is taken out; 10 not being a square number we take the root of 9 for the tens figure of the required root. Taking 9 from 10 leaves 1 in the hundreds place, making a remainder of 124; now twice the root (3) will be 6, which if we omit the units figure in 124 will go twice into the 12, hence 2 is the units figure of the root, which is 32.

In order to be certain that this process gives the root, we must bear in mind that the given number or power must contain, 1st, the square of the tens figure of the root. Secondly, twice the product of the units and tens, and, lastly, the square of the units figure. Having taken 9, the square of the tens figure, from the left hand period, the remainder 124 must contain twice the product of the units and tens, together with the square of the units, this is seen to be the case at a glance, 12 being twice the product of 3×2 and 4, the square of 2. The whole process may be exhibited

Thus, 10.24(32 root.
9
62)124
124

Here, by taking twice the tens figure of the root for a divisor, and the remainder (124) omitting the units for a dividend, the quotient gives the units figure of the root; then by placing the 2 in the root and also to the right of the 6, (thus 62,) and multiplying the 62 by this root figure (2,) the product gives the square of 2, (4,) and twice 3 multiplied by 2, that is, twice the product of the units and tens.

41. What is the square root of 2916 ?

2916(54 root.
25
104)416
416

25 is the greatest square number in the left-hand period (29.)

42. What is the square root of 6084 ?

6084(78
49
148)1184
1184

43. What is the greatest square number contained in 42 ?
Ans. 36.

44. What is the greatest square number in 54 ? 49

45. What is the greatest square number in 72 ? 64

46. What is the greatest square number in 22? 16

47. What is the greatest square number in 45, in 28, in 84, in 62, in 34, in 14, in 12, in 77, in 93, in 6, in 3, in 15, in 17, in 24, in 26, in 35, in 37, in 48, in 51, in 63, in 65, in 80, in 82, in 99, in 101, in 8, in 10 ? What is the *square root* of the greatest square contained in 19 ? Ans. 4.

48. What is the root of the greatest square in 57 ?
Ans. 7.

49. What is the root of the greatest square in 73, in 22,

in 3, in 5, in 8, in 10, in 14, in 18, in 23, in 27, in 34, in 38, in 46, in 53, in 59, in 67, in 74, in 86, in 95, in 103?

The radical sign, written thus, $\sqrt{}$ and read square root, is used to express the square root of any number to which it is prefixed, thus, $\sqrt{36}=6$; that is to say, the square root of 36 is 6, $\sqrt{64}=8$, &c.

The mental process, by means of which we are enabled to answer correctly the preceding questions, involves the principle (slightly modified) from which we derive the rule for extracting the square root of numbers generally. In order to perceive this clearly, we should (commencing at the left hand) consider the first ascertained figure of the root, as so many tens, (whatever may be its real local value,) and the second, units; but when the second figure of the root is found, regard *this* as the *tens* figure, and the third as units. Then the third, when ascertained, takes in reference to the fourth, the place of tens, while the fourth is units; again the *fourth* in turn takes the place of *tens*, and the fifth is units, &c. This is simply taking two figures at a time, and operating according to their relative local value without reference to other parts of the root.

RULE FOR EXTRACTING THE SQUARE ROOT.

1st. Point off the given number into periods of two figures each, counting from the units figure for whole numbers, and from the decimal point for decimals.

2d. Find the greatest square number in the left-hand period, and write its *root* in the quotient, subtract this square number from the left-hand period, and to the remainder bring down the next period for a dividend.

3d. Double the root already found, for a divisor, and try how many times this divisor is contained in the dividend, (excepting the right-hand figure,) and place the result in the root, and also at the right hand of the divisor. Multiply the divisor thus increased by the last figure in the root, and subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.

4th. Double the ascertained root for a new divisor, and proceed as before till all the periods are brought down.

NOTE 1.—If there be a remainder after all the periods are brought down, annex ciphers to this remainder for decimals, and proceed as before till the root is obtained to a sufficient degree of exactness.

NOTE 2.—To extract the square root of a vulgar fraction reduce it to its lowest terms, and extract the square root of both numerator and denominator, if each be a complete square, otherwise change it to a decimal, and extract the root of the decimal.

50. What is the square root of 451584?

```
    .  .  .
    451584(672 root.
    36
   ----
127)915
    889
   --------
1342)2684
     2684
```

Here the given number contains 6 figures, hence the root must contain 3; to designate this we place a dot over the second figure in each period. 45 is not a square number, the greatest square number that does not exceed 45 is 36; 6 being the square root of 36 we place it in the quotient, and subtract 36 from 45, and to the remainder (9) bring down the next period (15) for a dividend; we now double 6, the root figure for a divisor, and try how many times 12 is contained in the dividend 915, excepting the right-hand figure (5,) we find it will go 7 times, and place 7 in the root, and also to the right of 12, making the divisor 127, and then multiply this divisor by 7, (the last ascertained root figure,) and subtract the product from the dividend 915, and to the remainder (26) bring down the next period 84 for a new dividend; we now double the ascertained root (67) for a new divisor, and try how many times this new divisor 134 is contained in the new dividend (2684,) excepting the right-hand figure (4,) we find it will go 2 times, we place the 2 in the root, and also to the right of the divisor, making the entire divisor 1342, now multiply, and the work is completed.

51. What is the square root of 106929? Ans. 327.
52. What is the square root of 263169? Ans. 513.
53. What is the square root of 22071204? Ans. 4698.
54. What is the square root of 133225? Ans. 365.

55. What is the square root of 104976? Ans. 324.
56. What is the square root of 213444? Ans. 462.
57. What is the square root of 848241? Ans. 921.
58. What is the square root of 84180625? Ans. 9175.
59. What is the square root of 5499025? Ans. 2345.
60. What is the square root of 34058896? Ans. 5836.
61. What is the square root of 96864964? Ans. 9842.
62. What is the square root of 1679616? Ans. 1296.
63. What is the square root of 47692836? Ans. 6906.
64. What is the square root of 36372961? Ans. 6031.
65. What is the square root of 10342656? Ans. 3216.
66. What is the square root of 1452025? Ans. 1205.
67. What is the square root of 1008016? Ans. 1004.
68. What is the square root of 74770609? Ans. 8647.
69. What is the square root of 987656329?
Ans. 31427.
70. What is the square root of 119550669121?
Ans. 345761.
71. What is the square root of 572199960721?
Ans. 756439.
72. What is the square root of 86806731242169?
Ans. 9317013.
73. What is the square root of 419112517321?
Ans. 647389.
74. What is the square root of 10.4976? Ans. 3.24.
75. What is the square root of 97.476129? Ans. 9.873.
76. What is the square root of 1.447209? Ans. 1.203.
77. What is the square root of 10.? Ans. 3.16227+
78. What is the square root of 4795.217310?
Ans. 69.247+
79. What is the square root of 8.93? Ans. 2.98831+
80. What is the square root of .000729. Ans. .027.

81. What is the square root of $\frac{7056}{9216}$? Here the root of both terms of the fraction being extracted gives $\frac{84}{96}=\frac{7}{8}$ Ans.; or the fraction might have been reduced first, and then the root extracted, thus, $\frac{7056}{9216}=\frac{49}{64}$ the root of which is $\frac{7}{8}$. Ans.

82. What is the square root of $\frac{2704}{4225}$? Ans. $\frac{4}{5}$.
83. What is the square root of $\frac{5625}{9025}$? Ans. $\frac{15}{19}$.
84. What is the square root of $\frac{1521}{3249}$? Ans. $\frac{13}{19}$.
85. What is the square root of $\frac{361}{576}$? Ans. $\frac{19}{24}$.

86. What is the square root of $\frac{1296}{6561}$? Ans. $\frac{4}{9}$.
87. What is the square root of $\frac{196}{841}$? Ans. $\frac{14}{29}$.
88. What is the square root of $\frac{676}{2916}$? Ans. $\frac{13}{27}$.
89. What is the square root of $\frac{2601}{7225}$? Ans. $\frac{3}{5}$.
90. What is the square root of $\frac{3025}{9801}$? Ans. $\frac{5}{9}$.
91. What is the square root of $\frac{1225}{4096}$? Ans. $\frac{35}{64}$.
92. What is the square root of $6\frac{1}{4}$? Ans. $\sqrt{\frac{25}{4}}=\frac{5}{2}=2\frac{1}{2}$.
93. What is the square root of $5\frac{1}{16}$? Ans. $\sqrt{\frac{81}{16}}=\frac{9}{4}=2\frac{1}{4}$.
94. What is the square root of $3\frac{1}{16}$? Ans. $1\frac{3}{4}$.
95. What is the square root of $11\frac{1}{9}$? Ans. $3\frac{1}{3}$.
96. What is the square root of $21\frac{7}{9}$? Ans. $4\frac{2}{3}$.
97. What is the square root of $42\frac{1}{4}$? Ans. $6\frac{1}{2}$.
98. What is the square root of $156\frac{1}{4}$? Ans. $12\frac{1}{2}$.
99. What is the square root of $37\frac{36}{49}$? Ans. $6\frac{1}{7}$.
100. What is the square root of $27\frac{9}{16}$? Ans. $5\frac{1}{4}$.
101. What is the square root of $318\frac{1}{36}$? Ans. $17\frac{5}{6}$.
102. What is the square root of $\frac{13}{14}$?

This fraction is a surd; that is, its root cannot be found exactly, we must therefore reduce it to a decimal and extract the root of the decimal to 3, 4, or more places, which will give an approximation to the exact root.

```
Thus,   14|13.00000000
          .92857142+(.9636+
           81
         18)1185
            1116
         1923)6971
              5769
        19266)120242
              115596
                4646 remainder.
```

103. What is the square root of $\frac{29}{36}$? Ans. .89752+
104. What is the square root of $8\frac{5}{7}$? Ans. 2.9519+
105. What is the square root of $7\frac{9}{11}$? Ans. 2.7961+
106. What is the square root of $6\frac{2}{5}$? Ans. 2.5298+
107. What is the square root of $\frac{478}{549}$? Ans. .93309+

APPLICATION OF THE SQUARE ROOT.

108. A man bought a farm for which he gave as many dollars per acre as there were acres in the farm, the sum he gave was $2025; how many acres were there in this farm?
Ans. 45 acres.

109. The expense of a picnic party was found to be $79.21, which was just as many cents for each individual as the whole number of persons in the company; how many persons were there? Ans. 89 persons.

110. A merchant shipped a quantity of flour to Liverpool, and received for the whole $3317.76, which was just as many cents per barrel as there were barrels in the whole; how many barrels were there? Ans. 576 barrels.

111. If a drove of cattle be sold for $1369, and the price per head be as many dollars as the number of head the drove contains, what is the price per head? Ans. $37.

112. A farmer is said to have raised in one year, on a single field, 4624 bushels of oats, which was just as many bushels per acre as there were acres in the field; how many acres did the field contain? Ans. 68 acres.

113. In a certain field there are 104,976 hills of corn, and the number of hills in the length and width of the field are the same, that is, the field is square; how many hills are there in one row, or one side of the field? Ans. 324 hills.

114. If an army of 531,441 soldiers be drawn up in a perfect square; how many will there be in each rank?
Ans. 729 men.

115. In a certain square orchard there are 15,625 trees; how many trees in a row? Ans. 125 trees.

116. A certain number of men gave 30s. 1d. for a charitable purpose, each man gave as many pence as there were men; how many men were there? Ans. 19 men.

117. What is the length of one side of a square field containing 40 acres? Ans. 80 rods.

118. An oblong field being three times as long as it is wide contains 30 acres; what is the length and width of this field?

If the length of the field were divided into three equal parts, the whole would make three square fields, each con-

taining 10 acres, or 1600 square rods, the square root of which is 40 rods, the width of the field, and 3 times 40, or 120 rods must be the length.

10 acres.	10 acres.	10 acres.

Or, we might reason thus,—If the field were as wide as it is long, it would then be a square, and contain 90 acres, or 14,400 square rods, the square root of which is 120 rods, the the length of the field, and one-third of 120 will be the width, 40 rods.

119. An oblong field containing 10 acres, is 4 times as long as it is wide; what is the length and width of this field?

Ans. Length 80 rods, width 20 rods.

120. It is required to lay out 30 acres of land, in an oblong form, having the width $\frac{3}{4}$ of the length; what will be the length and width of the field?

Ans. Length 80 rods, width 60 rods.

121. It is required to lay out 50 acres in an oblong form, having the width $\frac{5}{9}$ of the length; what will be the length and width? Ans. Length 120 rods, width $66\frac{2}{3}$ rods.

If the width were made equal to the length, the field would contain $\frac{9}{5}$ of 50, or 90 acres.

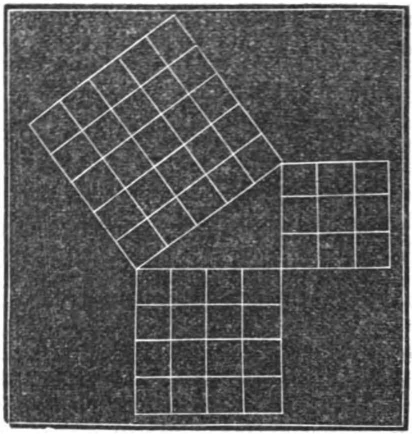
122. A field contains 24 acres, and the ratio of the length to the width is as 5 to 3; what is the length and width?

Ans. Length 80 rods, width 48 rods.

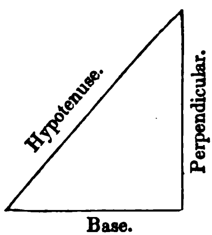
123. A carpenter wishing to support a perpendicular in a horizontal beam by a brace, has cut a mortice in the beam 4 feet from the perpendicular, and another in the perpendicular 3 feet from the beam; what must be length of the brace?

The annexed figure will show that the brace must be of such a length that its *square* will be equal to the *squares* of both the horizontal and perpendicular distances.

Thus, the square of the horizontal distance (4 feet) is 16, and that of the perpendicular distanee (3 feet) is 9, their sum is 25. The square of the brace is also 25, and the square root of 25 is 5 feet, the length the brace must be.



A triangle is a figure having three sides and three angles, when one of the angles would form a corner of a square the figure is called a *right angled triangle.*



The longest side of a right angled triangle is called the hypotenuse.

The square of the hypotenuse is equal to the sum of the squares of the other two sides.

Hence the square root of the sum of the squares of the base and perpendicular is equal to the hypotenuse.

2d. The square root of the difference of the squares of the

hypotenuse and base is equal to the perpendicular. (See the diagram on last page.)

3d. The square root of the difference of the squares of the hypotenuse and perpendicular is equal to the base.

124. In a right angled triangle the base is 12 feet in length, and the perpendicular is 16 feet; what is the length of the hypotenuse? Ans. 20 feet.

125. The wall of a certain fortress is 18 feet high, and is surrounded by a ditch 24 feet wide; how long must a ladder be to reach from the outside of the ditch to the top of the wall? Ans. 30 feet.

126. A ladder 40 feet long, resting on the ground at the distance of 24 feet from the bottom of a straight tree, and leaning against the tree, just reaches to the first limb; what is the length of the tree's trunk? Ans. 32 feet.

127. A certain castle which is 45 feet high, is surrounded by a ditch 60 feet broad; what length must a ladder be to reach from the outside of the ditch to the top of the castle? Ans. 75 feet.

128. Two ships sail from the same port, one goes due south 84 miles, the other due east 112 miles; how far are they from each other? Ans. 140 miles.

129. The distance between the lower ends of two equal rafters in the different sides of a roof is 32 feet, and the height of the ridge above the foot of the rafters is 12 feet; what is the length of a rafter? Ans. 20 feet.

130. If a hall be 48 feet long, and 36 feet wide, what is the distance between the opposite corners? Ans. 60 feet.

131. A line 27 yards long will exactly reach from the top of a fort to the opposite bank of a river, which is known to be 23 yards wide; what is the height of the fort? Ans. 14.142+ yds.

132. Suppose a ladder 60 feet long, be so planted as to reach a window 37 feet from the ground on one side of the street, and without moving it at the foot will reach a window 23 feet high on the opposite side; how wide is the street? Ans. 102.649+ feet.

133. A certain room is 28 feet long, 24 feet wide, and 18 feet high; how long must a line be to extend from one of the lower corners to an opposite upper corner? Ans. 41.036+ feet.

NOTE.—Take the square root of the sum of the squares of the three numbers.

134. The height of a tree growing in the centre of a circular island 100 feet in diameter, is 140 feet, and a line extending from the top of it to the further shore is 600 feet; what is the width of the stream, supposing the land on each side of the water to be level? Ans. 533.43+

135. A farmer wishes to set out a peach orchard, which shall contain 2400 trees, and to have the length of the orchard to the breadth in the ratio of 3 to 2; the distance between the trees is to be 7 yards, that is, from centre to centre; how many trees must there be in the length, and how many in the breadth of the orchard, and on how many square yards of ground will they stand?

Ans. 60 trees in length, 40 in breadth, stand on 112,749 sq. yds.

NOTE.—Observe that if the length were made equal to the breadth the orchard would contain but 1600 trees, and as one corner tree will belong to both length and breadth, the number of spaces between the trees will be one less than the number of trees in a row.

The square root of the product of any two numbers is a mean proportional between those numbers; 9 is a mean proportional between 3 and 27; that is, 9 has the same ratio to 27 that 3 has to 9.

136. Find a mean proportional between 2 and 8.

137. Find a mean proportional between 4 and 16.

138. Find a mean proportional between 2 and $12\frac{1}{2}$.

139. Find a mean proportional between 2 and $24\frac{1}{2}$.

140. Find a mean proportional between 5 and $9\frac{4}{5}$.

Ans. 7.

141. Find a mean proportional between 16 and 144.

Ans. 48.

142. Find a mean proportional between 25 and 169.

Ans. 65.

143. Find a mean proportional between 9 and 64.

Ans. 24.

144. Find a mean proportional between 6 and 24.

Ans. 12.

145. Find a mean proportional between 7 and 28.
Ans. 14.

146. Find a mean proportional between 8 and 72.
Ans. 24.

147. Find a mean proportional between $\frac{1}{2}$ and 8.
Ans. 2.

148. Find a mean proportional between $\frac{3}{4}$ and 3.
Ans. $1\frac{1}{2}$.

149. Find a mean proportional between $7\frac{1}{2}$ and 30.
Ans. 15.

150. Find a mean proportional between 24 and 96.
Ans. 48.

151. Find a mean proportional between $\frac{1}{2}$ and 98.
Ans. 7.

152. Find a mean proportional between 12 and 147.
Ans. 42.

153. Find a mean proportional between 180.625 and 10.
Ans. 42.5.

154. Find a mean proportional between 18 and 648.
Ans. 108.

155. Find a mean proportional between 17 and 153.
Ans. 51.

156. Find a mean proportional between .04 and .49.
Ans. .14.

157. Find a mean proportional between .09 and .64.
Ans. .24.

158. Find a mean proportional between .05 and 12.8.
Ans. .8.

EXTRACTION OF THE CUBE ROOT.

A cube is a solid body with six equal square sides, and containing equal angles.

A number is said to be cubed when it is raised to the 3d power, or is multiplied into its square.

The extraction of the cube root, is the finding of a number, which multiplied into its square, will produce the given

number. Thus, the cube root of 27 is 3, because 3 multiplied into its square (9) will produce 27; what is the cube root of 8, of 64, of 125, of 216, of 343, of 512, of 729?

Take a number of 2 figures, 24 for instance, and raise it to the 3d power, or cube it, writing each product in a separate column.

Thus,

T.	H.	tens.	units.	
		2	4	
		2	4	
		8	16	
	4	8		
	4	16	16	=2d power of 24.
		2	4	
	16	64		
	32	32		
8	48	96	64	

The left-hand column contains 8, the cube of 2, (the tens figure.) The next column contains 3 times the square of the tens figure (2) multiplied by the units figure, (4,) thus, 3 times $2^2 \times 4 = 48$. The next column to the right of this, contains 3 times the square of the units figure, (4,) multiplied by the tens figure, (2,) thus, 3 times $4^2 \times 2 = 96$. And the right-hand column contains the cube of the units figure, $(4)^3 = 64$.

These numbers when fully expressed will be,

First column,	64
Second column,	960
Third column,	4800
Fourth column,	8000
Complete cube	$=13824=(24)^3$.

The following proposition should be remembered by the learner:

The cube of any number of 2 figures contains the *sum* of the cube of the tens figure, 3 times the square of the tens figure multiplied by the units figure, 3 times the square of

the units figure multiplied by the tens figure and the cube of the units figure.

1. What numbers should occupy the column of tens, and hundreds to express all the *parts* of a complete cube in the following, and how will they stand when fully expressed, beginning with the left-hand column, and what is the complete cube and its root?

T.	H.	tens.	units.
64			27

The last question may be answered first, thus, the cube root of 64 is 4, (the tens figure,) and of 27 is 3, (the units figure,) hence the root is 43. Now the column of hundreds must contain 3 times the square of the tens figure multiplied by the units, thus, 3 times $4^2 \times 3 = 144$, and the column of tens must contain 3 times the square of the units figure, multiplied by the tens figure, thus, 3 times $3^2 \times 4 = 108$.

Ans. 144 should occupy the hundreds column, and 108 should occupy the tens column.

And the parts when fully expressed, beginning with the left-hand column will be,

Left-hand column,	64000
Second column,	14400
Third column,	1080
Right-hand column,	27
Complete cube,	=79507

2. What numbers should occupy the hundreds and tens columns in order to complete the following cube, and what will the parts be when fully expressed, and what will be the complete cube?

T.	H.	tens.	units.
343			64

3. Express the numbers required in the vacant columns to complete the following cube, also the parts in full, beginning with the left-hand column and the complete cube?

T.	H.	tens.	units.
125			512

4. What numbers are required in the vacant columns to complete the following cube, and what are the parts fully expressed, and also the complete cube?

T.	H.	tens.	units.
64	336		

Here the cube root of the left-hand period is 4, the tens figure of the root. Now the number in the hundreds column is composed of 3 times the square of the tens figure multiplied by the units figure; hence if we divide the number in the hundreds column by 3 times the square of the tens figure, the quotient must be the units figure of the root. Thus, 3 times $4^2=48$ divided into 336 gives a quotient of 7, which is the units figure of the root. Having now obtained the units figure, and knowing that the tens column must contain 3 times the square of the units figure multiplied by the tens figure, the number in the tens column we see must be 3 times $7^2 \times 4=588$, and in the units column 343, and the parts fully expressed will be, commencing at the left-hand column.

Left-hand column,	64000
Second column,	33600
Third column,	5880
Fourth or units column,	343
And the complete cube,	=103823

5. What numbers are required in the vacant columns to complete the following cube, and what are the parts fully expressed, and the complete cube, also the root?

T.	H.	tens.	units.
216	216		

6. Find in like manner, the parts to complete the following cube.

T.	H.	tens.	units.
343	294		

The following table of cube numbers should be remembered by the learner.

$1^3=1$	$4^3=64$	$7^3=343$
$2^3=8$	$5^3=125$	$8^3=512$
$3^3=27$	$6^3=216$	$9^3=729$

7. What is the greatest cube number that does not exceed 47? Ans. 27. The cube number next higher than 27 being 64 which exceeds 47.

8. What is the greatest cube number that does not exceed 180? . Ans. 125.

What is the greatest cube number that does not exceed 100, 200, 300, 400, 500, 600, 700, 37, 24, 7, 43, 162, 29, 816, 214, 48, 17, 12, 465, 379, 248, 875, 938, 2, 5, 19, 24?

9. What is the *cube root* of the greatest cube number that does not exceed 78? Ans. 4, the cube root of 64.

What is the cube root of the greatest cube number that does not exceed 249, 462, 314, 847, 363, 586, 435, 798, 914, 367, 632, 413, 275, 752, 687, 16, 9, 7, 5, 4, 13, 30?

The third power or cube of any number cannot contain more than three times the number of figures in the root, and at least but *two less* than three times the number in the root; take for instance the cube of the greatest number expressed by *two* figures, that is the cube of $99=970299$, (six figures,) being three times the number expressed in the root. Again, take the *least* number expressed by two figures, that is, the cube of $10=1000$, (four figures,) two less than three times the number expressing the root. The learner is now presumed to be prepared to understand the nature and application of the following rule for extracting the cube root.

RULE.

1st. Point off the given number into periods of three figures each, beginning at the units place.

2d. Find the *root* of the greatest *cube*, that does not exceed the left-hand period, which place in the quotient, and subtract the cube from the left-hand period, and to the remainder bring down the next period for a dividend.

3d. Take three times the square of the root for a defective divisor.

4th. Try how many times the defective divisor is contained in the dividend, omitting the two right-hand figures, and place the result in the quotient, and its square to the *right* of the defective divisor, supplying the place of tens with a cipher, if the square be less than ten. Now complete the divisor by adding thereto 30 times the product of the root previously found, by the figure last placed in the root.

5th. Multiply the divisor thus completed by the figure last placed in the root, and subtract the product from the dividend, and to the remainder bring down the next period for a new dividend, and thus proceed till the whole root is extracted.

NOTE 1.—Defective divisors after the first may be found, simply by adding to the last complete divisor, the number which completed it, and twice the square of the last figure in the root.

NOTE 2.—When upon trial it is found that the dividend does not contain the defective divisor, a cipher must be placed in the root, and two ciphers to the right of the defective divisor, and the next period brought down for a new dividend.

NOTE 3.—The cube root of a vulgar fraction is found by extracting the root of the numerator and denominator separately; but if the fraction be a surd, it must be changed to a decimal, and the root of the decimal extracted.

10. Extract the cube root of 389017?

	389017(73 root.
	343
Defective divisor, $7^2\times3=14709$	)46017
and square of 3. $7\times3\times30=630$	46017
Complete divisor$=15339$	

Here we find 343, the root of which is 7, to be the greatest cube that does not exceed 389. This cube being subtracted from 389 leaves a remainder of 46, to the right of which we bring down the next period 017, making 46017; now we take 3 times the square of 7, the root$=147$, for a defective divisor, and find it will go 3 times into 46017, if we omit the two right-hand figures (17,) we place this 3 in the quotient and its square (9) to the right of the defective divisor; and the square being less than 10, we write a cipher in the tens place. Now 30 times $7\times3=630$ completes the divisor.

11. Extract the cube root of 15252992.

	1216	15252992(248
30 times $2 \times 4 =$	240	8
Complete divisor,	1456	)7252
Number that completed it,	240	5824
Twice $4^2 =$	32	1428992
	172864	1428992
30 times $24 \times 8 =$	5760	
Complete divisor	178624	

12. Extract the cube root of 41421736. Ans. 346.
13. Extract the cube root of 206425071. Ans. 591.
14. Extract the cube root of 74088. Ans. 42.
15. Extract the cube root of 34328125. Ans. 325.
16. Extract the cube root of 373248. Ans. 72.
17. Extract the cube root of 21024576. Ans. 276.
18. Extract the cube root of 84604519. Ans. 439.
19. Extract the cube root of 65890311319. Ans. 4039.
20. Extract the cube root of 178263433152. Ans. 5628.
21. Extract the cube root of 890277.128. Ans. 96.2.
22. Extract the cube root of 451.217663. Ans. 7.67.
23. Extract the cube root of 164359469195433. Ans. 54777.
24. Extract the cube root of 12.977875. Ans. 2.35.
25. Extract the cube root of .001906624. Ans. .124.
26. Extract the cube root of 236029032. Ans. 618.
27. Extract the cube root of 2. Ans. 1.2599+
28. Extract the cube root of 171.46776406. Ans. 5.555+
29. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.
30. What is the cube root of $\frac{512}{729}$? Ans. $\frac{8}{9}$.
31. What is the cube root of $\frac{59319}{68921}$? Ans. $\frac{39}{41}$.
32. What is the cube root of $12\frac{19}{27}$? Ans. $2\frac{1}{3}$.
33. What is the cube root of $31\frac{15}{343}$? Ans. $3\frac{1}{7}$.
34. What is the cube root of $15\frac{5}{8}$? Ans. $2\frac{1}{2}$.
35. What is the cube root of $9\frac{1}{8}$? Ans. 2.092+

APPLICATION.

36. What is the difference between the cube root of 729 and the square root of the same number? Ans.

37. What is the difference between the cube root of 1331 and the square root of 121? Ans.

NOTE.—The cube root of any number which is a complete cube, may be found by a very simple mental process; when the root does not exceed two figures, thus in the cube of 1, 4, 6, and 9, the right-hand figure in the cube will be the same as the root. But 3 in the cube and 7 in the root reciprocate, and the same is true of 2 and 8. Hence if the units figure of the *cube* be 1, 4, 6, or 9, the units figure of the *root* will be the same; but if the units figure of the cube be 3, that of the root will be 7, and if the units figure of the cube be 7, that of the root will be 3, and 2 and 8 have the same property.

38. What is the cube root of 103.823? Here the *root* of the greatest cube in the left-hand period is 4, and the units figure of the cube is 3, therefore the units figure of the root must be 7, hence the root is 47.

39. What is the cube root of 389,017, of 132,651, of 658,503, of 970,299, of 2197, of 15,625, of 9261, of 531,441, of 175,616, of 148,877, of 238,328, of 405,224, of 24,389, of 97,336, of 185,193, of 17,576, of 474,552, of 91,125, of 195,112, of 912,673?

40. The contents of a cubical cellar is 21,952, what are the length, breadth, and depth? Ans. 28 feet.

41. A certain stone of a cubical form contains 474,552 solid inches, what is the superficial contents of one of its sides? Ans. 6084.

42. Required the side of a cubical vessel that shall contain 80 gallons, each 231 cubic inches. Ans. 26.43+in.

All solid bodies have the same ratio to each other, as the cubes of their diameters, or similar sides.

43. If a globe of gold one inch in diameter be worth $920, what is the value of a globe $3\frac{1}{2}$ inches in diameter?

Ans. $5145.

44. There are two marble statues of the same form but differing in size, one is 5 feet high and weighs 750 pounds, the other is 7 feet high; what will be its weight?

Ans. 2058 lbs.

46. If a ball 3 inches in diameter weighs 4 pounds, what will the weight of another ball of the same metal 9 inches in diameter be? Ans. 108 lbs.

$$\begin{array}{r|l} & \text{lb.} \\ & 4 \\ \cancel{3} & 9 \\ \cancel{3} & \cancel{9} \\ \cancel{3} & \cancel{9} \\ \hline \text{Ans.} & \$108\text{ lbs.} \end{array}$$

47. If a ball 4 inches is diameter weighs 9 pounds, what is the diameter of a ball weighing 72 pounds?

Ans. 8 inches.

$$\begin{array}{r|l} & \text{lb.} \\ & 4^3=64 \\ \cancel{9} & \cancel{72} \\ \hline \end{array}$$
$$\sqrt[3]{512}=8 \quad \text{Ans.}$$

48. There are two stacks of hay of precisely the same shape, one is $12\frac{1}{2}$ feet high, and contains $7\frac{1}{2}$ tons, the other is $18\frac{3}{4}$ feet high; how many tons does it contain?

Ans. $25\frac{5}{16}$ tons.

49. If a man can dig a cellar, which will measure 8 feet every way, in $2\frac{1}{2}$ days, how long time should he require to dig a similar cellar 12 feet every way? Ans. $8\frac{7}{16}$ days.

50. What is the difference between half a solid foot and a solid half foot? Ans. 648 cubic inches.

To find two mean proportionals between two given numbers, divide the greater by the less, and find the cube root of the quotient, then multiply this root by the least of the given numbers for the least mean, and the least mean by the same root for the greater mean.

51. What are two mean proportionals between 4 and 108?

Ans. 12 and 36.

52. What are two mean proportionals between 7 and 448?

Ans. 28 and 112.

53. What are two mean proportionals between 8 and 216?

Ans. 24 and 72.

54. What are two mean proportionals between 2 and 128? Ans. 8 and 32.

55. What are two mean proportionals between 17 and 2125? Ans. 85 and 425.

56. What are two mean proportionals between 56 and 12,096? Ans. 336 and 2016.

57. What are two mean proportionals between 76 and 166,972? Ans. 988 and 12,844.

58. The solid contents of a globe 21 inches in diameter, are 4849.0596; what is the diameter of a globe whose solid contents are 11494.0672 solid inches? Ans. 28 inches.

59. In a cubical foot, how many cubes of 3 inches, of 4 inches, of 6 inches, and 2 inches?

Ans. 64, 3 inch cubes; 27, 4 inch cubes; 8, 6 inch cubes; and 216, 2 inch cubes.

60. What must be the length of a side of a cubical box to contain just one bushel? Ans. 12.907+inches.

61. What are the inside dimensions of a cubical bin, that will hold 190 bushels of grain?

Ans. 6 feet, 2.204+inches.

NOTE—The roots of the 4th, 6th, 8th, 9th, and 12th powers may be extracted as follows:

For the 4th root take the square root of the square root.

For the 6th root take the square root of the cube root.

For the 8th root take the square root of the 4th root.

For the 9th root take the cube root of the cube root, and for the 12th root take the cube root of the 4th root.

In the 5th and 9th powers the *units* figure will be the same as the *units* figure of the root.

ARITHMETICAL PROGRESSION.

A series of numbers which uniformly increase or decrease by the addition or subtraction of the same number, is called an arithmetical progression, or progression by difference; and the number by which the terms increase is called the *common difference.*

The numbers composing the series are called the *terms* of the *series*, or progression.

The first and last terms are called the *extremes*, and the other terms the *means*.

When the numbers increase they form an ascending series, but when they decrease, a descending series, or progression. Thus, 1, 2, 3, 4, 5, 6, 7, &c., form an ascending series, and 7, 6, 5, 4, 3, &c., form a descending series.

In any series of numbers in arithmetical progression, the sum of the two extremes is equal to the sum of any two terms equally distant from the extremes, or to twice the middle term, when the number of terms is odd. Thus in the series 1, 2, 3, 4, 5, 6, 7, 8, 9, &c., the sum of the extremes 1 and 9, is equal to the sum of 2 and 8, or 3 and 7, or 4 and 6, or twice 5, the middle term.

The most important application of the principle of progression, is the finding of the sum of the series.

NOTE.—A curious *property* of *odd* numbers, and one that is not generally known, (nor has it *ever* that we are *aware* of, been applied to any practical purpose,) will be of considerable service, by way of enabling us to find the sum of any series in arithmetical progression. It is this, the sum of any series of *odd* numbers, commencing with 1, gives the square of the number of terms in that series. Thus, 1 and 3 are 4—the square of 2—(the number of terms,)—1, 3, and 5 are 9—the square of 3—(the number of terms,)—1, 3, 5, and 7 are 16—the square of 4—(the number of terms,)—1, 3, 5, 7, and 9 are 25, the square of 5—(the number of terms, &c.) Hence if we wish to find the sum, say of 13 terms of such a series, we have only to recollect that the square of 13 is 169, and we have the sum required.

What is the sum of 18 terms of the series, 1, 3, 5, &c., of 14 terms, of 17 terms, of 24 terms, of 35 terms, of 47 terms, of 29 terms, of 39 terms, of 34 terms, of 15 terms, of 45 terms, of 57 terms, of 75 terms?

Take now a series of numbers, commencing with 2, instead of 1, (increasing by the same common difference,) and compare the result.

$$\text{Thus, } \begin{cases} 2+4+6+8+10+12=42=\text{sum of the series.} \\ 1+3+5+7+\ \ 9+11=36=\text{sum of the series.} \end{cases}$$
$$\overline{6}\ \text{differ. of the sums.}$$

equal to the number of terms in either series.

Again, take a series, commencing with 3, instead of 1, increasing by the same common difference, and compare the results.

Thus, $\begin{cases} 3+5+7+9+11+13+15=63=\text{sum of series.} \\ 1+3+5+7+\ 9+11+13=49=\text{sum of series.} \end{cases}$

$\overline{14}$ difference is equal to twice the number of terms in either series.

Now observe that when the series commences with 2, instead of 1, the *sum* of any two terms in this series, will exceed *either* of those terms by *one* more than the excess of the sum of the corresponding terms in the first series above either of the terms added; and when the first term is 3, instead of 1, the sum of any two terms in *this* series will exceed either of *those* terms by 2 more than the excess of the sum of the corresponding terms in the *first* series above either of the terms added; and if the first term were 4, this difference of increase would be 3, and if the first term were 5, it would be 4, &c. Hence the *sum* of any number of terms of a series whose common difference is 2, will *exceed* the *sum* of an *equal number* of *terms* of the *natural* series of odd numbers, by as many times the number of terms in the series as the first term of the given series is greater than 1; or since the sum of the natural series of odd numbers is equal to the square of the number of terms, the sum of any *other* series whose common difference is 2, will exceed the *square* of the *number* of *terms*, by as many *times* the *number* of *terms* as the first term is greater than 1. From the above illustrations we deduce the following rule for finding the sum of any number of terms of an ascending series in arithmetical progression.

RULE.

To the *square* of the *number* of *terms add* the *number* of *terms* for the *sum* of the series, when the first term is 2, and the common difference the same; but when the common difference exceeds 2, increase the sum thus found in the ratio of that excess, and *to* or *from* the *result add* or *subtract*, as many times the number of terms, as the first term

is (units) greater or less than the common difference, and you have the sum required.

NOTE.—When the first term is 1, and the common difference 1, the sum of course will be half of that found by the first part of the rule; and when the first term is 1, and the common difference is 2, the sum will be the square of the number of terms.

1. Bought 15 yards of linen, at the rate of 2 cents for the first yard, 4 for the second, 6 for the third, &c.; what was the cost of the whole? Ans. $2.40.

Here we have only to recollect that the square of 15 is 225, and that if 15 be added to the square the sum will be 240, and we obtain the answer by a very simple mental operation.

NOTE.—We will again suggest to the learner never to use figures to solve a question, if he can possibly effect the solution by a purely mental operation, as by pursuing this course the *reasoning* and *retentive* powers of the mind will be constantly expanding and improving, whilst at the same time his progress in the acquisition of knowledge will necessarily be much more rapid than that of one who occupies so large a portion of his time, as is generally taken up in the mere mechanical process of making figures.

2. 25 persons agree to contribute to a charitable institution, in the following proportions, viz., the first is to give $2, the second $4, the third $6, and so on; what will be the whole sum contributed? Ans. $650.

3. The clocks of Venice are said to go to 24 o'clock; how many strokes must such a clock strike in 24 hours? Ans. 300 strokes.

4. How many strokes does a common clock strike in 12 hours? Ans. 78 strokes.

5. A farmer offers to sell 85 sheep, at the rate of 5 cents for the first sheep, 10 cents for the second, 15 cents for the third, and so on, increasing 5 cents on the price of each; what will he get for the whole at this rate? Ans. $182.75.

6. A butcher bought 48 oxen, and agreed to pay $1 for the first, $3 for the second, $5 for the third, and so on; what did he pay at this rate for the whole? Ans. $2304.

7. A man bought 50 yards of cloth, for which he was to pay 6 cents for the first yard, 9 cents for the second, 12 cents

for the third, and so on, increasing by the common difference 3; how much did he pay for the whole? Ans. $39.75.

8. 16 persons gave charity to a poor man, the first gave him 5 cents, the second 9, and so on, increasing by the common difference 4; how much did he received in all?

Ans. $5.60.

9. A merchant bought 36 yards of cloth, and gave for the first yard 8 cents, for the second 12 cents, and so on, increasing by the common difference 4; how much did he pay for the whole? Ans. $28.08.

10. Suppose 100 stones are placed in a right line 3 yards distant from each other, and the first 3 yards from a basket; what distance must a person travel to gather them singly into the basket? Ans. 17 miles, 380 yds.

11. A new road is to be marked out by 1000 stakes, to be driven into the ground at the distance of 22 feet from each other; the stakes are placed in a pile 22 feet from the point where the first one is to stand. Now suppose a man to start from this point and to take from the pile one stake at a time, and drive it into the ground, and continue the operation until he has driven the whole number in a direct line at the proper distances; what length of time would he require to complete the job, supposing him to walk 39 miles a day upon an average? Ans. $106\frac{1}{1}\frac{7}{8}$ days.

12. A merchant sold 1000 yards of linen, at 2 pins for the first yard, 4 for the second, and 6 for the third, &c., increasing two pins every yard; how much did the linen produce when the pins were afterwards sold at 12 for a farthing?

Ans. £86 17s. $10\frac{1}{6}$d.

13. If a number of dollars were laid in a straight line for the space of a mile, a yard distant from each other, and the first a yard from a chest, what distance would a man travel who, starting from the chest, should pick them up singly, returning with them one by one to the chest?

Ans. 1761 miles. (O.)

14. Suppose a number of stones were laid a rod distant from each other, for a distance of thirty miles, and the first stone a rod from a basket, what distance will a man travel who gathers them up singly, returning with them one by one to the basket? Ans. 288,090 miles 2 rods.

NOTE.—The entire distance from the basket to the furthest stone, will be one rod more than thirty miles. The solution requires 74 figures by the common method; our solution requires but 15.

15. A debt is to be paid at 16 different payments in arithmetical progression; the first payment to be \$14, and the last \$100; what is the common difference of the payments?
Ans. \$5.73⅓.

NOTE.—The common difference is equal to the quotient of the difference of the extremes divided by the number of terms less one.

16. A man agrees to travel from Philadelphia to a certain place in 16 days, and to go but 4 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 79 miles, required the daily increase, and what is the whole distance?
Ans. { Daily increase 5 miles, and whole distance 664.

17. A debt is to be paid at 8 different payments in arithmetical progression, the first payment to be \$21, and the last \$175, what was the common difference, and what is the whole sum?
Ans. { Common difference, \$22, and whole sum, \$784.

18. A man going a journey travelled 8 miles the first day, and increased his journey 4 miles every day, until the last day's journey was 56 miles; how many days did he travel?
Ans. 13 days.

NOTE.—Since the difference of the extremes divided by the number of terms less one gives the common difference, it follows that the difference of the extremes divided by the common difference will give the number of terms less one, to which if 1 one be added we have the number of terms.

19. A farmer sold a quantity of oats at the rate of 4 cents for the first bushel, and the price of every bushel after the first was to increase by the common difference of 3 cents, until the price of the last bushel should be 76 cents; how many bushels did he sell, and what was the average price per bushel?
Ans. { Sold 25 bushels. Average price 40 cents a bushel.

GEOMETRICAL PROGRESSION.

Any series of numbers, the terms of which increase by a common multiplier, or decrease by a common divisor, are said to be in geometrical progression.

As	2,	4,	8,	16,	32,	64,	128, &c.
Or	128,	64,	32,	16,	8,	4,	2.

The former is called an ascending series, and the latter a descending series.

The multiplier or divisor by which the series is increased or decreased is called the *ratio.*

The first and last terms of the series are called the *extremes*, and the other term the *means*.

In any geometrical series, the second term is found by multiplying the first term by the ratio, the *third* term by multiplying the *second* term by the ratio; the *fourth* term by multiplying the *third* term by the ratio, &c. Hence any term in an increasing series, may be found by multiplying the first term by that power of the ratio which is denoted by the number of terms less one, for instance the fifth term in the series, 2, 6, 18, &c., will be found by multiplying the fourth power of 3 (the ratio) by the first term, (2) thus, $3^4 \times 2 = 162$.

Again, take the series,	2,	6,	18,	54,
And multiply each term by the ratio,	6,	18,	54,	162.

Here the last term in the second series, that is, (162) is the result of multiplying the last term in the first series by the ratio; now if we subtract the first series from the second, the remainder will be twice the first, because the second is three times the first; if we divide by 2, therefore, we get the sum of the given series—and we see that to subtract the first from the second, we have only to diminish the last term in the second by (2) (the first term of the first.) Hence the

RULE.

Raise the ratio to the power whose index is one less than the number of terms which multiply by the first term, the

product will be the last or greater extreme 2—multiply the last term by the ratio, from the product subtract the first term, and divide the remainder by the ratio less one for the sum of the series, or raise the ratio to a power equal to the number of terms; subtract one from that power, multiply the remainder by the first term, and divide the product by the ratio less one for the sum of the series.

1. Sold 14 yards of broad cloth, at 2 cents for the first yard, 6 cents for the second, 18 cents for the third, and so on in geometrical progression; what did the whole amount to?

Here the ratio is 3.

Powers,	1st.	2d.	3d.	4th.	5th.	6th.
Ratio,	3,	9,	27,	81,	243,	729.

The square or second power of 81 is 6561=8th power, which multiply by 729 (6th power,) 6561×927 will give the 14th power=(47829.69) from which if we subtract 1, and multiply the remainder by the first term, and divide the product by the ratio less one, the result will be the sum of the series or answer to the question, but as the first term is 2 and the ratio less one (3—1) is also (2,) we have the answer, $47,829.68, simply by subtracting 1 from the 14th power of the ratio.

2. A thresher wrought 20 days, and was to receive 4 grains of wheat for the first day's labour, 12 for the second, 36 for the third, &c.; how much did his wages amount to, allowing 7680 grains to make a pint, and the whole disposed of at $1 a bushel?

Here 3 is the ratio, the 4th power of which is 81, and the second power of 81 gives the 8th power of 3 (the ratio)=6561; and the second power of (6561) gives the 16th power of the ratio, which being multiplied by 81 (the 4th power) gives the (20th.)

Thus, 6561×1656

43046721×81=16th power.

384|0)3486784440|0=20th power less 1.

908016+pints=14187¾ bushels, which at one dollar per bushel amounts to $14187.75.

NOTE.—After raising the ratio to the 20th power, we have (according to the rule) to subtract 1; multiply the remainder by the first term, and divide the product by the ratio less 1; but since the first term is (4) and the ratio less 1 is 2 = (3—1) it is only necessary to multiply by 2; but as the product after multiplying by 2 is to be divided by 7680, we omit the 2 as a multiplier, and divide the 20th power by 3840 = ½ (of 7680.)

See Pike's Arithmetic, page 175, for a solution of this question,—observe that in our solution it is not necessary to write down any figures (except by way of explanation) until we make the statement for finding the square of 6561; after which statement we have only to make 20 figures previous to performing the division for the purpose of changing the result to pints. Pike's solution requires 118 figures previous to performing this division, making a difference of 98 figures.

3. A farmer has 25 head of cattle which he offers to sell at the rate of 1 cent for the first, 2 cents for the second, 4 cents for the third, 8 cents for the fourth, &c.; what would the 25 head amount to at that rate?

Ans. 335,544.31. (8.)

4. A person married his daughter on new year's day and gave her one dollar towards her portion, promising to double it on the first day of every month for one year; what was her portion? Ans. $4095. (0.)

5. What will 24 acres of land amount to at 1 cent for the first acre, 2 cents for the second, 4 cents for the third, 8 cents for the fourth, &c.? Ans. $167,772.15. (0.)

6. A person offers to sell a farm of 30 acres, at two pins for the first acre, 6 for the second, 18 for the third, and so on. Now suppose he should sell the pins at a 1000 for a farthing, how much will he thus get for his farm?

Ans. £214,469,929 5s. 3½d.

7. A man offers to sell his horse by the nails in his shoes, which are 32 in number. He demands 1 mill for the first nail, 2 for the second, 4 for the third, and so on, demanding for each nail twice the price of the preceding. What would be the price of the horse at this rate?

Ans. $4294967.295.

ALLIGATION.

Alligation teaches how to form a mixture of several ingredients of different qualities, so that the mixture or composition may be of some intermediate quality or value. It is considered under two heads, Alligation Medial and Alligation Alternate.

Alligation Medial teaches how to find the mean price of several articles mixed, the quantity and value of each being given.

1. If a grocer mix 20 pounds of sugar worth 12 cents a pound, 30 pounds worth 9 cents a pound, and 24 pounds worth 10 cents a pound, what is a pound of the mixture worth?

$$\begin{array}{ll} & \text{Cents.}\ \text{lb.}\quad \text{Cents.} \\ \text{Thus,} & \left\{\begin{array}{l} 12\times20=240 \\ \ \ 9\times30=270 \\ 10\times24=240 \end{array}\right. \\ \hline & \quad 74\ )\ 750(10\tfrac{6}{37}\ \text{cents.}\quad \text{Ans.} \end{array}$$

Hence to find the mean value of a compound of several ingredients we may give as a

RULE.

Find the value of each ingredient, and divide the sum of their several values by the sum of the ingredients.

2. A farmer mixed together 5 bushels of rye, at 70 cents a bushel, 10 bushels of corn worth 60 cents a bushel, and 5 bushels of wheat worth $1.10 a bushel; what was a bushel of the mixture worth? Ans. 75 cents.

3. If 6 gallons of wine worth 77 cents a gallon, 7 gallons worth 82 cents a gallon, and 5 gallons worth $1.24 a gallon, be mixed together, what will 1 gallon of the mixture be worth? Ans. 92 cents.

4. If I mix 20 pounds of tea worth 75 cents a pound with 25 pounds worth 60 cents a pound, and 80 pounds worth 45 cents a pound, what is the value of 1 pound of the mixture? Ans. $52\frac{4}{5}$ cents.

5. A wine merchant mixes 12 gallons of wine, at 67 cents a gallon, with 24 gallons at 66 cents a gallon, and 16 gallons at 75 cents a gallon; what are 12 gallons of the mixture worth? Ans. $8.28.

6. A druggist bought 24½ gallons of syrup, at 34 cents a gallon, and 24½ gallons at 38 cents a gallon, and mixed both quantities and 14 gallons of water together, and sold the mixture at a profit of 75 per cent.; at what price per gallon did he sell it? Ans. 49 cents.

ALLIGATION ALTERNATE.

Alligation alternate teaches the solution of those questions in which the respective rates of the different ingredients are given to compose a mixture of a fixed rate.

NOTE.—Alligation alternate requires four cases, and a special rule for each case, according to the mode of treating the subject in the most popular arithmetics used in our schools; we give but one simple rule, which will be found applicable to all cases.

7. A merchant has spices, some at 18 cents a pound, some at 24 cents, some at 48 cents, and some at 60 cents; how much of each sort must he mix that he may sell the mixture at 40 cents a pound.

Operation.
Cents.

$$\left\{\begin{array}{r} 18 \\ 24 \\ 48 \\ 60 \end{array}\right.$$

150 cents.
160 cents.
10 difference.

Again.
Cents.

$$\begin{array}{r} 18\times2= 36 \\ 24\times2= 48 \\ 48\times2= 96 \\ 60\times3=180 \\ \hline 9\,)\,360=40 \end{array}$$

Here observe we have 4 ingredients, and the sum of the prices of these 4 is equal to 150 cents, but 4 at 40 cents is $(40\times4)=160$ cents, which is 10 cents more than the sum of the 4 given ingredients; now it is evident that we must

increase the quantity of one or more of the higher priced ingredients in order to make the average price 40 cents.

The difference between 40 and 60 (the highest price) is 20, if we divide the 10 by this (20) the quotient is $\frac{1}{2}$; now if this $\frac{1}{2}$ be added to the 1 ingredient at 60 cents, making $1\frac{1}{2}$ we would have the right proportion, but to avoid fractions we take twice ($1\frac{1}{2}$) or 3 at 60 cents, and 2 at each of the others, making 9 in all, viz., 2 at 18 cents, 2 at 24 cents, 2 at 48 cents, and 3 at 60 cents, the price of the whole amounts to 360 cents, which being divided by 9, gives 40 cents, the average price required.

RULE.

1st. Find the sum of the prices of the given ingredients.

2d. Multiply the *required price* or *mean rate*, by the number of the ingredients, and take the difference between this product and the above named *sum* and call it the *first difference*. If the *sum* of the given prices *exceeds* this *product*, we must find the difference between the required price and one or more of the given prices which are *less* than the required price, and divide the *first* difference by the *second difference*, or sum (of differences) and the quotient will show how many additional parts of the lower priced ingredients are required; but if the *product exceeds* the *sum* of the given prices, we must find the difference between the required price and one or more of the given prices which are *greater* than the *required* price, and then proceed as before.

NOTE.—Prove the results by alligation medial.

8. In what proportion must I mix sugars at 6 cents, 8 cents, and 12 cents a pound, that the whole may be worth 9 cents a pound?

	Cents.			Cents.
	Thus, 6	Thus,	3 at 6	=18
	8	Ans.	3 at 8	=24
	12		4 at 12	=48
	26 sum		10	90
$9\times3=$	27			
	1 1st dif.			
$12-9=3$)	2d dif. 3			

Now ($\frac{1}{3}$) added to 1, at 12 cents, will make $1\frac{1}{3}$ pounds at 12 cents, $1\frac{1}{3}=\frac{4}{3}$, take the numerator (4) for the number at 12 cents, and the denominator 3, for that of each of the others, (this is to avoid fractions.)

9. A grocer wishes to know in what proportions he may mix oils at 70 cents, 85 cents, and 95 cents a gallon, that the mixture may be worth 87 cents a gallon?

$$
\begin{array}{rl}
 & \text{Cents.} \\
\text{Thus,} & \left\{\begin{array}{l} 70 \\ 85 \\ 95 \end{array}\right. \\
\hline
 & 250 \text{ sum} \\
87\times3= & 261 \text{ cents} \\
\hline
 & 11 \text{ 1st difference} \\
\hline
(95-87)= & 8
\end{array}
$$

Here we have 3 ingredients, which at 87 cents, (the required price) amounts to $87\times3=261$ cents. Hence we see we must increase the highest price, viz., that at 95 cents; the difference between 95 and 87 being 8, we divide the first difference (11) by this 8, and add $\frac{11}{8}$ to the 1 gallon, at 95 cents, and to avoid fractions we take the denominator 8, to represent the unit, and our answer will be 8 gallons at 70 cents, 8 gallons at 85 cents, and 19 gallons at 95 cents, which prove true by alligation medial.

NOTE.—In all questions of this kind, when a fraction represents the quotient, the sum of its numerator and denominator will give the quantity of the ingredient or ingredients, which the nature of the question requires to be greater than the given quantity, and its denominator will express the proper quantity of the other ingredients.

10. How much corn at 52 cents, wheat at 90 cents, rye at 56 cents, and another quality of wheat at $1 a bushel, must be mixed together, that the composition may be worth 62 cents a bushel? Ans. 6 bushels at 52 cts., and 1 bushel at each of the other prices.

11. How much barley at 50 cents a bushel, rye at 75 cents, and wheat at $1, must be mixed that the composition may be worth 80 cents a bushel? Ans. 7 bushels at $1, and 4 bushels at 75 cts. and 50 cts.

12. It is required to mix several sorts of wine, viz., some at 60 cents, some at 80 cents, and some at $1.20, so that the mixture may be worth 75 cents a gallon; how much of each sort must be taken?

$$\text{Cents.}\quad \left\{\begin{array}{r} 60 \\ 80 \\ 1.20 \end{array}\right. \qquad \text{Ans.} \left\{\begin{array}{rr} 10 \text{ at} & 60 \\ 3 \text{ at} & 80 \\ 3 \text{ at} & 1.20 \end{array}\right. \text{Cents.}$$

$$\begin{array}{r} 2.60 \\ 75\times3=2.25 \\ \hline (75-60)=\frac{35}{15}=\frac{7}{3} \text{ 1st dif.} \end{array}$$

In this question we must increase the quantity at the lowest price.

Proof.

$$\begin{array}{rcl} 60\times10 & = & 600 \\ 80\times\ 3 & = & 240 \\ 1.20\times\ 3 & = & 360 \\ \hline 16 &) & 1200=75 \end{array}$$

13. A wine merchant has Canary wine at 50 cents a gallon, sherry at 76 cents, and claret at $1.75 a gallon; how much of each sort must he take to make a mixture worth 87 cents a gallon? Ans. 11 gals. at 50 cts. 11 gals. at 76 cts. and 6 gals. at $1.76.

14. A baker has flour worth 3 cents a pound, butter worth 20 cents a pound, and eggs worth 16 cents a dozen; what quantity of each must he take to make a cake worth 7 cents a pound? Ans. 11 lbs. of flour, 2 lbs. of butter, and 2 doz. eggs.

15. A man bought a tract of land of three different kinds, the first tillable land, at $120 per acre, the second meadow, at $84 per acre, and the third woodland, at $75 per acre; he wishes to know how many acres of each must be taken to make a farm worth $100 per acre? Ans. 41 acres of tillable, 20 acres of meadow, and 20 acres of woodland.

16. A goldsmith has gold 12, 16, 17, and 22 carats fine;

what proportion of each kind must he take to make a compound 18 carats fine? Ans. 9 of 22 carats, and 4 of of each of the other kinds.

NOTE—If an ounce, or any other quantity of pure gold be divided into 24 equal parts, these parts are called carats. But gold is frequently mixed with some baser metal, which is called *alloy*, and the mixture is said to be so many carats fine, according to the proportion of pure gold contained in it; thus if 20 parts of pure gold be mixed with 4 parts of *alloy*, the compound is said to be 20 carats fine, alloy is considered of no value.

17. In what proportion may gold of 15 carats fine, 19, 23, and pure gold, (that is, 24 carats fine,) be mixed that the compound may be 20 carats fine? Ans. 2 of 19 carats fine with 1 of each of the others.

18. In what proportion must I mix gold of 12, 14, 17, 19, and 22 carats fine that the compound may be 18 carats fine? Ans. 5 of 22 carats fine, and 2 of each of the other kinds.

19. A trader has 90 pounds of tea worth 40 cents a pound, which he would mix with some at 50 cents, some at 85 cents, and some at 90 cents. How much of each of the other sorts must he mix with the 90 pounds to make the mixture worth 60 cents a pound?

Ans. 40 pounds of each of the other sorts.

Cents.	Cents.
40	90
50	40
85	40
90	40

$$\begin{array}{r} 265 \\ 60\times4=240 \\ 240 \\ \hline 25 \end{array}$$

$$(60-40)=\frac{25}{20}=\tfrac{5}{4} \text{ first dif.}$$

The quotient $(\frac{5}{4})$ gives 9 at 40 cents, and 4 at each of the other sorts; but since the quantity at 40 cents must be 90 pounds, we must annex a cipher to each result to obtain the true result.

20. In what proportion may coal worth \$4.50, \$4.25, \$3.75, and \$3.50 a ton be mixed that the mixture may be worth \$4.12½ a ton? Ans. 5 tons at \$4.25 with 1 ton at each of the other prices.

21. A miller has corn worth 60, 70, and 80 cents a bushel, and wheat worth \$1.10 and \$1.20 a bushel, in what proportion must he mix these several qualities of corn and wheat that the mixture may be worth \$1 a bushel?

Ans. 4 bushels of wheat at \$1.20 a bushel, with 1 bushel at \$1.10, and 1 bushel of each quality of corn.

22. A grocer would mix 30 pounds of sugar, at 14 cents a pound, with some at 9 cents, 10 cents, and 13 cents a pound; how much of each sort must he mix with the 30 pounds that the mixture may be worth 12 cents a pound?

Ans. 30 pounds at 13 cents, 18 pounds at 9, and 18 at 10 cents; or 15 pounds of each sort.

Cents.	Cents.	Cents.	Cents.
14	30	14	30
9	18	9	15
10	18	10	15
13	30	13	15
46		46	
$12\times4=48$		$12\times4=48$	

$$\left.\begin{matrix}14-12\\13-12\end{matrix}\right\}\ \frac{2}{3} \qquad\qquad (14-12)=\frac{2}{2}=1$$

If we take the difference between 12 (the mean rate) and 14, and divide the first difference thereby, we get 1 to add to the 1 at 14 cents, making 2, which we must multiply by 15 to make 30; this gives 15 pounds at each of the other prices, but if we take the difference between 12 and 14 and also between 12 and 13, we have 3 to divide the first difference by; this gives 5 pounds at 14 cents, and also at 13 cents, and 3 pounds at 9 and 10 cents, but since the 5 must be multiplied by 6 to make 30, we must multiply the 3 also by 6.

NOTE.—Observe that when one of the ingredients is limited in quantity, we find the proportions as before, and increase this result if necessary to the required quantity.

23. If 10 bushels of wheat at 96 cents a bushel be mixed

with rye at 72 cents, barley at 48 cents, and oats at 24 cents, what quantity of each must be mixed with the 10 bushels of wheat that the mixture may be worth 56 cents a bushel?

Ans. 30 bushels of barley, 10 of rye, and 10 of oats.

Or 15 bushels of oats, 10 of rye, and 10 of barley.

And as a third answer, we may give 14 bushels of barley, 14 of oats, and 10 of rye.

NOTE.—We obtain the first answer by taking the difference between the mean rate and 48 for a divisor, the second by taking the difference between the mean rate and 24 for a divisor, and lastly by taking the sum of those differences for a divisor.

24. A grocer would mix teas at 50 cents, 60 cents, and 100 cents per pound, with 20 pounds at 80 cents a pound; how much of each sort must he take to make the composition worth 75 cents a pound?

Ans. 20 pounds at 100 cents, 15 pounds at 50 cents, and 15 pounds at 60 cents.

25. How much gold at 16, 20, and 24 carats fine, and how much alloy must be mixed with 10 ounces of 18 carats fine, that the composition may be 22 carats fine?

Ans. 10 ounces of 16 carats fine, 10 ounces of 20 carats fine, 170 ounces of pure gold, and 10 ounces of alloy.

26. A farmer wishes to mix 14 bushels of wheat at $1.20 per bushel, with rye at 72 cents, barley at 48 cents, and oats at 36 cents; how much of each must be taken to make the mixture worth 64 cents a bushel?

Ans. 14 bushels of rye, 14 bushels of barley, and 24 bushels of oats.

27. A brewer has beer at 6 cents, 8 cents, and 10 cents a gallon; how much of each sort must he take to make a mixture of 30 gallons, worth 7 cents a gallon?

$$\text{Ans.}\left\{\begin{array}{rr} 6 & 4 \\ 8 & 1 \\ 10 & 1 \end{array}\right\} \begin{array}{c} = \\ \times 5= \\ = \end{array} \left\{\begin{array}{l} 20 \text{ at } 6 \text{ cents.} \\ 5 \text{ at } 8 \text{ cents.} \\ 5 \text{ at } 10 \text{ cents.} \end{array}\right.$$

$$24$$

$$\frac{7\times 3=21}{(7-6)=\tfrac{3}{1}=3}$$

Here we get in the first place, 4 gallons at 6 cents, 1 at 8

cents, and 1 at 10 cents, making 6 gallons in all, instead of 30; now by multiplying the first answer by 5 we get the required quantity.

NOTE.—Observe that when the whole compound is limited to a certain quantity, if we divide that quantity by the *sum* of the quantities *first* obtained, and multiply *each* of those quantities by the *quotient*, the product will be the proportion of each required.

28. A goldsmith has gold 15, 17, 20, and 22 carats fine, and would melt together of all of these sorts so much as to make a mass of 40 ounces 18 carats fine. How much of each sort is required?

Ans. 12 of 15 carats fine, 12 of 17 carats fine, 8 of 20 carats fine, and 8 of 22 carats fine.

29. A merchant has sugar at 8 cents, 10 cents, 12 cents, and 20 cents a pound; with these he would fill a hogshead that would contain 200 pounds; how much of each sort must he take that the mixture may be worth 15 cents a pound?

Ans. $33\frac{1}{3}$ at 8 cents a pound, $33\frac{1}{3}$ at 10 cents, $33\frac{1}{3}$ at 12 cents, and 100 pounds at 20 cents.

30. A man has different kinds of apples, some worth 31 cents a bushel, some 37 cents, some 46 cents, and some 74 cents; what quantity of each kind must he take to fill a bin containing 54 bushels, that the whole may be worth 50 cents a bushel?

Ans. 12 bushels at 31 cents a bushel, 12 at 37 cents, 12 at 46 cents, and 18 at 74 cents.

31. How much sugar at 14 and 16 cents a pound, must be mixed with 6 pounds at 19 cents, and 12 pounds at 22 cents, that the composition may be worth 20 cents a pound?

$$\begin{array}{rr} 1 & 14 \\ \underline{1} & 16 \\ 19\times 6 = & 114 \\ 22\times 12 = & 264 \\ \hline & 408 \\ 20\times 20 = & 400 \\ \hline 20-16 = & \frac{8}{4}=2 \end{array} \qquad \text{Ans.} \left\{ \begin{array}{l} 1 \text{ pound at 14 cents.} \\ 3 \text{ pounds at 16 cents.} \end{array} \right.$$

In this question 2 of the ingredients being limited in quan-

tity, we find their value, and also that of one of each of the others, the *sum* being the value of 20 pounds. The difference between this sum and 20 pounds at the mean rate is 8 cents; hence we see that the quantity of one or more of the lower priced ingredients must be *increased without* increasing those that are limited; it is desirable therefore that a whole number be added to the quantity which is to be increased, although a fraction will also give the correct proportion, hence we take the difference between the mean rate (20) and 16, which gives us 4 to divide into 8, the quotient being 2, which we add to the 1 pound at 16 cents making 3 pounds.

32. A tobacconist would mix 14 pounds of tobacco at 42 cents a pound, 24 pounds at 36 cents a pound, with two other kinds, one at 24 cents and the other at 18 cents a pound; how much of each of the two latter kinds must he take to make the mixture worth 30 cents a pound?

Ans. 50 pounds at 24 cents, and one pound at 8 cents a pound; or $17\frac{1}{3}$ pounds of each of the latter kinds.

33. A brewer would mix 28 gallons of ale at 18 cents a gallon, and 18 gallons at 9 cents a gallon, with two other kinds, one at 16 cents, and the other at 12 cents a gallon; how much of the 16 and 12 cent ale must he take that the mixture may be worth 11 cents a gallon.

Ans. 20 gallons of each.

34. I have 18 gallons of wine at 48 cents a gallon, 8 gallons at 52 cents, and 4 gallons at 85 cents, and would mix the whole with two other kinds of wine, one at $1.26 and the other at $2.12 a gallon; how much of the wine at $1.26, and of that at $2.12 must I mix with the other three that the mixture may be worth $1.00 a gallon?

Ans. 10 gallons of each.

35. A lady purchased 14 yards of calico at 22 cents a yard, and 12 yards at 20 cents a yard; she wished to know how many yards of two other kinds, one at 15 cents, and the other at 17 cents a yard, she must purchase to make the average price of the whole 18 cents a yard?

Ans. 20 yards of each of the other two kinds.

EQUATION OF PAYMENTS.

Equation of payments consists in finding a mean time for the payment at once of several debts, payable at different times, so that no loss of interest shall be sustained by either party.

1. A owes B $46 dollars, $12 of which is to be paid in 8 months, $16 in 5 months, and $18 in 3 months; at what time may the whole debt be paid at once, without injustice to either party?

Explanation.—The interest of $12, for 8 months, is the same as the interest of $1 for 96 months; the interest of $16 for 5 months is the same as that of $1 for 80 months; and the interest of $18 for 3 months is the same as that of $1 for 54 months; hence the interest of $12 for 8 months, of $16 for 5 months, and of $18 for 3 months is the same as the interest of $1 for 230 months, which is equal to the interest of $46 for 5 months; therefore 5 months is the medium time for the payment of the whole debt. From the above illustration we infer the following

RULE.

Multiply each payment by the time at which it is due, and divide the sum of the products by the sum of the payments, and the quotient will be the equated time.

2. E owes G $2400, of which $480 are to be paid at 5 months, $960 at 8 months, and the rest at 12 months; but they agree to make one payment of the whole; what is the equated time?

Operation.

$$
\begin{array}{l}
\$ \\
480\times\ 5=\ 2400 \\
960\times\ 8=\ 7680 \\
960\times12=11520 \\
\hline
2400\ \ )21600(\text{Ans. 9 months.}
\end{array}
$$

3. A owes B $380, of which $100 are to be paid in 6

months, $120 in 7 months, and $160 in 10 months, but they agree that the whole shall be paid at one time; when must it be paid? Ans. In 8 months.

4. A merchant sold goods amounting to $1500, to be paid as follows, $250 in 2 months, $500 in 5 months, and $750 in 8 months; what is the mean time of paying the whole? Ans. 6 months.

5. A merchant has due him $300, to be paid in 60 days, $500 to be paid in 120 days, and $750 to be paid in 120 days; what is the equated time for the payment of the whole? Ans. $108\frac{12}{31}$ days.

6. A owes B $640, $150 due in 30 days, $200 due in 60 days, and the remainder in 90 days; what is the equated time for the payment of the whole? Ans. $66\frac{9}{16}$ days.

7. C owes D the following sums, viz., $180 now due, $120 due in 3 months, $200 due in 6 months, and $300 due in 12 months, and they have agreed to make one payment of the whole; at what time must it be made? Ans. $6\frac{9}{20}$ months.

8. A merchant purchased goods to the amount of $2000, of which $400 are to be paid at present, $800 at 6 months, and the rest at 9 months; but it is agreed to make one payment of the whole; when must it be paid? Ans. 6 months.

9. G owes K $420, which will be due 6 months hence; it is agreed that $60 shall be paid now, and that the rest remain unpaid a longer time than 6 months; when must it be paid? Ans. In 7 months.

REDUCTION OF CURRENCIES.

Previous to the act of Congress establishing a mint, and regulating the coin of the United States, (in the year 1792,) each State had its own particular currency. Under the *colonial* government the several States issued bills of credit, to supply the want of specie, and to answer as a medium of trade; but as these bills were not received by the British

merchants in payment for goods at their par value, holders of the bills had to pay more. Thus in New York they had to pay in bills of that State, at the rate of 8 shillings for 4s. 6d. sterling, (4s. 6d. sterling being taken as the value of a dollar.) In New England 6 shillings were taken as the value of a dollar.

The following table shows the value of a dollar of this currency in the several States:

New England, Virginia, Kentucky, and Tennessee,	6s.=$1.
New York, North Carolina, and Ohio, - -	8s.=$1.
Pennsylvania, New Jersey, Delaware, and Maryland, - - - - - -	7s. 6d.=$1.
South Carolina and Georgia, - - -	4s. 8d.=$1.
In Canada and Nova Scotia the dollar is valued at 5s. sterling, or English money, - -	4s. 6d.=$1.

1. Change £218 7s. 4d. New England currency to Federal money.

$$7s.—4d. = \tfrac{11}{30} \qquad \text{Thus, } \begin{array}{r|l} & £ \\ 3\cancel{0} & 6551 \\ \cancel{6} & \cancel{2}\cancel{0} \\ \hline \end{array}$$

Ans. 727.88\frac{8}{9}$

Here we express the given sum in shillings, and since 6 shillings make a dollar, we place 6 on the left of the line, &c.

2. Change 727.88\frac{8}{9}$ to New England currency.

$$\text{Thus, } \begin{array}{r|l} & \$ \\ \cancel{9} & 6551 \\ \cancel{2}0 & \cancel{6} \\ \hline \end{array}$$

Ans. £218 7s. 4d.

3. Change £46 16s. 6d. of the old currency of New York to Federal money. Ans. 117.06$\frac{1}{4}$.

4. Change 117.06\frac{1}{4}$ to the old currency of New York.

5. Change £387 of the old currency of Pennsylvania to Federal money. Ans. $1032.

6. Change $1032 to the old currency of Pennsylvania.

7. Change £187 9s. 10d. of the old currency of Georgia to Federal money. Ans. $803.53\frac{4}{7}$.

8. Change 803.53$\frac{4}{7}$ to the old currency of Georgia.

9. Change £25 10s. 6d. Canada currency to Federal money. Ans. $102.10.

10. Change $109.10 to Canada currency.

11. Change £97 8s. sterling to Federal money. Ans. 432.88\frac{8}{9}$.

12. Change 432.88\frac{8}{9}$ to sterling money. £1=$\$\frac{40}{9}$.

NOTE.—The comparative value of the dollar and the pound sterling, as given above, is called the *nominal par value*. The actual value of the pound is more than is here given. This difference is usually estimated in trade by adopting the nominal value as given above, as the basis of the calculation, and then adding or subtracting a certain per cent. to compensate for the inequality of value.

In calculating *ad valorum* duties in the custom-houses of the United States, on goods imported from England, the pound sterling is estimated at $4.84, according to act of Congress, passed January 18, 1837.

EXCHANGE.

Exchange is the act of paying or receiving the money of one country for its equivalent in the money of another country, by means of bills of exchange. It comprehends both the reduction of money and the negotiation of bills.

A bill of exchange is a written order for the payment of a certain sum of money, at an appointed time. It is a mercantile contract, in which four persons are generally concerned, viz.

1st. The *drawer*, who receives the value, and is also called the *maker* and *seller* of the bill.

2d. The person upon whom the bill is drawn is called the *drawee*. He is also called the acceptor, when he accepts the bill, which is an engagement to pay it when due.

3d. The person who gives value for the bill, is called the *buyer*, *taker* or *remitter*.

4th. The person to whom it is ordered to be paid, is called the *payer*, who may, by endorsement, pass it to any other person.

It is customary, as a precaution against accidents or miscarriage, to draw three copies of a foreign bill, and to send them by different posts. They are denominated the *first*, *second*, and *third* of *exchange*, and when any one of them is paid, the rest become void and of no value.

1. A merchant in Philadelphia imports goods from London, amounting per invoice to £1525 16s. 9d. sterling; what sum in money of the United States must be given for the bill of exchange to that amount, exchange being at $4.84 per pound sterling?

$$\begin{array}{r|l} \text{£} & \\ \text{Thus, } \cancel{80} & 122067 \times 121 \\ 20 & \underline{\cancel{4}.\cancel{8}\cancel{4}} \\ & \overline{14768.107} \\ \text{Ans.} & \overline{\$7384.05\tfrac{7}{20}} \end{array}$$

2. A merchant in Boston exports goods to France, which are sold there for 7576 francs; what is the value of a draft on his factor for that amount at $18\frac{3}{5}$ cents per franc?

Ans. $\$1409.13\frac{3}{5}$.

3. Taylor & Co. of Philadelphia, have purchased goods in Liverpool, to the amount of £18,761 10s.; what is the value of the goods in Federal money, the par value of the pound sterling ($\$\frac{40}{9}$) being at a premium of $8\frac{1}{2}$ per cent.?

Ans. $\$90,472.12\frac{2}{9}$.

4. United States on London.—Change £4109 11s. 10d. sterling to United States currency, exchange at 7 per cent. above par. Ans. $\$19,543.39\frac{4}{27}$.

5. London on the United States. Change $4287.50 to sterling; exchange at 4 per cent. above par.

Ans. £1003 5s. 6d.

6. An English gentleman died in Philadelphia, and left an estate of $176,984, which his executors wish to remit to London; for how many pounds sterling must a bill of exchange be drawn, at 8 per cent. premium, in favour of England?

Ans. £36,871 13s. 4d.

7. A merchant in Philadelphia wishes to remit to London £9648; what must he pay for a bill of exchange, when sterling money is at a premium of 12 per cent.?

Ans. $48,025.60.

8. New York on Paris.—Change 48,764 francs to Federal money, exchange being 5 francs, 25 centimes per dollar? Ans. $9288.38\frac{2}{21}$.

NOTE.—1 franc = 100 centimes.

9. Paris on the United States.—Change $4093.80 to money of France; exchange at 5 francs 30 centimes per $1. Ans. $21,697 francs 14 centimes.

10. United States on France.—Change 20,828 francs 67 centimes to Federal money; exchange at $1 for 5 francs 38 centimes. Ans. $3871.50.

AMSTERDAM AND ANTWERP.

Accounts are kept throughout the kingdom of the Netherlands in florins or guilders, and cents.

100 cents = 1 florin or guilder.

The par value of the florin in Federal money is 40 cents.

11. United States on Amsterdam.—Change 47,395 florins to Federal money; exchange at par. Ans. $18,958.

12. United States on Antwerp.—Change 6281 florins, 88 cents to Federal money; exchange at par. Ans. 2512.75\frac{1}{5}$.

13. Antwerp on the United States.—Change $47,632.78 to Dutch money; exchange at 39 cents per florin. Ans. $122,135$\frac{1}{3}$ florins.

14. United States on Antwerp.—Change 21,883 florins, 50 cents to Federal money; exchange at 42 cents per florin. Ans. $9191.07.

15. Amsterdam on the United States.—Change $1362.68 to money of Amsterdam; exchange at 38 cents per florin. Ans. 3586 florins.

RUSSIA.

In Russia accounts are kept in roubles and copecks.

10 copecks = 1 gruvener,
10 gruveners, or 100 copecks = 1 rouble.

16. United States on Russia.—Change 32,499 roubles to Federal money; the value of the rouble in the United States being 75 cents. Ans. $24,374.25.

17. Russia on the United States.—What is the value of $5763.60 in Russian currency?

Ans. 7684 roubles 80 copecks.

18. A of New York draws on B of St. Petersburg for 2347 roubles, 50 copecks; what is the value in Federal money, at $62\frac{1}{2}$ cents per rouble? Ans. $1467.18\frac{3}{4}$.

19. Boston on St. Petersburg.—Change 4952 roubles 25 copecks to Federal money; exchange being 77 cents per rouble. Ans. $\$3813.23\frac{1}{4}$.

PRUSSIA.

Accounts are kept in Prussia in thalers or rix dollars, good groschens, and pfenings.

12 pfenings=1 good groschen,
24 good groschens=1 rix dollar.

20. United States on Prussia.—Change 4285 rix dollars, 18 good groschens to Federal money; exchange at 67 cents per rix dollar. Ans. $\$2871.45\frac{1}{4}$.

NOTE.—The par value of the rix dollar in the United States is $66\frac{2}{3}$ cents.

21. Prussia on the United States.—Change $15,741.60 to Prussian money, exchange at 64 cents per rix dollar.

Ans. 24,596 rix dollars, 6 good groschens.

PERMUTATION.

Permutation or variation means the different ways in which the order or relative position of any given number of things may be changed; thus a and b are susceptible of the *two* positions ab and ba, which may be expressed by 1×2; a, b, and c, are susceptible of six *different relative* positions, thus, abc, acb, bac, bca, cab, and cba, and this number of variations is expressed by $1\times2\times3=6$; hence the following

RULE.

Multiply the terms of the natural series of numbers, from 1 up to the given number of things continually together, and the product will be the answer required.

1. How many different arrangements may be made in seating a class of 14 pupils. Ans. 87,178,291,200. (21.)

2. How many days can 7 persons be placed in different positions at dinner? Ans. 5040 days.

3. In what length of time may the greatest possible number of changes be rung upon 12 bells, allowing 3 seconds to every change. Ans. In 45 years, 195 days, 18 hours.

4. What time will it require for 8 persons to seat themselves in the greatest possible number of different ways at dinner? Ans. 110 years, 142 days.

COMBINATION.

Combination consists in taking a less number of things out of a greater without any regard to the order in which they stand. Thus, out of the letters a, b, and c, three different combinations may be made, viz. ab, ac, bc; no two combinations having the same letters. This number of combinations may be expressed

Thus, 1 | 3 Ans.
$\cancel{2}$ | $\cancel{2}$

RULE.

Take for a dividend a series of numbers, the first term of which is equal to the *number* of things out of which the combinations are to be made, and decreasing by 1 till the number of terms is equal to the number of things to be taken. Then take for a divisor the natural series, 1, 2, 3, 4, &c. up to the number of things to be taken at a time, and the quotient will give the answer required.

1. How many combinations can be made of 7 letters out of 10, the letters all being different?

$$\begin{array}{r|l} 1 & 10 \\ \cancel{2} & \cancel{9} \\ \cancel{3} & \cancel{8} \\ \cancel{4} & \cancel{7} \\ \cancel{5} & \cancel{6} \\ \cancel{6} & \cancel{5} \\ \cancel{7} & \cancel{4} \\ \hline \text{Ans.} & 120 \end{array}$$

2. How many combinations can be made of 6 letters out of 12? Ans. 924.

3. What is the value of as many different dozens as may be chosen out of 24, at 1d. per dozen? Ans. £11,267 6s. 4d.

MISCELLANEOUS QUESTIONS.

1. The sum of two numbers is 4638; one of the numbers is 2469, what is the other? Ans. 2169.

2. The difference of two numbers is 468; one of the numbers is 376, what is the other? Ans. 844.

3. The product of two factors is 19,475; one of the factors is 1025, what is the other? Ans. 19.

4. If the quotient be 4298 and the divisor 437, what is the dividend? Ans. 1878226.

5. 468 is $\frac{9}{14}$ of what number? Ans.

6. What number is that which being divided by $23\frac{3}{4}$, the quotient will be $=\frac{3}{4}$ of $\frac{2}{3}$ of $9\frac{1}{3}$? Ans. $110\frac{5}{6}$.

7. What is the difference between six dozen dozen, and half a dozen dozen? Ans. 792.

8. If the fourth of 20 be 3, what will be the ninth of 63? Ans. $4\frac{1}{5}$.

9. What number is that to which if its $\frac{1}{2}$ and its $\frac{1}{3}$ be added the sum will be 792? Ans. 432.

10. What number is that to which if its $\frac{1}{3}$, its $\frac{1}{4}$, and 89 more be added, the sum will be 19,887? Ans. 12,504.

11. What number is that which being divided by $\frac{3}{4}$, the quotient will be 48?

12. What number is that which being diminished by $\frac{1}{4}$ and $\frac{1}{5}$ of itself, the remainder will be 572? Ans. 1040.

NOTE.—$\frac{1}{4}$ and $\frac{1}{5}$ added together make $\frac{9}{20}$; hence we perceive that the required number consists of 20 parts, and since 9 parts have been taken away, the remainder (572) must be 11 parts.

13. A person after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, found he had $275 left; how much had he at first?

Ans. $660.

14. A school teacher being asked how many scholars he had, answered, if to double the number I add $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of them, I shall have 222; how many had he? Ans. 72.

Solution.—The least common denominator for $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, is 12; hence we consider the number of scholars to consist of 12 equal parts; then double the number will be 24 parts, $\frac{1}{2}$ of the number 6 parts, $\frac{1}{3}$ 4 parts, and $\frac{1}{4}$ 3 parts, making in all 37 parts; hence the number of scholars must be

$$\begin{array}{r|l} \cancel{37} & \cancel{222} \\ & 12 \\ \hline \text{Ans.} & 72 \text{ scholars.} \end{array}$$

15. A person after spending $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, of his money, had $216⅔ left; how much had he at first? Ans. $1000.

Solution.—$\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, added together make $\frac{47}{60}$; hence, we consider the number of dollars he had to consist of 60 parts; and having spent 47 parts, the remainder ($216⅔) must be 13 parts, and therefore he must have had at first

$$\begin{array}{r|l} \cancel{3} & \cancel{650} \\ \cancel{13} & \cancel{60} \\ \hline \text{Ans.} & \$1000 \end{array}$$

16. A farmer being asked how many sheep he had, answered, that he had them in 5 fields; in the first $\frac{1}{4}$ of his flock, in the second $\frac{1}{6}$, in the third $\frac{1}{8}$, in the fourth $\frac{1}{12}$, and in the fifth 450; how many had he? Ans. 1200 sheep.

17. If my horse and saddle are worth $98, and my horse be worth 6 times as much as my saddle? what is the value of the horse? Ans. $84.

18. Bought 45 barrels of beef at \$3.50 per barrel, among which are 16 barrels, whereof 4 are worth no more than 3 of the others; how much must I pay for the whole?

Ans. \$143.50. (0.)

19. A person bought a carriage, horse, and harness, for \$720, the horse came to twice the price of the harness, and the carriage to twice the price of the horse and harness; what did he give for each?

Ans. For the carriage \$480, for the horse \$160, and for the harness \$80.

NOTE.—Since the carriage cost twice as much as the horse and harness, it must have cost $\frac{2}{3}$ of \$720, and the horse and harness $\frac{1}{3}$; again, since the horse cost twice as much as the harness, he must have cost $\frac{2}{3}$ of \$240, and the harness $\frac{1}{3}$.

20. A person dying leaves $\frac{1}{4}$ of his property to his wife, $\frac{1}{6}$ to each of two daughters, and the remainder, which is \$1750 to the poor; what is the whole amount of his property?

Ans. \$4200.

21. An estate of \$7500 is to be divided among a widow, two sons, and three daughters, so that each son shall receive twice as much as each daughter, and the widow \$500 more than all the children; what was her share, and what the share of each of the children?

Ans. { Widow's share, \$4000.
Each son, \$1000.
Each daughter, \$500. }

22. After paying away $\frac{1}{5}$ of my money, and $\frac{1}{4}$ of the remainder, I had \$495 left; how much had I at first?

Ans. \$825.

23. What number must $7\frac{3}{5}$ be multiplied by that the product may be $6\frac{2}{3}$? Ans. $\frac{50}{57}$.

24. A teacher being asked how many scholars he had, answered that if the number were multiplied by one-half of itself the product would be 648; how many scholars had he?

Ans. 36.

NOTE.—If the number be multiplied by itself, the product will be twice as much as if multiplied by its half.

25. A man being asked how much he gave for his horse,

answered that if the number of dollars he gave were multiplied by $\frac{3}{4}$ of itself the product would be \$6912; how much did he give for his horse? Ans. \$96.

26. The yearly interest of Charlotte's money at 6 per cent. exceeds $\frac{1}{10}$ of the principal by \$356, and she does not intend to marry any man who is not scholar enough to tell her fortune; what is her fortune? Ans. \$8900.

Remark.—$\frac{1}{10}$ being 10 parts of 100, and 6 per cent. being 6 parts, and their difference 4 parts, therefore \$356 must be $\frac{4}{100}$, or $\frac{1}{25}$ of her fortune.

27. A trader begins the world with \$1200, and finds that he can clear \$1200 in 6 years by his store, and \$1200 in 8 years by joining a manufacturer, and likewise that he expends \$1200 in 3 years by gambling; how long will his estate last if he follow all three? Ans. 24 years.

28. It was observed of a certain legislature that when 19 more than half of the members were present, 16 more than $\frac{1}{4}$ were absent; how many members were there in the legislature? Ans. 140.

29. A manufacturer finds he wants \$37 of being able to pay off his hands, at \$10 each per week, he gives them \$8 a-piece and has \$19 left; how many men had he employed? Ans. 28.

Suggestion.—If he had had \$37 in addition to the \$19 which he had left, making \$56, he could have paid them \$2 a-piece more.

30. A person willing to distribute some money among a number of beggars, found he wanted 43 cents to give them 12 cents a-piece, he therefore gave them 9 cents a-piece and had 14 cents left; how many beggars were there? Ans. 19 beggars.

31. A farmer being asked how many head of cattle, and how many sheep he had, answered that if the number of cattle were multiplied by 13, the product would exceed the number of sheep by 19, but if multiplied by 9, the product would be less than the number of sheep by 37; how many head of cattle, and how many sheep had he?

Ans. 14 head of cattle and 163 sheep.

32. \$1800 are to be divided among three persons, A, B, and C, in such a manner that A shall have twice as much as

B, and B three times as much as C; what will each one receive? Ans. A \$1080, B \$540, C \$180.

NOTE.—The money must evidently be divided into a certain number of shares of which C receives 1, B 3, and C 6, making in all 10 shares.

33. A farmer bought a horse, a yoke of oxen, a cow, and a sheep, for \$207; the cow cost 8 times as much as the sheep; the oxen 3 times as much as the cow, and the horse $1\frac{1}{2}$ times as much as the oxen; what was the cost of each?

Ans. The sheep cost \$3, the cow \$24, the oxen \$72, and the horse \$108.

34. A man had £7 17s. 6d. to pay among his labourers, to every boy he paid 6d., to every woman 8d, and to every man 16d.; there was 1 boy to 3 women, and 1 woman to 2 men; how many were there of each?

Ans. 15 boys, 45 women, and 90 men.

35. If £79 4s. 10d. be divided among 4 men, 6 women, and 9 boys, so that each woman shall receive twice as much as a boy, and each man twice as much as a woman, what will be the share of each?

Ans. Each boy must have £2 2s. 10d., each woman £4 5s. 8d., and each man £8 11s. 4d.

36. Purchased for a cloak $5\frac{1}{4}$ yards of broadcloth, that was $1\frac{7}{8}$ yards wide, to line this I purchased flannel that was $\frac{7}{8}$ yard wide; but on being wet it shrunk 1 nail in width, and 1 yard in every 20 yards in length; how many yards of flannel was it necessary to buy? Ans. $12\frac{186}{247}$ yds.

The solution requires but 8 figures in the work.

37. A man covered 14 pair of window shutters with tin, each shutter requiring $8\frac{1}{2}$ sheets, for which he was to have $11\frac{1}{2}$ cents per sheet; what will he receive when they are finished? Ans. \$27.37. (0.)

38. When 100 boxes of prunes cost \$2.10 each, and by selling them at \$3.50 per cwt., the gain is 25 per cent., the weight of each box is required. Ans. 84 lbs. (0.)

39. What is the cost in Federal money, of 940 bales of cotton, averaging 340 pounds per bale, at $\frac{3}{8}$ of a penny per pound? Ans. \$2219.44$\frac{4}{9}$. (13.)

40. Bought a quantity of cloth for \$750, $\frac{5}{20}$ of which I found to be inferior, and which I had to sell at \$1.25 per

yard, and by so doing I lost \$100; at what rate per yard must I sell the rest, that I may lose nothing by the whole?

Ans. \$3.15$\frac{10}{21}$. (6.)

41. Divide \$840 between A and B, so that A may have \$168 more than B.

Ans. A must have \$504, and B \$336.

NOTE.—Take out \$168 and divide the remainder equally.

42. What is the age of a person who says that if $\frac{3}{4}$ of the years I have lived be multiplied by 7, and $\frac{2}{3}$ of them be added to the product, the sum will be 426?

Ans. 72 years.

NOTE.—12 is the smallest common denominator, $\frac{3}{4}=\frac{9}{12}$, and $\frac{2}{3}=\frac{8}{12}$; now 7 times 9 parts added to 8 parts equal 71 parts.

43. A certain sum of money is to be divided among 4 persons in such a manner that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth the remainder, which is \$135; what was the sum? Ans. \$540.

44. Borrowed a sum of money, for which I paid 6 per cent. a year, simple interest; in 12 years it amounted to \$860; what was the sum borrowed? Ans. \$500.

45. A house being let upon a lease of 5 years, at \$60 a year, and the rent being in arrear for the whole time; what was the sum due at the end of the term, simple interest being allowed at 6 per cent. a year? Ans. \$336.

46. A certain gambler lost 3 bets successively, and each time lost half of his money and \$1.50 more; how many dollars had he at first, admitting that he had \$17 after he lost the third bet? \$157.

47. A and B have the same income; A saves $\frac{1}{5}$ of his, but B, by spending \$240 per annum more than A, at the end of 8 years finds himself \$720 in debt; what is their income?

Ans. 750.

48. A man directed in his will that $\frac{1}{2}$ of his estate should be given to his wife, $\frac{1}{5}$ of the remainder to his oldest son, and $\frac{1}{4}$ of the residue to his oldest daughter, and $\frac{1}{2}$ of what then remained to four other children, who received \$450 each; what was the value of his estate? Ans. \$12,000.

49. A son asked his father's age, the father replied, "Your age is 12 years, to which if $\frac{5}{8}$ of both our ages be added, the sum will be equal to mine;" what was the father's age?

Ans. 52 years.

50. Three men, A, B, and C, undertake to dig a ditch for \$57.50. Now A and B together are supposed to do $\frac{3}{4}$ of the work, A and C $\frac{9}{10}$, and B and C $\frac{13}{20}$, they are to divide the money according to the proportion of the work done by each? how must it be divided?

Ans. $\begin{cases} \text{A } \$25.00. \\ \text{B } \$12.50. \\ \text{C } \$20.00. \end{cases}$

51. Thomas sold 150 pineapples at $33\frac{1}{3}$ cents a piece, and took no more money than Harry did for melons at 25 cents a piece; how many melons had Harry?

Ans. 200 melons.

52. A person having spent in one year all his income and $\frac{1}{4}$ as much more, found that by saving $\frac{1}{15}$ of his income afterwards, he could in 4 years make good the deficiency, and have \$20 left; what was his income? Ans. \$1200.

53. A gatekeeper is to receive 6 cents for every wagon, 4 cents for every gig, 2 cents for every horseman, and 1 cent for every footman that passed the gate; at the end of the year he found that 3150 gigs had passed, and that 7 gigs passed when 5 wagons did, and 4 horsemen passed when 6 footmen did, and 5 footmen passed when three gigs did; what number of wagons, horsemen, and footmen passed, and how much did the gatekeeper receive?

Ans. $\begin{cases} 3150 \text{ gigs,} \\ 2250 \text{ wagons,} \\ 5250 \text{ footmen,} \\ 3500 \text{ horsemen.} \end{cases}$

Amount of toll \$383.50.

54. A trader having increased his estate annually 10 per cent, for 7 years, finds himself worth \$19487.171; what was his capital at the commencement of the term?

Ans. \$1000.

55. A market-woman having bought a quantity of apples, at the rate of 250 for a dollar, and sold one half of them at 2 for a cent, and the remaining half at 3 for a cent, finds she has gained 14 cents by the transaction; how many apples did she buy? Ans. 840.

56. A man who was but slightly versed in numbers, bequeathed his estate, valued at $6500, to his wife and two children, as follows: to his wife $\frac{1}{2}$, to his eldest son $\frac{1}{3}$, and to the other $\frac{1}{4}$; how much must they severally receive to fulfil the design of the testator?

Ans. Wife $3000, eldest son $2000, youngest son $1500.

57. A man and his wife usually consumed a barrel of flour in 36 days, but when the man was from home it lasted the woman 90 days; how long would the man be alone in consuming it? Ans. 60 days.

NOTE.—The smallest common multiple of 36 and 90 is 180; then, since they both together consume 1 barrel in 36 days, in 180 days they will consume 5 barrels, and in the same time the woman will consume 2 barrels, leaving 3 barrels for the man to consume in 180 days; hence he will consume 1 barrel in 60 days.

58. A, B, and C can complete a piece of work in 15 days; A can do it in 30 days, and B in 40 days, in what time can C perform it? Ans. 120 days.

59. Joseph and Samuel together can do a piece of work in 18 days; Joseph alone can do it in 30 days; in what time can Samuel do it alone? Ans. 45 days.

60. A man meeting a boy driving a flock of geese, said, Good morning, sir, with your hundred geese; I have not a hundred, said the boy; but if I had as many more, and half as many more as I now have, and $2\frac{1}{2}$ geese besides, I should have a hundred; how many had he? Ans. 39 geese.

61. Suppose a man contracts to dig 80 rods of canal, 30 feet wide at the bottom, 40 at the surface, and $4\frac{1}{2}$ feet deep; what would it amount to, at 20 cents per cubic yard?

Ans. $1540. (0.)

62. What will the digging of a cellar 40 feet long, 30 feet wide, and 6 feet 6 inches deep amount to at 22 cents per cubic yard? Ans. 63.55\frac{5}{9}$.

63. A sheepfold was robbed three nights successively; the first night, half the sheep were stolen and half a sheep more; the second night, half the remainder and half a sheep more; and the last night, they took half the remainder and half a sheep more, which reduced the number to 42; how many were there before any were stolen? Ans. 343 sheep.

64. Bought 1000 barrels of flour at $4.25 a barrel, and sold 500 barrels at $5.25, and 500 barrels at $5.12½ per barrel; what was the gain per cent. on the whole transaction?

Ans. $22\frac{1}{17}$ per cent. (4.)

65. If 3000 pounds of beef serve 340 men 15 days, how many pounds will serve 120 men for 25 days?

Ans. $1764\frac{12}{17}$ pounds. (0.)

66. What is the discount of $513.82½, due 21 months hence at 6 per cent. a year? Ans. $48.82½. (9.)

67. If 18¾ per cent. is lost by selling shoes at 75 cents a pair, at what price should they be sold to gain 12½ per cent.?

Ans. 1.03\frac{1}{13}$. (4.)

68. A grocer gains 25 per cent. by selling tea at 87½ cents a pound; how much would he gain per cent. by selling 1 cwt. 1 quarter and 13 pounds for $126?

Ans. $17\frac{1}{7}$ per cent. (6.)

69. If 73 men can do a piece of work in 73 days, how many men will be required to do the same in 15.75 days?

Ans. 292 men. (0.)

70. A man exchanged 760 gallons of molasses, at 37½ cents per gallon, for 66½ cwt. of cheese, at $4 per cwt.; how much will the balance in his favour be? Ans. $19.

71. Sold 342 pounds of beef, at 6 cents per pound, and received his pay in molasses, at 37½ cents per gallon; how many gallons did he receive? Ans. $54\frac{18}{25}$ gals.

72. A gentleman left his son a fortune, $\frac{5}{16}$ of which he spent in 3 months, ¾ of ⅚ of the remainder lasted him 9 months longer, when he had only $2640 left; what sum did the father bequeath him? Ans. $10,240.

73. At what time between 5 and 6 o'clock are the hour and minute hands of watch together?

Ans. $27\frac{3}{11}$ minutes past 5.

$$
\begin{array}{r|l}
 & 12 \\
11 & 5 \\
\hline
 & 60 \\
\hline
 & 5\text{—}27\frac{3}{11} \text{ minutes.}
\end{array}
$$

The minute hand must pass over 12 spaces to gain 11, but less to gain 5.

74. A man dying left a son and a daughter travelling in a foreign country; and making his will, ordered that if the son alone should return, $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother; but if the daughter alone should return, the mother was to have $\frac{2}{3}$, and the daughter the remainder; now it so happened that the son and daughter both returned, by which the widow lost in equity $2400 more than if the daughter only had returned; what would have been her dowry had the son only returned?

Ans. $1200.

75. A Greek epitaph designed for the tomb of Diophantus, is said to have stated that he passed $\frac{1}{6}$ of his life in childhood, $\frac{1}{12}$ in adolescence; that after $\frac{1}{7}$ and 5 years more had been passed in a married state, he had a son who lived to $\frac{1}{2}$ his own age, and whom he survived 4 years; what was then the age of Diophantus? Ans. 84 years.

76. A person being asked the time of day, said that the time past noon was equal to $\frac{7}{8}$ of the time till midnight; what was the time? Ans. 36 min. past 5.

77. A person being asked the time of day, replied, the day is now 16 hours long, and the sun rises at 4 o'clock; now if you add $\frac{1}{2}$ of the hours that have passed since the sun rose to $\frac{3}{4}$ of those which must elapse before he sets, you will have the exact time of day; what was the time?

In order to solve this question we reason thus,—as the whole time that has passed since the sun rose, *added* to the *whole* time that must elapse before he sets is 16 hours, it follows that *half* the time that has passed since the sun rose, *added* to *half* the time that must elapse before he sets must be 8 hours, and therefore $\frac{1}{2}$ the time that has passed since the sun rose, *added* to $\frac{3}{4}$ the time that must elapse before he sets, will be *more* than 8 hours; hence the time passed is evidently greater than the time yet to elapse.

Again, since $\frac{1}{2}$ of the time passed added to $\frac{3}{4}$ of the time yet to elapse, will give the time of day, or the whole number of hours that have passed since the sun rose, it is readily inferred that $\frac{1}{2}$ of the time passed must be equal to $\frac{3}{4}$ of the time yet to elapse, because $\frac{1}{2}$ of the time *passed added* to $\frac{1}{2}$ of the time passed must also give the time of day or number of hours that have elapsed since the sun rose.

Wherefore 2 parts of the time passed are equal to 3 parts of the time yet to elapse, and the time yet to elapse is consequently equal to but $\frac{2}{3}$ of the time passed, it is therefore evident that the whole time passed added to $\frac{2}{3}$ of the time passed, (that is, $\frac{5}{3}$ of the time passed,) must be equal to 16 hours.

$$\begin{array}{r|l} & 16 \\ 5 & 3 \\ \hline & 48 \\ \hline & 9 \text{ hours } 36 \text{ minutes.} \end{array}$$

And of course the time passed is 9 hours 36 minutes.

That is, the time of day was 36 minutes past 1 o'clock.

78. A gentleman has 2 horses, and a saddle worth \$60; now if the saddle be put on the first, it will make his value double that of the second horse; but if it be put on the second, it will make his value triple that of the first; what was the value of each horse?

Ans. { First was worth \$36. Second worth \$48.

79. A and B invested equal sums in trade; A gained a sum equal to $\frac{1}{5}$ of his stock, and B lost \$378; then A's money was double of B's; what sum did each invest?

Ans. \$945.

80. Three towns are so situated that A lies 90 miles south of B, and C 120 miles west of A; what is the distance from A to C? Ans. 150 miles.

81. A lady purchased silk for a dress at \$1.50 per yard, and lining for the same at 80 cents a yard; the whole number of yards of both silk and lining was 16, and the whole cost 20; how many yards were there of each?

Ans. { $10\frac{2}{7}$ yards of silk, $5\frac{5}{7}$ yards of lining.

82. A farmer having driven his cattle to market, received for them all \$576, being paid at the rate \$24 per ox, \$16 per cow, and \$6 per calf; there were as many oxen as cows, and four times as many calves as cows: how many were there of each sort? Ans. 9 oxen, 9 cows, 36 calves.

83. There is a fish, whose head is 16 inches long, his tail is as long as his head and half the length of his body, and his body as long as the head and tail; what is the whole length of the fish? Ans. 128 inches.

84. Three men, A, B, and C, playing at cards, staked

\$324, but they happened to disagree, each seized as many of the dollars as he could; A got a number unknown, B as many as A and 15 over, C got a fifth part of their sums added together; how many dollars did each get?

Ans. A \$127$\frac{1}{2}$, B \$142$\frac{1}{2}$, and C \$54.

85. A, B, and C can perform a piece of work in 4 days, B can do it in 12 days, C can do it in 15 days; in what time would A and B perform the labour?

Ans. In 5$\frac{5}{11}$ days.

86. A merchant bought 9 packages of cloth for \$34,560; each package containing 8 parcels, each parcel 12 pieces, and each piece 20 yards; how much would [illegible] of $\frac{3}{4}$ of $\frac{5}{6}$ of 39 yards cost? Ans. \$[illegible].50. (7.)

87. With 12 gallons of Canary at 76 cents per gallon, I mixed 18 gallons of Madeira wine at 58 cents per gallon, and 12 gallons of cider at 37 cents per gallon; at what rate must I sell a quart of this mixture so as to clear 10 per cent.?

Ans. 15$\frac{5}{7}$ cents.

88. A and B commence trade in company; A advanced at first \$500, and B \$750, at the end of 4 months A adds \$600 to his stock; what sum must B advance at the same time to entitle him at the end of the year to equal profits with A? Ans. \$225.

89. A and B would barter; A has 150 bushels of wheat at \$1,25 per bushel, for which B gives 65 bushels of barley worth 62$\frac{1}{2}$ cents per bushel, and the balance in oats at 37$\frac{1}{2}$ cents per bushel; what quantity of oats must A receive from B? Ans. 391$\frac{2}{3}$ bushels.

90. Bought a cask of oil, containing 77 gallons, for 91 cents per gallon, but by accident 28 gallons leaked out; how must I sell the remainder, per gallon, so as to sustain no loss?

Ans. \$1.43.

91. A merchant receives on commission three kinds of flour; from A he receives 50 barrels, from B, 25 barrels, and from C, 40 barrels. He finds that A's flour is worth 10 per cent. more than B's, and that B's is worth 20 per cent. more than C's. He sells the whole at \$6.80 per barrel; what in justice, should each man receive?

Ans. $\begin{cases} \text{A } \$379.50. \\ \text{B } \$172.50. \\ \text{C } \$230.00. \end{cases}$